AEROPHOTOGRAPHY
AND
AEROSURVEYING

BY

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First Edition
Fifth Impression

McGRAW-HILL BOOK COMPANY, Inc.
NEW YORK AND LONDON
1941

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THE MAPLE PRESS COMPANY, YORK, PA.

To

AGNES STEVENS BAGLEY

PREFACE

There is no longer any need to preach for aerial photography—not in the United States—for so widespread has become its use and so great its value that even the farmer who plants his fields in a remote corner of the country knows its value. A progress map issued by the U.S. Department of Agriculture as of Oct. 1, 1940, indicates that aerial photography completed, in progress, or approved by that department alone extended into every state and covered approximately three-quarters of the entire continental area. Combined with areas covered by other organizations the total area that has been photographed from airplanes is over 2 million square miles. The number of photographs taken is probably not less than 3 million, and a large percentage of these have been taken during the last decade.

The pioneer work of aerophotography in the United States has been done chiefly by the Army. Aerial photographs have been employed not only to improve methods of gathering military information but also for making contoured maps and mosaics. A notable recent activity has been the making of controlled mosaics by quadrangles of areas not covered by standard maps. Although use by the Army has been extensive, it has been exceeded by that of other government organizations for making standard maps, controlling crops, conserving the soil, and making studies and plans pertaining to better utilization of land.

This book is intended for students who desire to make a systematic study of aerophotography and aerosurveying. It is the result of studies carried out through many years and an arrangement of the substance of lectures given at the Institute of Geographical Exploration of Harvard University during the last four years. The student's interest may be centered in the particular field of air work, that of extreme refinement and precision in making moderate-scale or larger scale maps of important areas under special investigation, or the more general use of aerial photographs in making exploratory or reconnaissance maps of large areas. The book is intended to give him a correct start in any of these directions.

The aim has been to hold closely to practices and methods that are fundamentally sound and have been proved in standard surveying and to exclude those which would not stand under mathematical examination. However, many approximate methods are adequate for practical purposes and hence more valuable in some respects than rigorous solutions that cannot be applied without excessive labor or the use of elaborate and expensive equipment. A method that eliminates 85 to 95 per cent of the error and can be used without great expense or labor is in many cases preferable to another that is more accurate but not so adaptable. Mathematical analyses are employed only to the extent necessary for solutions of important problems and to determine the soundness of methods.

Aerial surveys are indispensable in explorations. No present-day expedition is satisfactorily equipped without an aerial camera, an airplane suitable for photography, and a short-wave radio set. The importance of aerophotography and radio communication as well as their feasibility in remote and difficult tropical regions was established by the Hamilton Rice Expedition into the Amazonas in 1924 and 1925. Geographic positions fixed by observations with the prismatic astrolabe along routes gave adequate control stations for the maps of vast regions which could not otherwise have been reliably controlled.

It has been possible to prepare this book only because of the support and interest given to the undertaking by Dr. A. Hamilton Rice, director of the Institute of Geographical Exploration, who furnished the opportunity and facilities. I desire to express my appreciation of the assistance and valuable aid given by Mr. Edward S. Wood, Jr., who has so ably assisted in the courses on which we have labored together. My hope is that the many manufacturers and others who have so liberally contributed information and photographs of their instruments and equipment will find some indication of my appreciation in an understanding treatment of the subjects concerned.

JAMES W. BAGLEY.

CAMBRIDGE, MASS.,
November, 1941.

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AEROPHOTOGRAPHY AND AEROSURVEYING

CHAPTER I

INTRODUCTION

1. Chief difficulties of aerophotography and aerosurveying.—In aerial photography it is possible neither to level the camera precisely nor to select the exact position from which an exposure is to be made. These two handicaps greatly increase the work of aerosurveying. The high speed of the airplane and wind, which nearly always tends to drive the airplane off course, complicate the work of obtaining photographs that have the proper relationship with one another and with the area to be photographed. Well-taken photographs require expert operation of the camera, and well-arranged photographs require thorough teamwork between pilot and photographer. If registered on a stable material, such as glass, photographs that have been well taken with a proper aerial camera have the properties of true perspective and may be employed under the principles that govern in this type of projection. At the scales ordinarily employed in map making, any small part of a map may be considered to be an orthographic projection. The first task in utilizing aerial photographs for making maps is to convert the images from the perspective projection to the orthographic projection. The second task relates to obtaining from the photographs themselves enough accurately located points (objects) to provide for bringing together into proper relationship the details that make up the body of the map. Lastly, there are the difficulties that arise in extracting the mass of detailed information from the photographs and transferring it to the map.

2. Map information furnished by aerial photographs.—Aerial photography furnishes more completely than any other means the information required for making maps. The vertical photograph particularly approaches the ideal for this purpose. It registers in their proper perspective relation all objects and features not obscured or screened by other objects above or near them. Of areas free of trees and structures that cover details, a fair vertical photograph gives practically 100 per cent of all surface information required for a map. It is only the invisible items, such as political boundaries and names and details obscured by woodlands, bluffs, and buildings, that are not shown. Even the low oblique photograph furnishes a high percentage of required surface data.

3. Controlled photography.—(*a*) That the image produced by a precise camera is a nearly perfect perspective is strictly true only when there is no movement of the camera or of the subject. This is speaking of the camera in its usual form, the image being caught on a flat surface without any transparent substance other than air between the lens and the focal plane. First-class lenses are so well corrected for distortion and give such sharp definition that defects in the perspective quality of the image as a whole can be detected only by making careful measurements with the aid of special measuring devices equipped for magnification. In the laboratory the large plate-bearing camera is used extensively in the reproduction of maps and in the copying of designs that must be kept true in scale. In this work the subject has but two dimensions—length and breadth. It is necessary only to have the plane of the subject and the photographic plate perpendicular to the optical axis of the lens to avoid distortion of the image to be registered. Then the scale of the photograph is determined by the distances of the subject and plate from the optical center of the lens, and sharp definition is assured when these distances conform to the fundamental law of optics that

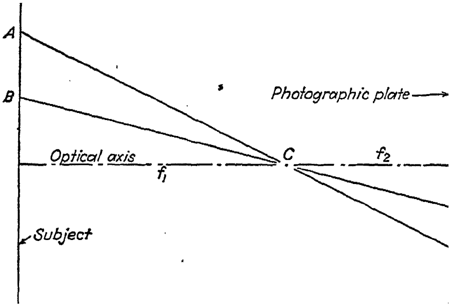


Fig. 1.—Setup of a camera for photography without distortion.

$$1 + 1 \tag{1}$$

F being the equivalent focal length of the lens and f_1 and f_2 being the distances.

In this statement the complex character of the photographic lens is left out of consideration. It is supposed that the lens has an optical center at a point through which rays of light pass in going from every part of the subject to the plate. Figure 1, which is drawn in a plane containing the optical axis of a lens and is at right angles to the plane of the subject and to the photographic plate, shows the setup for photographing a map or drawing at a definite scale and without distortion. In this instance the photograph will be a copy of the subject at a smaller scale. That under this arrangement there will be no distortion is evident because any distance AB on the subject will be reduced to ab on the plate in the ratio f_2/f_1, the triangle ABC being similar to the triangle abC. In photographing a map it is necessary to take into consideration its scale and to have the ratio $f_2:f_1$ proper to give the wanted scale in the photographic copy of it. Suppose the scale of a map to be 1 inch equals 1,000 feet, and suppose that it is desired to have a copy of it at the smaller scale of 1 inch equals 1,200 feet. Then f_2/f_1 must equal 1,000/1,200, or $f_2 = \frac{5}{6}f_1$, and every line on the map 12 inches long would have to be 10 inches long on the copy. If the focal length of the lens of the camera is 30 inches, the fundamental equation (1) requires that

$$\frac{1}{30} = \frac{1}{f_1} + \frac{6}{5f_1}$$

whence $f_1 = 66$ inches and $f_2 = 55$ inches. Then the camera would have to be set with its focal plane 55 inches, and the map 66 inches, from the center of the lens. It is customary to draw a line along the margin of the map ending at short crosslines and to show by another crossline the length that it is to have in the photograph. The photographer then can check the camera setting by measuring the length of the image of the scale line on a ground glass. This is completely controlled photography under advantageous conditions; results obtained under these circumstances leave little to be desired.

Two significant positions for the lens are worth noting. If the distance to the subject is very large (several thousand feet), $1/f_1$ in equation (1) becomes so small that it may be stricken out. Then f_2 equals F practically. This is the position of the lens with respect to the sensitive plate or film for taking vertical and distant oblique aerial photographs. It is called *setting the lens for infinity*, or *universal focus*. The second position is that for making a photograph the same size as its negative or for copying a subject (map or photograph) at its full scale. In this setup f_1 must equal f_2. Substituting either f_1 for f_2 or f_2 for f_1 in equation (1), both are found to equal $2F$.

(*b*) In surveying with the camera on the ground, leveling and orientation can be accomplished with considerable accuracy; the only difficulties to be overcome arise from weather and obstructions on the terrain. But even the most favorable stations offer limited views of the surrounding areas, so that, except of limited areas and in special cases, there is never the assurance that every important feature will be photographed completely enough to make a thorough map. Mapping of mountains or bare regions and of inaccessible areas that can be seen but not penetrated has been done by ground surveys with the camera set at suitable stations, many advantages in field work and marked improvement in results being realized therefrom because of knowledge of perspectives and the availability of methods and instruments for proper utilization of the information furnished by photographs. The principles involved are simple, but the labor is great.

4. Conditions for aerial photography.—In aerial photography, conditions are quite different from those encountered in photography on the ground or in the laboratory. Movement of the airplane along its course, vibrations, and turning motions must all be overcome sufficiently to keep the photographs near enough to true perspectives to permit them to be used for making reliable measurements. It is necessary that the camera be cushioned and that the exposure time be of such short duration that movements of the image cast by the lens will be negligible or at least so small that they will not seriously impair the perspective properties of the photographs. It is never possible to have the camera precisely level or held at any desired angle. Nevertheless, in the face of these handicaps, because the aerial photograph furnishes a picture of the terrain with its innumerable features so nearly complete and true, surveying and mapping have been advanced and improved far beyond the great cost and labor spent in developing methods and making instruments for its better and more satisfactory utilization.

Photographs taken with a camera pointed downward toward the earth and approximately leveled, which can be obtained in a fraction of the time required to make the most elementary type of map by ground survey, in many instances furnish expeditiously and at moderate cost all information required for special purposes. If the terrain involved is not too mountainous, such photographs may be used as maps are used for making preliminary studies, investigations, and estimates.

5. Quality of a map.—Quality in a map is comprised of four components: accuracy, completeness, clarity, and appearance. The last two are the responsibility of those engaged in putting the map into its final condition for reproduction. At present only the first two items are to be considered. The map made up of symbols is the product of the mapmaker whose task consists of placing each line as near as practicable to its proper place relative to the other lines of the ensemble. A measure of accuracy in a map is the precision attained in plotting its details. A few points of the map, called *control points*, are fixed in position by accurate ground survey, and numerous other points are located with special care. The bulk of the lines, or symbols, are brought together by placement with respect to this net of points. In the nature of the process exactitude is seldom attained in all details. In standard maps it may be assumed generally and confidently that all control points resulting from triangulation or transit traverses are so accurately fixed in relative position one with another that, when they are properly plotted, no error in their positions can be detected by ordinary means of measurement. With respect to the details of the map a large percentage will be accurately placed, but some will be misplaced. High quality exists in the map if the percentage of misplaced details is small and the amount of the displacement is in no case excessive. The great advantage to mapping brought by aerial photography lies in the reduction all over the map of the amount of displacement of details so that a more nearly uniform standard of quality is attained in maps of all scales.

A measuring stick of draftsmanship is given by Table 1, which also indicates the care necessary to be taken in the survey processes. The table is based on the assumption that lines making up a map can and usually will be drawn with a width of 1/200 inch. Some lines of maps necessarily have to be wider than this to form certain symbols as standardized, but at present the concern is only with respect to plotting or drawing the pencil lines over which the symbols are traced in ink.

TABLE 1.—ACCURACY REQUIRED IN PLOTTING MAPS

Scale of Map	Number of Feet Covered by a Line 1/200 Inch Wide
1:4,800	2
1:12,000	5
1:20,000	8.3
1:24,000	10
1:40,000	16.7
1:48,000	20
1:62,500	26
1:63,360	26.4
1:125,000	52
1:250.000	104

This table may be considered to be an indicator of perfection in the placement of details of a map. If, in making a map at a particular scale, no feature is misplaced more than the number of feet given opposite that scale, it will be impossible for the user to detect any error or to utilize greater accuracy; and the map will serve its purpose thoroughly in so far as accuracy is concerned.

6. Perspective quality of the photograph.—The image produced by a first-class lens is true enough to meet the requirements of mapping at all scales; but unless this image is registered on glass or on a film base that retains its size and shape, the quality is impaired for actual use. Glass plates are objectionable for use in aerial cameras because of their bulk, weight, and fragility. Moreover, unless the negative or a plate positive is used throughout the mapping process, the advantage of the stable medium is lost. Fortunately, there is now available commercial photographic film in rolls which in stability approximates fairly closely the glass plate. Diapositives on glass made from negatives of this material provide perspectives which, although not perfect, do give reliable results in the determination of point positions and of elevations. Film usually shrinks a little by going through the processes of development, fixing, washing, and drying. Measurements made on a number of negatives about 3½ years after development showed an average shortening in length of 0.1553 per cent and an average shortening in width of 0.1322 per cent, the difference between the two percentages being 0.0231 per cent. Assuming no further differential change to arise from making diapositives, the maximum error in locating points on a map and in determining elevations from using such diapositives would be of the order of magnitude of 1 part in 4,300.

For mapping at moderate scales this quality is amply adequate. However, much mapping is done directly from prints made on photographic paper, which is far from being so satisfactory a medium as film or glass. Prints on double-weight (twice as thick as ordinary) paper are changed in size less than those made on ordinary photographic paper. Measurements made on double-weight prints from the same negatives mentioned above showed a differential change between length and width ranging from 1 part in 400 to 1 part in 22,000, an average being about 1 part in 800. A distortion of the photographic image as much as 1 part in 800 has no serious effect on the location of points, except when large-scale plottings are made, but does have an appreciable effect on the determination of elevations. For photographs commonly employed in standard mapping, the maximum error from this cause liable to occur in an elevation determination is on the order of 15 feet.

A report on dimensional changes in aerial photographic films and papers by members of the National Bureau of Standards and printed in *Photogrammetric Engineering*, July-August-September, 1938, indicates that the humidity of the air surrounding film negatives (both nitrate and safety aero) has appreciable effect on their dimensions. Relative humidity of 65° is recommended for rooms under humidity control. This report cites 20 samples of nitrate film measured at 65° relative humidity in which the differential shrinkage ranged from 0 to 0.03 per cent and of five samples of safety aero film in which the differential shrinkage was 0.01 per cent or less. These tests were made under conditions of controlled humidity. Such low shrinkage values will not be found under usual conditions of great range of humidity.

Excellent results were also obtained from waterproof photographic papers measured under controlled humidity. Differences were found between the machine direction and the cross direction in both films and paper. It is recommended that the machine direction of both cut film and paper be indicated by the manufacturer and that printing be done with the direction of maximum change in the film at right angles to the direction of maximum change in the paper.

In the July-August-September, 1937, issue of the same journal, results of measurements of a laminated bromide paper made of two sheets of paper with a thin sheet of aluminum between and of a waterproof paper indicated that the laminated sheets gave practically as good results as the safety aero film and that the waterproof paper gave as good results as the nitrate film; however, the change in one direction was less than in the other.

Measurements made at the Institute of Geographical Exploration on prints made from Eastman Aero Enlarging Mapping paper under normal interior conditions with steam heat during dry winter weather showed a differential change varying from 0 to 0.09 per cent as between camera dimensions and photograph dimensions. This was without regard to run of either film or paper in manufacture. The negatives were made with nitrate film, and about four years had elapsed since development. A remarkably high percentage of these photographs showed no appreciable differential, so that it would not be prohibitive to exclude those most affected and to reprint to obtain photographs consistently in keeping with requirements for standard surveys. The paper used had been in stock about one year.

Tests made of prints on the same type of paper received only a few days prior to use and printed from the same negatives gave a differential change varying from 0 to 0.1 per cent. For this test several prints were made from each negative; for half the prints the paper as taken from the package was turned at right angles to the other half. Normal indoor conditions without heat during mild spring humid weather existed. About 70 per cent of the prints were under 0.01 per cent of differential change. These tests indicate that aging of the paper gave little if any improvement and that the difference between interior conditions from winter to spring had little effect.

7. Photogrammetry.—This term, which literally means photo-measurement, is synonymous with the older term *iconometry* (image measurement). Photogrammetry may be defined further as that science which consists in utilizing photography for making reliable measurements in all fields of application. Some of these are surgery, astronomy, engineering, architecture, agriculture, archeology, geography, criminology, aerotechnology, military operations, surveying, and mapping. At the present time the field of mapping overshadows all others in importance and volume.

8. Scale of a vertical aerial photograph.—Disregarding distortions caused by tilt of the camera and displacements brought into the image by roughness of terrain, the scale of a vertical photograph in connection with its use as a map may be expressed as its focal length divided by the height of the camera above the terrain at exposure. Although not strictly accurate, this expression is reliable enough for general use in making estimates and in approximate mapping. This manner of expressing scale is commonly referred to as the *representative fraction.* Designating the focal length of the photograph by f and the height of the camera aboveground by H, the scale becomes f/H, both terms of the fraction being in the same unit. A scale expressed in this manner is convertible to feet to the inch, by dividing both numerator and denominator by the numerator to obtain a fraction with unity as the numerator, then dividing the denominator by 12. Thus if f is 0.75 feet and H is 12,000 feet, $f/H = 1/16{,}000$, or 1 inch $= 16{,}000/12 = 1{,}333$ feet.

9. Usage of important words and terms.—The following important words and terms are explained according to their usage in this book.

10. The focal length of a fixed-focus camera is the shortest distance from the nodal point of emergence (optical center) of the lens to the focal plane. It is therefore equal to the length of the perpendicular drawn to the focal plane from the optical center of the lens. It is usually designated by the letter f.

11. The focal length (principal distance) **of a photograph** corresponds quite closely to the focal length of the camera with which it was taken but differs from the latter according to the amount of shrinkage or expansion of the negative and the print. The focal length of a photograph can be readily determined by simple proportion if the focal length and dimensions of the focal-plane frame of the camera are known. Inasmuch as in aerial photography the lens is usually fixed in position for focus on distant objects, it is assumed that there is never any appreciable variation in the focal length of the camera.

12. The optical center of a photograph is the point where the optical axis of the lens strikes it. In accurately constructed cameras this point is coincident with the foot of the perpendicular dropped from the optical center of the lens to the focal plane. In this case the optical center is identical with the principal point of the photograph, and the two terms are used synonymously. This point is commonly indicated by marginal marks registered on the photograph.

13. The perspective center of a photograph is on the perpendicular to the photograph erected at its principal point and is that point which is at a distance from the photograph equal to its focal length. Wherever the perspective properties of a photograph are to be dealt with, it is necessary to determine the position of the perspective center.

14. The vertical point of an aerial photograph is that point where a plumb line held at the optical center of the lens at exposure of the camera would have pierced the negative from which the photograph was printed. It is also called the *plumb point.*

15. The center of distortion.—The images in aerial photographs are subject to systematic distortions in scale because the camera is practically always tilted from a level position. These distortional effects radiate from a common point which has been designated the *center of distortion.* The point lies approximately midway between the optical center of the photograph and the vertical point. Precisely, it is that point on the line connecting the optical center and the vertical point which is at a distance from the optical center equal to the focal length of the photograph multiplied by the tangent of one-half the angle of tilt of the camera at exposure. It is also called the *isocenter.*

16. Nodal points of a lens.—In considering the action of lenses in forming photographic images, it is convenient to employ the conception of nodal points. In most of the present-day photographic lenses the nodal point of incidence (admission) lies a short distance in front of the nodal point of emergence. It is conceivable that lenses departing from the usual in design may have the positions of their nodal points reversed from this arrangement. Focal distance is measured from the emergence node. The exact positions of the nodal points of any lens can be determined only by a proper optical test. An interesting and useful characteristic of the photographic lens is that slight rotation of the lens about an axis, which is perpendicular to its optical axis and passes through a nodal point, causes no movement of the image formed at the focal plane. Advantage is taken of this fact in the construction of panoramic cameras of wide angular field, the lens being made to revolve on a shaft whose axis passes through one of the nodal points.

Nodal points are also called *Gauss* points. It is customary to speak of the front and rear nodal points, the former meaning that of admission, the latter that of emergence. A *nodal plane* is perpendicular to the optical axis of the lens and passes through a nodal point.

17. The focal plane is the plane of the camera at right angles to the optical axis in which the image of the object photographed is brought into sharp focus. It is the plane in which the sensitive surface of film or plate rests.

18. The aperture of a photographic lens is the approximately circular opening that allows a controlled amount of light to pass through the lens. Lenses are ordinarily equipped with an iris diaphragm which provides for regulating the diameter of the aperture. The system customarily used in the United States to designate apertures is based on the quotient of the focal length of the lens divided by the diameter of the aperture, and a series of quotients is arranged in a scale such that each number indicates an aperture affording one-half the amount of light of that having the next smaller number. The smallest number usually corresponds to the lens wide open. This is known as the *f system.* The area of the aperture of the smaller number is therefore twice that of the next larger number in the scale. This regularity is, however, interrupted at the starting end of the scale in those lenses having a slightly larger or slightly smaller effective diameter than *f*/4. For example, if the scale begins with *f*/4.5, the lens will admit at 5.6 a little more than half as much light as will be admitted at 4.5, but from 5.6 on the reduction is by regular steps.

Another system, called the *uniform system,* starts at 1 and continues by doubling to make up the complete scale. The scales in these two systems are given below in an arrangement that shows the correspondence of their respective numbers:

f System	Uniform System
4	1
5.6	2
8	4
11.3	8
16	16
22.6	32
32	64
45	128
64	256

In the uniform system unity was made to correspond to f/4 because at the time f/4 lenses were generally the speediest made. To form a scale in it for a lens having an aperture larger than f/4 would require starting with a fraction less than unity.

19. The speed of a lens, or the relative amount of light that it transmits, is indicated by the smallest number given in the range of apertures. Thus in the f system, 4 indicates a fairly speedy lens, *i.e.*, one capable of admitting a relatively large amount of light to act on sensitized film or plate.

20. Aberrations of lenses tend to distort or blur the images. *Distortion* is of special interest; for if not kept within narrow limits, it will seriously impair the quality of photographs taken for measurement purposes. Elimination of blurring to afford sharpness of definition is likewise of prime importance. Because of difficulties in overcoming undesirable aberrations in lenses, large in proportion to their focal length, most aerial cameras at this time are fitted with lenses of speed f/4.5 or less.

21. The vertical photograph is one taken with an aerial camera leveled as well as practicable to keep the optical axis vertical.

22. The oblique photograph is one taken with the optical axis of the camera inclined considerably from the vertical position.

23. The composite photograph is a vertical photograph taken with a multiple-lens camera. Ordinarily it consists of three or more parts, one of which is a common vertical, the other parts being verticals made from obliques by transformation in printing. Negatives made with the five-lens camera consist of one vertical and four obliques. Photographs made from the oblique negatives are transformed during projection at printing to bring them into the plane of the common vertical, upon which they may be joined together in accurate relationship and the composite employed as an ordinary vertical.

24. Transformation printing is the process of printing photographs by projection into a plane not parallel to the plane of the negative. Proper transformation printing provides for maintaining complete perspective qualities of the photograph.

25. A mosaic is a map of terrain made by joining two or more vertical photographs together to give a related picture of the entire area.

26. A line map is a designation of drawn maps in contradistinction to mosaics. It is merely a new designation of the common type of map.

27. Resection is the process of determining a position by orienting along a foresight, then observing one or more lateral objects whose positions are known. It is commonly employed in surveying with the planetable to find the position at which the planetable is set either on the line of a foresight or at a previously sighted object. It is also employed in triangulating with aerial photographs to orient them properly and to locate the positions of their centers.

28. Crab is a term used to express the skewing of an airplane with respect to a straight course. Crabbing is necessary to prevent a side wind from driving the airplane off its course. The word is also employed in connection with orientation of aerial photographs on a map or plotting sheet, when the outlines of a photograph fail to square with the flight course or with the outlines of adjacent photographs.

29. A stereogram, or *stereo pair of photographs,* consists of two photographs of the same subject taken from different positions for the purpose of examination with a stereoscope. The photographs may be mounted together on a card or placed side by side on a table or stand, as may be most convenient for adjustment to the stereoscope.

30. The aerial photograph at exposure.—In order to utilize the aerial photograph to the limit of its possibilities it is not only necessary to know the meaning of the terms employed about it and to recognize the objects it pictures, but the photograph must be envisaged in relation to certain reference planes, lines, and points. Figure 2 represents a photograph set up in space at the position of the camera when the exposure was made to get it. S is the perspective center of the photograph; it also represents the position in space from which the exposure was made. The angle i between the optical axis and the plumb line is the angle of tilt of the photograph. The dihedral angle between the plane of the photograph and a horizontal plane is also equal to i. The axis about which the photograph is tilted passes through the perspective center S; this axis is always horizontal, and the line of intersection of the photograph and any horizontal plane is parallel to it. C, at the foot of the optical axis, is the center (principal point) of the photograph, and V is the vertical point. The principal plane of the photograph is defined by the optical axis and the plumb line; therefore, in every case it is a vertical plane which is perpendicular to the photograph and to the axis of tilt. It cuts from the photograph the principal line VB which passes through both the center C and the center of distortion M. Inasmuch as practically all aerial photographs are tilted more or less, it is necessary to deal with the principal plane, a horizontal plane, the angle and axis of tilt, the principal line,

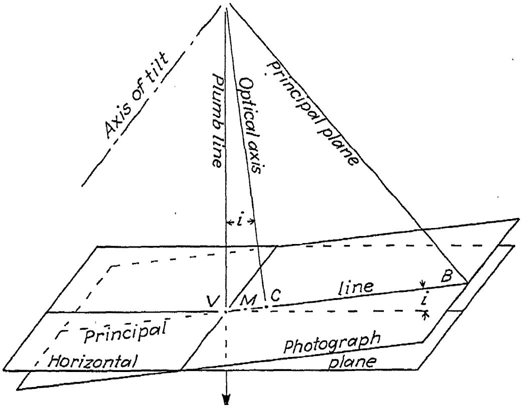


FIG. 2.—An aerial photograph set up as at exposure.

and the four points S, V, M, and C in order to make complete and satisfactory use of them in the process of mapping.

Important significant relationships that are employed in dealing with an aerial photograph as a perspective, therefore, are:

(*a*) The principal plane is that vertical plane which includes the plumb line and the optical axis.

(*b*) The principal line is the intersection of the principal plane and the photograph; it therefore includes the center (principal point), the vertical point, and the center of distortion.

(*c*) The intersection of the photograph and a horizontal plane is perpendicular to the principal plane; the axis of tilt is parallel to this line and therefore is always horizontal and perpendicular to the optical axis and the plumb line at the perspective center.

It should be noted that a true vertical has no principal plane, no principal line, and no axis of tilt. In it C, M, and V are one point, *viz.*, the center of the photograph.

CHAPTER II

AERIAL CAMERAS AND ACCESSORY INSTRUMENTS

31. Types of aerial camera.—There are two general types of aerial camera: single lens and multiple lens. Aerial cameras may also be classified as between-the-lens, focal plane, or louver, depending on the type of shutter with which they are equipped; or as film cameras or plate cameras, depending on whether they are provided with magazines to take roll film or glass plates. Both single-lens and multiple-lens cameras are customarily made up of a base and a detachable magazine or magazines. In one form the single-lens camera consists of a body, a cone, and a magazine. This arrangement provides for a series of cameras of various focal lengths, the body and magazine being used with two or more cones fitted with lenses of different sizes. Multiple-lens cameras are almost wholly employed for making exposures that result in composites or related views that take the place of composites; they are seldom used to obtain obliques, in the manner that single-lens cameras are used, but may be so employed by proper mounting in airplanes that afford the required unobstructed view. Practically all the cameras of both types constructed in the United States are made for loading with roll film, and the majority of them have magazine capacities that range from 100 to 300 exposures. A single exception to the general rule is the Brock and Weymouth camera, which employs glass plates instead of film. Nearly all the single-lens cameras have readily removable and readily replaceable magazines, but the multiple-lens cameras are not arranged for magazine exchange. Single-lens cameras are equipped either for manual operation or for automatic operation. Those equipped for automatic operation may also be operated manually. Some of them may be employed to obtain either verticals or obliques by changing attachments.

32. Requirements for aerial photography.—The airplane is the only aerial vehicle that is both suitable and readily available for aerial photography. It must provide room for the photographer to operate and give the necessary unobstructed view for the camera, have the capacity to stay aloft long enough to complete a task, and be capable in speed and climbing ability to reach the necessary vantage positions.

33. Oblique photography requires only moderate performance of the airplane, but it does call for the exercise of judgment in selecting the direction of aim of the camera to obtain the maximum advantage of lighting of the subject. With a camera rigged properly for taking obliques and an airplane available that affords the necessary unobstructed view, it is necessary only to fly to a position that gives a favorable view of the subject and make the exposure. However, this requires skillful maneuvering by the pilot to permit a suitable shot to be obtained. He should throttle down to the minimum speed practicable with full maintenance of control and fly steadily without bank or turn at the critical time of exposure. No accessories are indispensable, although it makes the operation easier to have an oblique mount to steady the camera. Usually the camera is pulled back against the chest to cushion it against vibration when making exposures. Best results are obtained by making the exposure when the airplane is moving toward or away from the subject; for speed of the ship is then less liable to cause blurring of the image than when taking a beam shot. This is especially true when the airplane is fairly close to the subject. For use as illustrations obliques may unavoidably have to be taken through the glass of a window, but photographs so taken will not be satisfactory for surveying uses. Whenever aerial photographs are to be employed for making measurements, no glass or other transparent material should be interposed between the lens and the subject except a color filter properly designed and fitted. Because of the few requirements attendant upon getting oblique photographs it is readily possible at most airports to hire an airplane suitable for this work. Many airplanes have windows that can be opened for oblique exposures, and in some cases a special door may be installed to afford the required opening. In all aerial photography it is necessary to use shutter speeds that are fast enough to arrest the ever-moving image.

34. Vertical photography requires the camera to be set up in a special mount which is provided with leveling mechanism and shock absorbers. An opening in the floor of the airplane large enough to give a clear view below is necessary. Multiple-lens cameras, having great angular field, are usually employed in airplanes equipped with retractable landing gear, which is drawn in during flight to avoid obstruction of the view by the wheels; or in flying boats, which afford the desired scope without interference. Numerous accessory instruments are also required. Of these the most important are the stop watch, level bubbles, viewfinder, altimeter, and flight map. Other useful accessories are listed in Art. 60. In photography at high levels oxygen and equipment for breathing it are essential. For automatic operation an intervalometer and a battery or windmotor are required. Usually special arrangements must be made at airports to prepare for taking verticals, and airplanes having special capacities often are required. Moreover, verticals more often than not are taken of comparatively large areas for mapping purposes. This task can be well done only through proper training and experience. Because of the highly complicated character of the air work, vertical photographs for mapping purposes are usually taken by concerns that have the facilities and trained personnel to carry out such undertakings under contract and according to specifications. The Army and Navy have pioneered in the development of aerial photography in the United States. These organizations maintain operating units that are trained both in the taking of aerial photographs and in their utilization for making maps and for other purposes.

35. Manual and automatic operation.—Some cameras are equipped for either manual or automatic operation. There is scarcely enough advantage in automatic operation when taking obliques to justify the cost of the required accessories. For taking verticals, however, it has two notable advantages. First in importance is that it provides for remote control in starting and stopping the camera, which is required in military operations when space and weight-carrying limitations of the airplane do not admit of having a photographer. In the second place it regulates the intervals between exposures more exactly than can be done manually. This is of value in certain mapping methods where it is desirable to determine quickly and reliably the distance traveled between exposures. A third advantage, which is less important than the first two, is that its employment leaves the photographer free to devote more time to keeping the camera leveled. In automatic operation it is necessary only to set the intervalometer to give the desired exposure interval, switch on the power to commence photography, then turn it off at the end of the run. If a photographer is in attendance, he adjusts the camera for crab, keeps it as well leveled as practicable, and constantly checks the action of all parts to detect failure to function.

In the United States most of the vertical photographs taken up to the present time have been obtained with manually operated semiautomatic cameras. There are two reasons for this. In the first place, motive power and the intervalometer are not required, so there is less equipment to install and fewer pieces to keep in running order. In the second place, contract specifications have not held aerial photography to close tolerances with respect to overlap or to distance traveled between exposures. Aerial cameras employed for most of this work are equipped with interior regulating mechanisms which stop the winding of film at the proper place, apply the film-flattening pressure or suction, and set the shutter for the next exposure. Once started on a run to take photographs, it is readily practicable for the photographer to adjust the camera from time to time, make an exposure near enough to the exact instant to give the specified overlap, and wind fresh film into place before the next exposure must be made. In practically all photography for aerial mapping it has been necessary to employ a photographer to perform essential preliminary work involving computations of altimeter reading corrections, determining the effects of wind, making compensations for crab, and finding the interval of time between exposures. As his services are necessary in these respects, he is available to operate the camera, so there has been no convincing argument for automatic operation. As tolerances are narrowed in contract specifications there will be an increasing advantage in the wholly automatic camera over the one operated by hand.

36. Single-lens aerial mapping cameras.—The cameras described in Arts. 37 to 42 inclusive are intended particularly for taking aerial photographs to be used in photogrammetric instruments and for making maps of the highest quality. They are designed to eliminate as many as practicable of the defects that disturb the perspective properties of photographs and to reduce to a minimum unavoidable minor causes of errors. They are examples of the application of technical skill necessary to the production of cameras that take a place in modern surveying along with transits, theodolites, telescopic alidades, and other first-class instruments used in surveying on the ground. They also have been designed to take advantage of excellent wide-angle lenses now available.

37. The Fairchild photogrammetric aerial camera.—This camera, shown in Fig. 3, is for taking verticals only. It is made by the Fairchild Aviation Corporation at Jamaica, N.Y. The magazine and base are included in a single compartment. Its most unusual feature is its exceptionally fast between-the-lens shutter, which has a range of speeds from $\frac{1}{50}$ to $\frac{1}{300}$ second, intermediate speeds being $\frac{1}{100}$ and $\frac{1}{200}$ second. The shutter is arranged for ready removal to permit inspection and cleaning.

The camera is fitted with either a $5\frac{1}{8}$-inch lens giving a total angular field of 97°, a 6-inch lens giving a field of 93°, or an $8\frac{1}{4}$-inch lens giving a field of 76°. Diaphragm stops range from full opening of the lens to f/22. It holds a roll of film $9\frac{1}{2}$ inches wide of any length up to 450 feet and has a capacity for about 600 exposures. . Negatives are 9 by 9 inches. Constructed with great care, its parts are finished for close fitting; the adjustment of the optical axis into perpendicularity to the focal plane and the planeness of the focal plane are given especial attention. A separate focal-plane plate with cross marks is fitted and doweled in place after exact setting with relation to the optical axis. This provides for accurate and facile setting of negatives or diapositives made from them in photogrammetric plotting machines. The camera is arranged for either semiautomatic or wholly automatic operation. It is provided with a special mount supported on three feet with shock absorbers.

Leveling is accomplished by turning the handwheels on two flexible shafts extending upward from the feet of the mount and with two cross-level vials placed on top. A stop watch, an exposure counter, a film punch, a shutter-speed-setting knob, and a diaphragm-control knob are also placed on top of the camera. The film is held flat by vacuum which is applied automatically immediately before tripping the shutter and released just before winding of the film. The entire back panel is removable for loading films. The camera may be turned in azimuth on the mount.

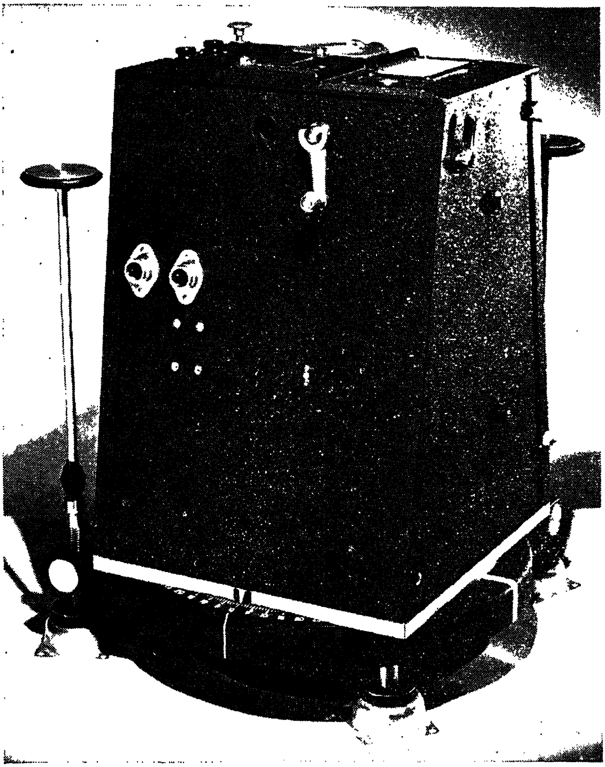


FIG. 3.—The Fairchild photogrammetric aerial camera. (*Courtesy of Fairchild Aviation Corporation.*)

The weight of this camera, equipped for automatic operation and including intervalometer, cables, signal lamps, and Venturi tube, is about 75 pounds; for semiautomatic operation, 61 pounds. Its over-all dimensions are 26½ inches high by about 13¾ inches square. The mount is 19 inches in diameter, the mounting feet protruding a little outside its perimeter.

38. The Mark Hurd photogrammetric aerial camera.—This camera, which is equipped for both vertical and oblique photography, consists of a conical base drawn out to rectangular form at the top and a magazine.

It is a product of the Mark Hurd Instrument Company of Minneapolis, Minn., and has been developed in recent years. Figure 4 shows it installed in its special mount and ready for taking vertical photographs. The camera is unique in the United States in that air under pressure is employed to flatten the film against a focal-plane backing plate. A socket is provided for attaching a hose (not shown in the figure) through which air is carried into the cone. Air under sufficient pressure may be caught by a funnel in the slip stream or provided by a small air compressor. The camera is operated manually, but it is equipped with automatic film-measuring, shutter-setting, air-pressure-regulating, and focal-plane locking mechanism. The cone is made in two sizes: one is equipped with an 8¼-inch lens; the other, with a 10-inch lens. Each of these may be used interchangeably with the magazine. The weights of the camera are as follows: with the smaller cone, 35 pounds; with the larger one, 36 pounds.

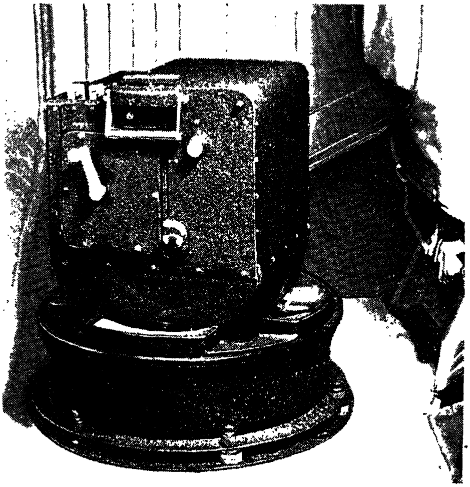


FIG. 4.—The Mark Hurd photogrammetric aerial camera. (*Courtesy of Mark Hurd Air Mapping Corporation.*)

The cone is equipped with the lens in a special between-the-lens shutter; a focusing device to compensate for changes in the focal distance that are caused by extreme ranges in temperature; shutter-tripping mechanism; and the focal plane at its top, which carries registration marks to furnish the principal point of every photograph.

The magazine carries inside seats for the two film spools, the focal-plane pressure plate, and a gear-box unit which includes mechanism for operating the shutter and the pressure plate and for measuring the film to be wound for an exposure. This gear box is attached to a plate that carries the film-winding crank and may be removed from the magazine by loosening a few screws. On top of the magazine are placed the instrument panel with two cross-level bubbles, a stop watch, and two automatic exposure counters; the shutter trigger; and projecting from the top a handle on each of two opposite sides. The shutter trigger is placed adjacent to the handle on the right side, where it may be readily tripped without disturbing the position of the camera. The stop watch is reset at zero at each tripping of the shutter, and the counters record another exposure. Each counter has a knob for setting it at zero. One counter is used to record the total of exposures of any job or mission; the other, for recording the number on any run. The shutter may be set and operated at speeds ranging from 1/65 to 1/150 second. The diaphragm may be set at any desired aperture by means of a knob handily placed on the side of the cone. The lens cells are fitted to the shutter in a manner that permits them to be removed readily for cleaning and replaced without danger of disturbing either the lens alignment or its focus. Lenses working at f/6.8 are used on these cameras.

To equip the camera for oblique photography a rear sight consisting of a bead is mounted on a face of the magazine, and a front sight consisting of a negative lens is attached to the cone. When the bead is aligned with the eye and the center of the negative lens, the optical axis of the camera is pointed toward the object seen.

Another distinctive feature of this camera lies in the fact that it provides for using protective film containers so that it may be unloaded and reloaded with film in daylight and during flights. The containers themselves must be loaded in the darkroom, but once inside them even supersensitive film is said to be safely guarded against fogging. Loading of film may be done with or without the containers. Without using the containers any of the standard rolls of film on spools of the proper length (about 9½ inches) up to 180 feet in length may be used. Employment of the containers requires that the 180-foot roll of film be used. This gives about 225 exposures.

The mount for this camera is entirely different from the mounts of other cameras made in the United States. As Fig. 4 shows, it not only supports the camera and permits it to be rotated round the optical axis, to overcome crab of the airplane, and tilted in all directions for leveling, but also furnishes the view-finder elements necessary to determine the amount of required rotation and the interval between exposures. There are four negative lenses, one below and aligned with each upright face of the camera, as mounted. The camera occupies a position in the mount such that the center of each view-finder lens, as marked by the intersection of two etched lines and a bead near the top of each upright face, furnishes sights parallel to the optical axis of the camera, thereby providing for aiming the camera in whatever direction it may be oriented or tilted. In this mount the camera may be tilted as much as 25° in every direction. By so tilting the camera it is possible to find out if an object some distance ahead is centered under the line of flight and thereby to determine if the ship is set satisfactorily on the line along which it is intended to take the photographs.

The mount consists of two parts: an outer ring which is attached to the floor of the cockpit on rubber shock absorbers and an inner ring which carries the camera and may be manipulated on the outer ring through bearing surfaces of the ball-and-socket type. Ball bearings permit the inner ring to be rotated easily all the way round the circle. Another and special vertical mount of this type affords tilting of the camera through 60° so that low obliques may be taken as well as verticals.

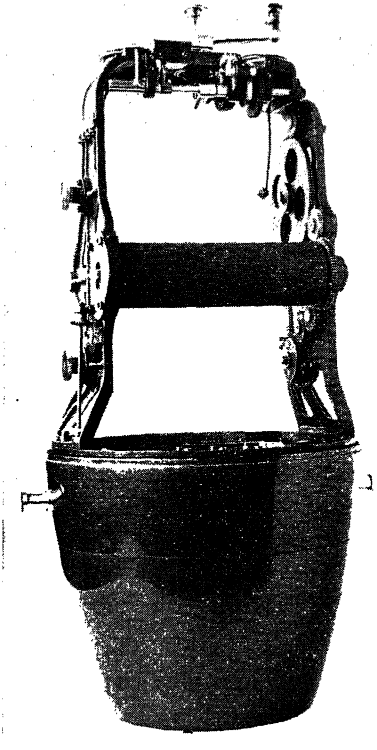


FIG. 5.—The Abrams photogrammetric aerial camera with magazine cover removed. (*Courtesy of Abrams Aerial Survey Corporation.*)

39. The Abrams photogrammetric aerial camera.—In design this camera is distinctive; in appearance it differs from others, presenting a smooth and rounded exterior. It is also exceptional because of its great film capacity—enough for 650 exposures. Figure 5 shows the camera with film-protecting cover removed to display the gear train and other operating parts. Figure 6 shows it ready for taking verticals as set in a special pneumatic mount. The camera consists of a cast-aluminum framework which supports, below, a separate skeleton camera-proper base made of invar steel and, above, the operating mechanism and film. Light-spun duralumin casings complete the assembly. The invar-steel base carries the lens and ends at its top in the focal-plane frame, which is equipped with marginal marks to indicate the principal point of photographs. It is mounted in the framework on rubber shock absorbers. The design and construction assure that neither the focal length of the camera nor the size of the focal-plane frame will be changed by low temperatures or changes in the other parts.

The operating parts consist of film-winding gears and crank, a stop watch, an exposure counter, a level bubble, a shutter trigger, and a film-metering mechanism. A film gauge indicates the amount of unexposed film on the supply roll, its dial giving the number of exposures remaining for use. It also has a punch to mark the film for clipping. Revolving parts are mounted on ball bearings. Gears are constructed of steel and fiber, so that meshing is by one material to the other. The camera may be equipped with lenses ranging in focal length from 5 to 10 inches, and special adapters permit the mounting of lenses of different sizes. Negatives are 9 by 9½ inches. Shutters are of the self-setting between-the-lens type. They are provided with an iris diaphragm having a stop scale and with a speed-setting arm. The film is flattened in the focal plane by either blown air or suction or both, an automatic valve regulating the air pressure or suction. Rolls of film up to 500 feet long may be used.

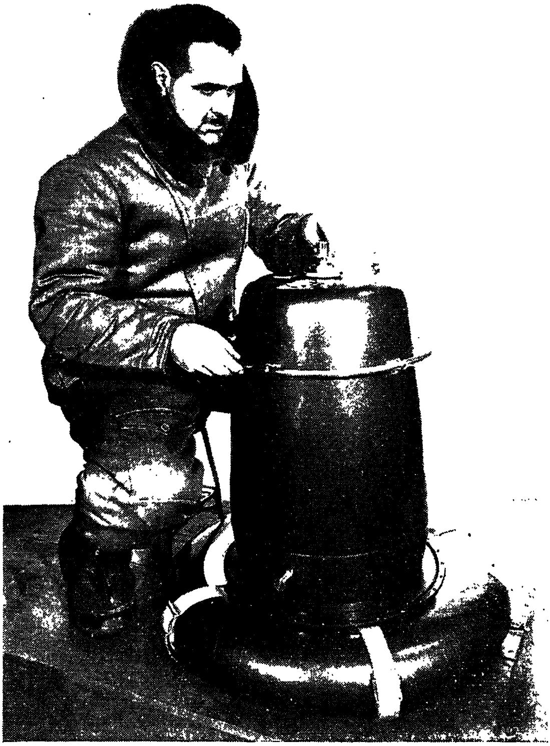


FIG. 6.—The Abrams photogrammetric aerial camera mounted for use. (*Courtesy of Abrams Aerial Survey Corporation.*)

Encircling the upper casing is a ring that serves as a handwheel for turning the camera for orientation and for leveling it. Short cylindrical stubs project from the side of the camera at or near its center of gravity to serve for mounting in the gimbal type of mount. A door is provided near the middle of the casing to permit inspection of the lower parts. Empty, the camera weighs 48 pounds; loaded with 500 feet of film, it weighs 71 pounds. The height is 31 inches; diameter, 16 inches.

The mount shown in Fig. 6 permits the camera to be turned for orientation and adjusted for leveling, as does the gimbal type of mount. When set in proper orientation the camera is held to level position with one hand and the shutter tripped with the other.

40. The Park photogrammetric aerial camera.—This camera consists of two principal parts, a cone and a magazine, both of which are made of aluminum castings and held together securely by four lock screws. Figure 7 shows the two parts separated. The cone carries the lens in a between-the-lens shutter; the shutter-resetting mechanism; a punch which when pressed pricks the film at its edge, giving a mark that can be recovered in the darkroom by touch; and the focal-plane frame, which is equipped with marginal marks to indicate the principal point of photographs and to provide for accurate setting in photogrammetric instruments.

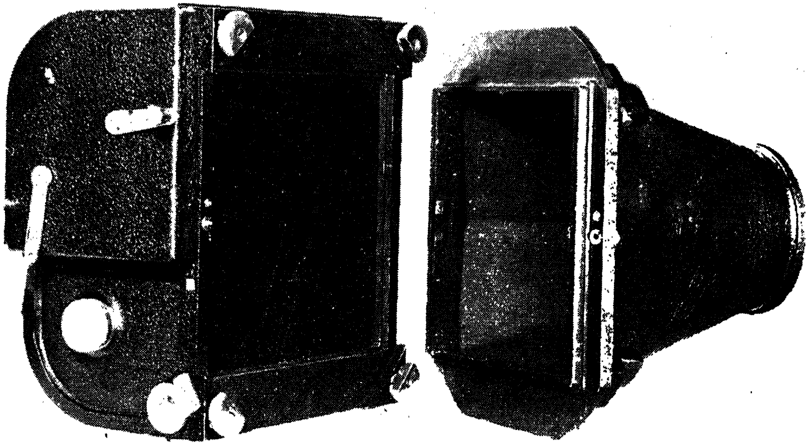


Fig. 7.—The Park photogrammetric aerial camera, the two parts separated. (*Courtesy of Park Aerial Surveys, Inc.*)

The shutter and the lens are mounted separately, and the shutter may be removed quickly even during flight. Knobs and indicators for setting the shutter at its various speeds and the diaphragm at its stops are placed on the front of the camera beside the lens. The cone protrudes sufficiently to protect the lens and to provide for attaching a windbreaker apron which can be battened to the floor of the ship.

The magazine carries the operating mechanism consisting of a gear box attached on one side with film-winding crank and shutter trigger extending from its cover; a film-spacing roller and spindles for the film spools, inside; a stop watch, two cross-level bubbles, and an exposure counter on top. The stop watch is reset at zero at each tripping of the shutter, and it can also be reset independently of the shutter trigger by pressing a button immediately above the trigger. A special feature of the magazine is its division into two compartments by a lightproof partition, with a door for each compartment, so that the opening of one door admits no light to the other side. This arrangement permits a loaded magazine, having part of the film already exposed, to be removed; the film clipped; and a leader attached to the unexposed end. Exposed film then may be removed from the magazine in daylight, an empty spool inserted, and the remainder of the film threaded for operation with the loss of but a small length of film. Another special feature is a button on the gear box which on being pushed releases the film stop and permits film to be drawn off without tripping the shutter. It also serves to run

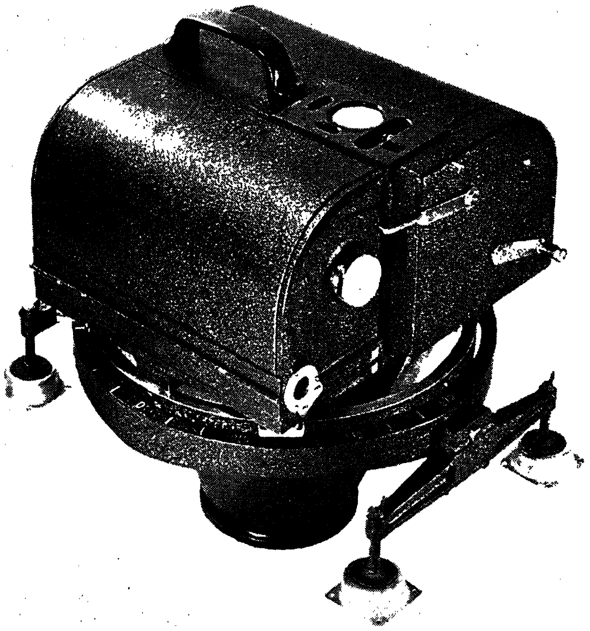


Fig. 8.—The Park photogrammetric aerial camera on its mount. (*Courtesy of Park Aerial Surveys, Inc.*)

off the leader and trailer of the film without operating the exposure mechanism. The capacity of the magazine is sufficient to accommodate a roll of film 150 feet long, giving about 190 negatives 9 inches square. The film is flattened against the focal plane by suction. The film-spool spindles and all rollers over which the film passes are equipped with ball bearings.

Figure 8 shows the camera assembled and placed in its mount ready for operation. The mount consists of two concentric gimbal rings, the inner ring being mounted on the outer one; the latter is mounted on bars with rubber shock absorbers set directly on the floor of the ship. The camera rests on the inner ring and may be turned on it to any desired orientation. The outer ring is graduated with 10° marks, and a plate attached to the cone of the camera is graduated to even degrees. These provide for orienting the camera at any angle of crab from 0 to 90° both clockwise and counterclockwise.

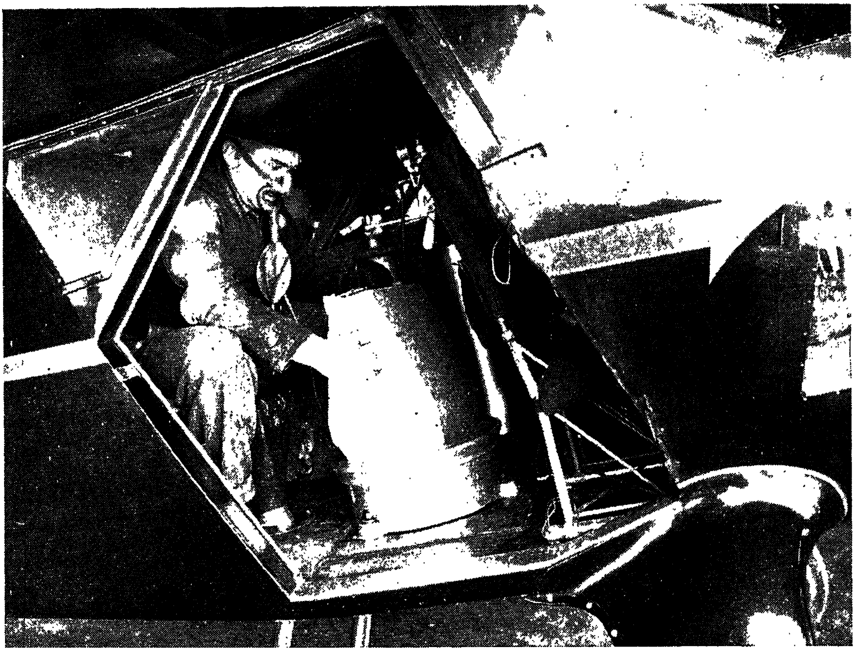


FIG. 9.—The Aero Service Corporation photogrammetric aerial camera installed in an airplane for use. (*Courtesy of Aero Service Corporation.*)

The camera is so constructed that the same magazine can be interchanged with cones containing lenses of different focal lengths. With the cone for an 8¼-inch lens, the dimensions of the camera are: width 15 inches, front to back 13½ inches, and height 18½ inches. Complete with this cone, magazine mount, and shock absorbers, the weight is about 42 pounds.

41. The Aero Service Corporation photogrammetric aerial camera.—Although a complete description of this camera is not available, Fig. 9 shows it as mounted in an airplane ready for use. It furnishes negatives that are 9 by 9 inches in size, and a comparatively large roll of film is accommodated. Photographs made with one of these cameras indicated exceptionally fine definition, and enlargements up to several times the size of the negatives give remarkably sharp details. This camera has been used extensively by the corporation to carry out contracts for furnishing photographs to Federal agencies for use in standard mapping.

42. The Brock aerial plate camera.—This is the only aerial camera employing glass plates that has been made and used systematically in the United States. It is the result of early efforts in aerial mapping in this country and is an item in the Brock process of topographic mapping. The camera and all the instruments of this process are now owned by the Aero Service Corporation of Philadelphia.

The following description of the camera is taken from an article by E. A. Schuch in *Photogrammetric Engineering* for April-May-June of 1940, page 55:

> The camera is of the full automatic type and is provided with daylight loading magazines which hold 48 glass plates, 6.5 by 8.5 inches of $\frac{1}{16}$-inch thickness. These plates are placed on individual shelves in the magazine and moved into the focal plane and back into the magazine by septums which act only as guides and cannot cause any deviation from the optical axis.
>
> The cone with the lens mounted and doweled therein constitutes a unit interchangeable in the camera proper. When the glass plate is in position for exposure, it is pressed firmly against the machined surface of the cone and held in place by four springs located near each corner of the cone. The camera is hand operated. Turning of a crank charges the shutter at the same time the plate moves into position. When the plate is in position, the mechanism automatically locks and is released only after the plate is exposed. After the plate is exposed and returned to the original compartment from which it came, the magazine rises and places the next plate in position to resume the operation. As the magazine rises, the light door on the magazine closes by this same amount. This operation continues until the last plate is exposed. The camera door cannot be opened unless the magazine door is closed, which prevents exposure of plates to daylight.
>
> The next magazine is merely placed in the camera, which is then ready to operate. The time required for an experienced cameraman to change magazines is about 10 seconds. The number of plates that can be carried is limited only by the hours of photographic light available.
>
> The camera is suspended in the plane by a gimbal mount, the oscillation being damped by hydraulic dashpots. An easy means is also provided to compensate for the unequal weights of the loaded magazine.

The plates are exposed with the longer axis of the plates in the direction of the flight.

The camera swung on its mount is shown in Fig. 10.

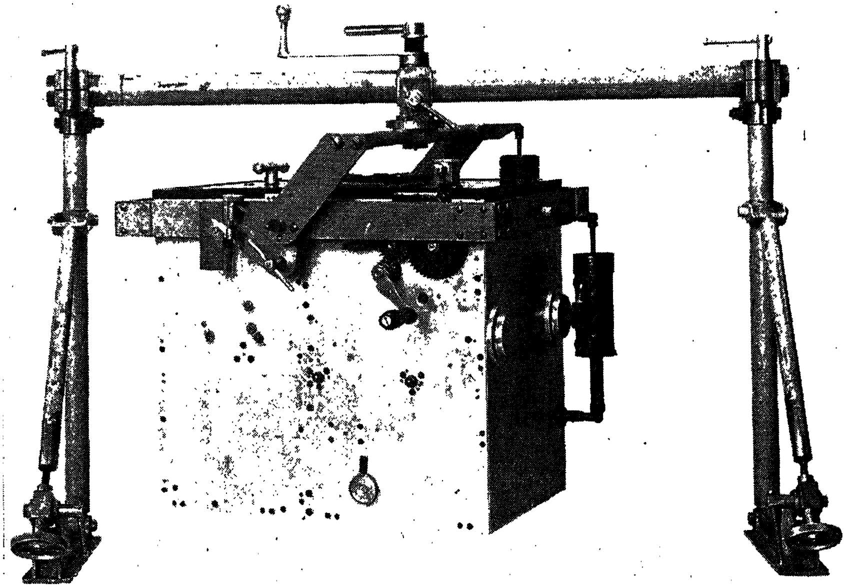


FIG. 10.—The Brock aerial plate camera. (*Courtesy of Aero Service Corporation.*)

43. Single-lens aerial cameras for general and special purposes.—In this category are placed aerial cameras designed primarily for uses other than photogrammetric, such as for taking obliques, for military reconnaissances and intelligence purposes, or for providing lightness and compact size where extreme precision is not essential. However, with the new vacuum-back magazine the K-3C camera qualifies well for mapping purposes and quite appropriately might have been placed with the cameras described in Arts. 37 to 42. All these cameras are made by the Fairchild Aviation Corporation and are constructed of suitable alloys to provide strength, durability, and lightness in weight. They are here separated into two classes: (*a*) those equipped with the between-the-lens shutter and (*b*) those equipped with the focal-plane shutter. In addition a special machine-gun camera is described.

44. Special-purpose aerial cameras equipped with between-the-lens shutters. (*a*) *The Fairchild K-3C aerial camera.*—Figure 11 shows a single-lens camera equipped for both vertical and oblique photography. This model has been used extensively in the United States in past years by the Army and the Navy and by many private organizations that have been engaged in the work of furnishing verticals for mapping purposes under contracts and in the taking of miscellaneous obliques for use as illustrations. The camera is shown ready for installation in its mount to take vertical photographs manually. An attachable 12-volt electric motor makes it wholly automatic in operation when used with an intervalometer and battery. Two grip handles and an open sight, which are put on for oblique photography, are shown beside it. It consists of a body (the rectangular middle section), a magazine on top, and a cone at the bottom.

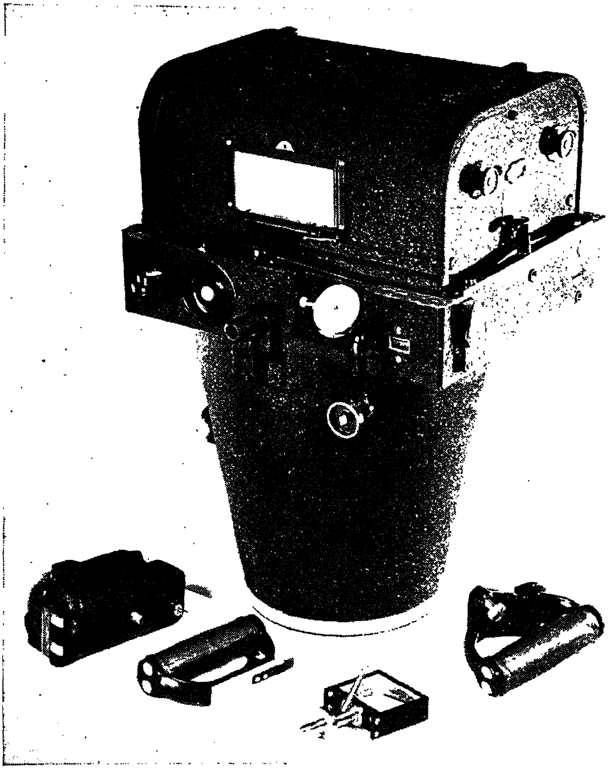


FIG. 11.—Fairchild K-3C (K-3B) aerial camera equipped with 12-inch lens cone. Handgrips, electric motor, and oblique (direct vision) view finder are shown detached. (*Courtesy of Fairchild Aviation Corporation.*)

Inside the body is installed the combined shutter and magazine-drive mechanism. On its front face are placed a universal level bubble, a shutter-speed-setting knob, a stop watch, a film-winding crank for manual operation, and an exposure counter. On its right face there is a shutter trigger for use when taking vertical photographs, threaded recesses for attaching the right-hand grip, and a slotted recess which engages a second shutter trigger, attached to the right-hand grip for use when taking obliques.

On its back face there is a place to fasten the open sight. This is the upper face when oblique photographs are taken. On the left face there is a place to attach the electric motor and sockets for plugging in cables that connect with a battery and an intervalometer when rigged for automatic operation.

In the cone are installed the lens mounted in its between-the-lens shutter and rods that control the speed of the shutter and trip it. There is a place on its left side for attaching the second grip handle, and on the front and back are attached the two short tubular projections by which the camera is swung in its mount. The illustration shows the cone that carries a lens whose focal length is 12 inches. Four other cones are made for this camera. These are installed with lenses ranging in focal length from 6¼ to 24 inches, the speed of the lenses ranging from f/4 to f/11. All the cones are attachable to the single body, and one may be replaced readily by another. Shutter speeds of this camera range from $\frac{1}{50}$ to $\frac{1}{200}$ second for lenses between 6¼ to 12 inches in focal length and from $\frac{1}{35}$ to $\frac{1}{100}$ second for 24-inch lenses.

Two types of magazine are made for this camera. One accommodates rolls of film 9½ inches wide, 200 feet long, giving 250 exposures and negatives 9 by 9 inches in size. This magazine is equipped with a suction back which holds the film flat in the focal plane, the necessary vacuum for the purpose being supplied through a rubber hose which connects the reservoir of the back with a Venturi tube placed in the slipstream. The back is so arranged that it is automatically lifted a trifle before fresh film is wound into place and returned to its seat at the end of the movement, thereby preventing undesirable friction. The second type of magazine accommodates rolls of film 9½ inches wide and 75 feet long, giving 100 exposures and negatives 7 by 9 inches in size. This magazine carries a glass plate which supports the film and a backing plate which presses the film against the glass plate. The pressure plate is automatically lifted from the film when winding is begun and returned against the film at the end of the winding cycle. Both types of magazine are provided with connections to the body mechanism that mesh automatically when a magazine is set in place. Magazines are held fast by a simple sliding bar equipped with a locking lever. Also available for use with this camera is an adapter which provides for using standard-cut film or plate holders that may be readily changed during flight. The K-3C camera ranges in weight from about 41 to a little over 60 pounds, depending on the size of lens and cone.

(*b*) *The Fairchild K-8A aerial camera* is similar to the K-3C camera but is equipped with a small auxiliary lens which records on each negative along an edge the position of a level bubble; the dial of a clock; the number of the exposure; and a space wherein a special note as to the locality, date, and other items may be inserted before photography is begun. This camera is rigged for wholly automatic operation. Interchangeable cones of different sizes and lenses mounted in the between-the-lens shutter are furnished to range in focal length between 8¼ and 24 inches. The camera ranges in weight from about 44 to 68 pounds.

This camera is equipped specially for remote-control operation to take verticals without the services of a photographer and is intended primarily for military reconnaissances and intelligence purposes. It is also suitable for taking verticals for use in making mosaics or photographic atlas sheets.

(*c*) *The K-7C camera* is designed particularly for taking verticals for military use at high altitude and under extreme ranges of temperature but may also be used for obliques. It is made up of a cone with an f/6 24-inch lens and a magazine that accommodates regular rolls of film on 9½-inch spools and 75 feet long. Negatives are 9 by 18 inches (24 by 48 centimeters) in size, so each roll of film gives 45 to 47 exposures. The magazines are standardized, and each is furnished with a lightproof slide; loaded ones may be removed and replaced by others loaded before starting a flight. The speeds of its shutter are $\frac{1}{30}$, $\frac{1}{50}$, and $\frac{1}{100}$ second. Its weight is 58 pounds.

(*d*) *The F-1 camera* is made for general oblique photography. It is furnished either with or without a recording device and has a 20-inch telephoto lens. The parts consist merely of the cone and the magazine. The magazine, which takes roll film and gives negatives 5 by 7 inches in size, may be detached while loaded and replaced with another loaded magazine or by an adapter which accommodates a combination cut-film and plate magazine. It holds either 12 sheets of film or 12 plates. The camera is semiautomatic, film being wound and the shutter set by turning one of the handles. Exposures may be made with this camera at a very rapid rate—about one every 3 seconds. Its weight is 31 pounds.

(*e*) *Aerial camera specially rigged for night photography.*—This camera was designed for photography at night to obtain military information. It operates automatically at the peak of intensity of a photoflash flare. It accommodates cut film or plate holders that may be exchanged during flight. Negatives obtained are 8 by 10 inches in size. The lens is an f/3.5 of 13½-inch focal length and of exceptional speed in this size. Shutter speeds are $\frac{1}{20}$, $\frac{1}{30}$, and $\frac{1}{40}$ second. The mount for vertical photography is similar to the mount of the K-3C camera. Complete equipment consists of the camera, a photoelectric cell, and a control box with surge relay which occupies a place on top of the magazine. Photoflares are furnished as extra equipment (*f*) *The particular purpose of the F*-14 *camera* is to provide a compact and lightweight camera suitable for both vertical and oblique photography. Its weight is only 30 pounds. It is rigged for either wholly automatic or semiautomatic operation. It is equipped with an f/4 8¼-inch lens and a detachable magazine with a self-contained vacuum-back focal plane and a capacity for 200 exposures, giving negatives 7 by 6⅝ inches in size. Loaded magazines are interchangeable during flight. Shutter speeds are 1/35, 1/50, 1/100, and 1/150 second.

45. General- and special-purpose aerial cameras equipped with the focal-plane shutter are included in this article.

(*a*) *The F*-11 *camera* is designed particularly for long-range obliques. It is equipped with a 40-inch telephoto lens and takes a 5- by 7-inch picture, being made up of a cylindrical base and roll-film magazine with capacity for 40 exposures. A ring round the base near its middle forms the bearing surface for the mount. The roll-film magazine may be interchanged with a cut-film and plate magazine. Its weight is 42 pounds.

(*b*) *The F*-8 *camera* is designed particularly for oblique and general photography, although a special suspension mount provides for taking verticals. It is light, handy, and well balanced. Its special features consist of a ring by which the lens can be focused for distances from 8 feet to infinity, a cap that covers the back of the lens and is automatically lifted for exposures and returned to its position afterward, a combination shutter and film-winding knob, and an exposure counter. An exposure cannot be made until fresh film has been brought into the focal plane and the shutter wound. Magazines both for roll film and for cut film or glass plates are provided. Both types are interchangeable when loaded. Rolls of film give 40 exposures; the capacity of cut-film-glass-plate magazines is 12 of either. The negatives are 5 by 7 inches in size. Grip handles and an open sight provide for accurate and steady aim. Rigged for oblique photography with the roll-film magazine, this camera weighs but 18 pounds.

46. Fairchild machine-gun cameras.—The machine-gun camera (or camera machine gun) is employed in training machine gunners for aerial combat. Permitting the gunner to examine and study the defects of his aim and operation of the gun and the effectiveness of his firing, it has been an important item in military training. The photographic record brought in by the trainee enables the instructor to analyze mistakes in the operation of the gun and to instruct accordingly. The camera gun is made for operation as the real machine gun is operated; but in place of the regular magazine a small cartridge of film is loaded, and a small motion-picture camera incorporated in the gun takes pictures at the rate of 16 per second. Photography is started by moving a trigger and stopped by releasing it. The image of a reticle appears in each photograph. This aids in the study of the "shots" to determine the percentage of hits. The dial of a watch installed in the camera is photographed at the end of every burst of "fire."

The camera is equipped with an f/3 lens, a special rotary-disk shutter such as used on motion-picture cameras, and an iris diaphragm with apertures ranging from f/3 to f/16. The magazine accommodates enough 16-millimeter film for 700 shots at each loading. The lens is set at a position to focus on objects from 25 feet to infinity. When fully wound up, the driving mechanism will operate the camera long enough to expose an entire loading of film. The sight consists of a front bead and a rear double ring, the latter being fitted with crosshairs. The camera is easy to load, and the gun simple to operate. The gun may be mounted for flexible hand operation or in a fixed position. It closely resembles the Colt or the Browning machine gun in size, weight, and operation. Extra equipment consists of a bore sight for aligning the gun in a fixed mount, a film-developing outfit, a film-viewing device, and a 16-millimeter projector.

A second model of the camera machine gun is designed with a streamlined housing for attachment in a fixed position on top or at the leading edge of a wing. This gun is electrically operated by a trigger switch mounted on the control stick of the airplane. The regular rate of exposure of 16 per second may be retarded as desired to 12 per second.

47. Multiple-lens cameras.—The first multiple-lens camera made in the United States had a three-chambered base and a single magazine with capacity for a large roll of film which provided for all exposures. An oblique chamber was placed at each side of a central chamber, and the camera was mounted in the airplane for taking pictures aligned across the flight course. These were used during the period 1918 to about 1929, when they were superseded by a four-lens camera. This was developed from the tri-lens camera by adding a fourth chamber, having a separate magazine, which photographed the terrain behind that covered by the vertical chamber, thereby giving to the photographs much greater strength with respect to orientation. Some of these four-lens cameras are still in use. About 1930 the first of the present five-lens cameras was completed, and since their availability in numbers a little later the greater part of the mapping work with multiple-lens cameras carried out in the United States and some Latin-American countries has been done with this camera.

48. The T-3A (five-lens) camera.—Figure 12 is a photograph of the five-lens camera, made by the Fairchild Aviation Corporation, as it appears when ready for placement in its mount. It consists of a base that contains five separate chambers. Grouped round a central chamber which takes vertical views are four symmetrically placed inclined chambers which take oblique views. The angle of obliquity of each of the inclined chambers is exactly 43° with respect to the central chamber; *i.e.*, each oblique axis crosses the axis of the vertical chamber at that angle. As a consequence of this arrangement the angle between the axes of each pair of opposite oblique chambers is 86°, and the total angular field of the camera measured through a pair of oblique chambers is about 140°. Each chamber is equipped with an f/6.3 lens, and all lenses are matched closely in focal length, this being about 6 inches. Figure 13 is a front view of the camera showing the five lenses. Each lens is mounted in an electrically operated between-the-lens shutter, and exposures are made by an electric surge. The inside dimensions of the focal-plane frames of the oblique chambers are 5½ by 6 inches, and those of the frame of the vertical chamber are 5½ by 5½ inches. Notches in the edges of the frames indicate accurately the position of the principal point of every photograph. Set in the focal plane of the vertical chamber are cross-bubble vials which are recorded as shadowgraphs. Conduits inside provide the connecting links between the electric plug-in socket and the shutters. A ring at the bottom both guards the lenses from

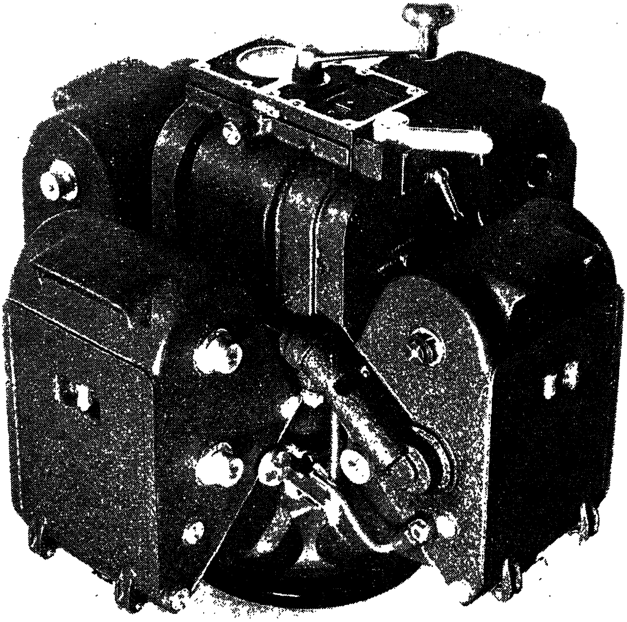


FIG. 12.—Fairchild T-3A 5-lens aerial camera. (*Courtesy of Fairchild Aviation Corporation.*)

damage and provides a stable footing for the camera when loading and handling.

There is a detachable magazine for each oblique chamber and a fixed magazine for the vertical chamber. Each of these accommodates a roll of film 6 inches wide and 120 feet long, giving 200 exposures. Every magazine is equipped with a suction back having a shallow air space in which a partial vacuum is formed to hold the film flat in the focal plane. Idler rolls placed at the ends of the magazine facilitate the winding of the film and prevent unwanted friction. A seat for the roll of film and another for the take-up spool occupy the upper part of each magazine. Each take-up spool is geared into a single winding crank, placed on top of the central magazine, which brings unexposed film into all focal planes at the same time. When a magazine is placed in its proper position, these gears are brought into mesh and the vacuum conduit makes connection with a pipe to which the vacuum hose is attached. Vacuum is provided by a Venturi tube placed in the slipstream. The vacuum action is fully automatic, being controlled by the film-winding and shutter-tripping mechanism. Immediately after each exposure the air connection is broken to permit easy winding of the film, then restored at

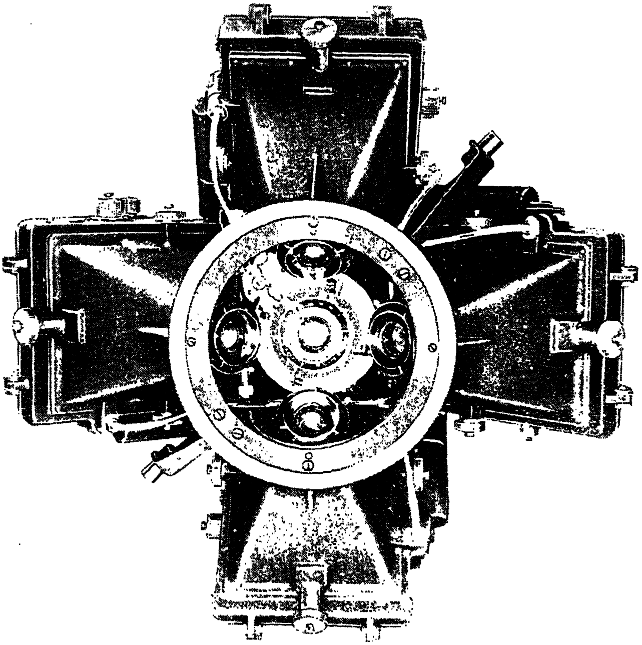


FIG. 13.—Front of Fairchild T-3A 5-lens aerial camera. (*Courtesy of Fairchild Aviation Corporation.*)

completion of the winding. A film-metering device automatically pays out the proper amount of film and stops the winding when enough film for another exposure has been brought into place. A stop watch, two cross-level bubble vials, the shutter-tripping lever, and an exposure counter, which are placed on top of the control magazine, complete the equipment.

T-3A cameras are sturdy and well constructed and are calibrated with precision. The focal planes are so machined that the dihedral angles between the oblique chambers and the central chamber are all equal, within 1 minute of arc. The lenses are set in position and aligned under this same tolerance. Characteristics of the lenses on one of these cameras illustrate the success attained in matching. Designation of this camera is H U 35. The equivalent focal lengths of the lenses as determined by optical test are

Oblique lenses	*A*	149.86 mm.	Central lens	*B* 150.01 mm.
	C	149.88 mm.		
	D	149.90 mm.		
	E	149.89 mm.		

The lenses are set at positions to give the actual focal length of oblique chambers of 149.88 millimeters. Photographs made with this camera confirm the care of its construction by the results obtained with them in locating secondary control points by the radial-line method and in obtaining elevations by the displacement method.

The mount for the T-3A camera is similar to the mount for the K-3C camera, being of the gimbal type, which provides for leveling the camera and rotating it about a vertical axis to compensate for crab of the ship or to orient the camera at any desired angle to the line of flight. It is supported on sponge-rubber shock absorbers.

49. The nine-lens camera of the U.S. Coast and Geodetic Survey.—This is the largest aerial camera that has been made in the United States, and photographs taken with it closely approximate the maximum of combined field and scale practicable to attain in aerial photography. The camera is intended primarily for making coastal charts at the scales 1:10,000 and 1:20,000. Instead of having oblique chambers inclined to a central chamber, all the lenses are set with their axes parallel, and oblique rays of light are reflected into eight of the lenses by mirrors. In flight the camera is placed with the lenses pointing downward, and the mirrors under the outer lenses are inclined about 13° to the vertical, outwardly at their lower edges, so that each obstructs the light coming directly upward but sends to its lens the oblique rays between 32 and 65° from the terrain lying on its side of the scene below. Light passing through the "oblique" lenses not coming from the desired areas is prevented from reaching the film by black fins attached to the walls of the chambers.

There is a separate chamber for each lens but only one focal plane, all nine of the images being registered on a single frame of the film. The film is held flat by a vacuum back. Four small auxiliary lenses register in the corners of the negative four collimation marks for use in setting it for printing, the dial of a watch, the face of a counter, two cross-level bubbles, and data pertaining to a mission as prepared for the occasion and inserted in the space provided.

The field of the camera is 130 by 130°. Negatives are about 23 inches square; composite photographs, about 35.4 inches square. The magazine has a capacity for rolls of film 200 feet long and about 23 inches wide, which give 100 exposures with the desired space between. The magazine can be unloaded and reloaded in flight but is ordinarily loaded in the darkroom. The camera is equipped with Ross wide-angle X-pres lenses having a focal length of about 8¼ inches; all of them are matched to $\frac{1}{10}$ millimeter. Their corrected field in which distortion does not exceed 0.05 millimeter is 68°. They work at f/4. Each lens is mounted in an electric shutter, and all the shutters are operated simultaneously by solenoids, the speeds being $\frac{1}{80}$ and $\frac{1}{50}$ second. The camera is 29 inches wide, 27 inches from front to back, and 31 inches high. Its net weight is 306 pounds; its gross weight with all equipment for photography, 750 pounds. The mirrors, the framework supporting them, and the lenses and the lens barrels are all made from a single billet of chromium steel. The mirrors are coated with evaporated aluminum, their reflective efficiency being about 85 per cent.

A special mount of the gimbal type is provided for the camera. It permits the camera to be leveled and oriented as required to square it with the flight course. The accessories used with the camera are an exposure interval-crab indicator, an intervalometer, a vacuum pump, and a signal lamp. Power is taken from the ship's power line or may be supplied by a separate storage battery. Operation is automatic. A rubber-tired man-drawn truck serves to move the camera about from laboratory to airplane. A special hoist is used to install it in the airplane.

50. Special printer for the nine-lens camera.—Composite photographs are made from the negatives with a bi-lens projection printer designed and constructed for the purpose. A negative holder provides for accurate centering and placement on the printer. It is first placed on a turntable to print the outer parts of the photograph by transformation, then put into position to print the central part by straight projection through the second lens. The image plane is equipped to rotate in step with the negative turntable, but actual positions are determined by dowels. Printing of the oblique parts is done with the emulsion face of the negative turned away from the lens; of the central part, with that face toward the lens. This procedure restores the true orientation of the parts which is disturbed by the mirrors. Printing may be done on paper, film, or glass plates. A vacuum device holds paper or film flat in the focal plane. During the period of printing, masks are placed over those parts of the sensitive surface which are to be shielded from stray light. Both camera and printer are of such good quality and adjustments in assembly of these instruments have been so well executed that the parts of the composite photographs join in a highly satisfactory manner and the perspective quality maintained is of high order.

51. Camera mounts.—Various methods of mounting cameras in airplanes are employed. For vertical photography the commonest form and the type most generally used in past years consists of gimbal rings and bearings. This provides for orienting the camera by revolution about a vertical axis and for leveling it. The mount is constructed so that the lower ring may be clamped to bars at four points of cushioned support. The upper ring rests directly on the lower ring in a manner to permit easy rotation of the upper with respect to the lower ring. From the upper ring, arms extend upward to end in cylindrical-shaped seats with clamps. A frame with short tubular projections rests in these seats and may be clamped at sufficient pressure to hold it at any desired position for leveling. At 180° in direction to the line between the tubular projections, seats similar to those of the upper ring are attached to the frame. These also have clamps, and they serve to receive the camera, which once placed is securely held and may be adjusted for leveling. When mounted, the camera therefore can be rotated and tilted in two directions. The clamps are screwed to that point which permits ready adjustment of the camera as level bubbles indicate to be necessary yet will hold the camera wherever set. Circular graduations on one of the rings and an index mark on the other provide for setting the camera with respect to the axis of the ship to square it with the course being flown. This mount is ordinarily used with a framework that rests on the floor of the cockpit and provides the required supporting bars.

Other forms of mount for vertical photography are shown in Figs. 3, 4, 6, 8, 9, 10, and 18. The advantage of some of these lies in the added accessibility that they give to the camera during operation. Some also fairly effectively close the opening against cold air. The essential elements of insulation against vibration and provisions for leveling and turning the camera in azimuth are incorporated in all of them.

In some mounts provision is made for raising and lowering the camera. This is a desirable feature and is particularly advantageous for installation in the hull of a hydroplane to permit the cover of the opening to be taken off after leaving the water and replaced before landing. Vertical adjustability also assists in safeguarding the lenses and camera front from damage at take-off.

Descriptions of some of them will be found in the articles about the cameras with which they are used. Figure 14 shows the mount used with the Fairchild photogrammetric camera (Art. 37).

For oblique photography a cushioned swivel mount made up of a fork or ring to receive the camera and attached to the side of an open cockpit is often used. The machine-gun turret which permits traversing a wide arc is adapted for oblique photography by the Army. Much oblique photography is done without a mount, the camera being cushioned against the body for steadiness. If the camera is too heavy to hold by hand, a mount must be employed; and if there is much work to be done, it is important to have a suitable mount that will both diminish fatigue and aid in getting satisfactory photographs.

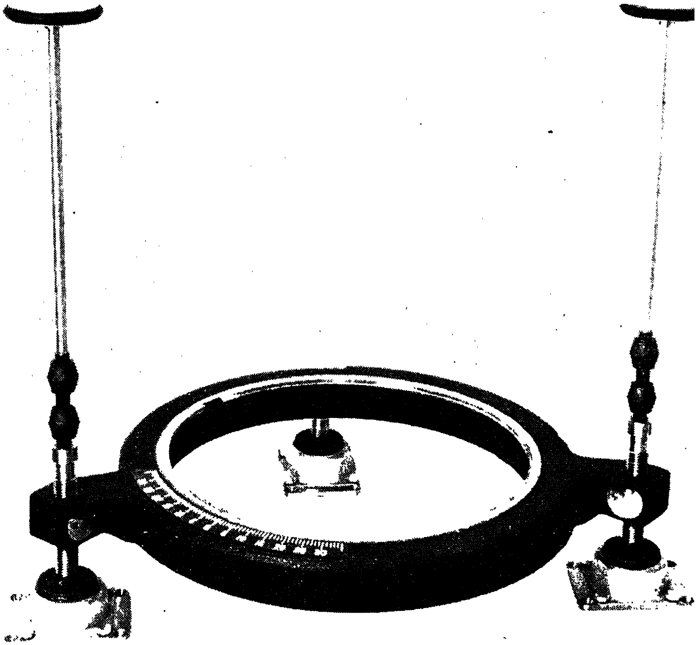


FIG. 14.—Mount for the Fairchild photogrammetric aerial camera. (*Courtesy of Fairchild Aviation Corporation.*)

52. Vertical view finders.—In photography for mapping, the view finder is the most important of the accessory instruments. In fact it is virtually indispensable. It serves four purposes: (*a*) to determine the correct interval between exposure to give the desired overlap between consecutive photographs; (*b*) to determine the crab of the ship to the flight line so that the camera may be turned to the proper position to square it with that line; (*c*) to determine the line that the airplane is actually flying along or if a particular chosen line is being followed; and (*d*) the proper instant to start photography and to stop; *i.e.*, with a map or advance knowledge of an area it enables the photographer to tell when its border is being neared and when the ship is passing from it.

Figure 15 is a general view of the Fairchild vertical view finder with covers open. It has been used in the Army for many years and is widely used throughout the United States. It consists essentially of a lens, a ground glass, a level bubble, and a ball-and-socket joint with a clamp to apply pressure. In effect it is a camera without a magazine. It may be rotated and tilted as required. With the clamp set at proper pressure the socket holds the superstructure in any desired position. It is equipped with an 8½-inch f/4.5 lens, and the ground glass is 5¼ by 6½ inches in size. The height open is 17 inches; closed, 13 inches; it weighs 9½ pounds as installed. It requires a hole in the floor 4½ inches in diameter. The covers are held upright by springs when the strap is unbuttoned. These partially shield the ground glass from light above and heighten the brilliance of the image.

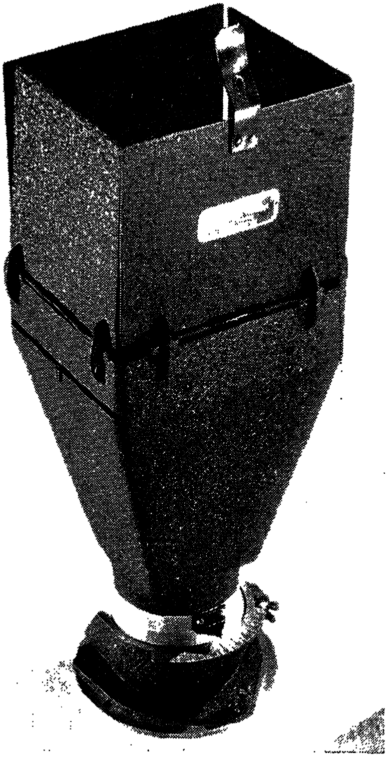


FIG. 15.—Fairchild vertical view finder. (*Courtesy of Fairchild Aviation Corporation.*)

Figure 16 is a top view of the finder showing the ground glass ruled with lines and the universal level. The many parallel lines across the glass serve to set the view finder square to a flight line, for when so set the images of objects will appear to travel along or parallel to these lines. Across them are two lines, equidistant from the center, spaced to accord with the overlap desired in the photographs to be taken —usually 60 per cent. These are the exposure-interval lines. The distance between them also depends on the focal length of the camera to be used and the width of the photographs. For example, if the focal length of the camera is 8¼ inches and the width of the photographs 7 inches, the lines would be 2.8 inches apart to indicate a 60 per cent overlap if the focal length of the view-finder lens were the same as that of the camera. But for a view-finder focal length of 8½ inches, the lines must be 2.8 × 8.5/8.25 = 2.9 inches (nearly) apart. For a camera having a focal length of 12 inches and taking photographs 7 inches wide the exposure-interval lines of the same view finder should be 1.98 inches apart.

With the exposure-interval lines properly spaced for the camera, the interval between exposures is given by the time required for images of objects to travel from one to the other when the finder is set at level position and the airplane is being flown on an even keel at the height for photography. The center mark of the ground glass serves to determine the line along which the ship is flying and the instant that it reaches a position directly over a particular object. With the view finder level, the image of the object directly below is at the center of the ground glass. It is important that the center mark of the glass be placed on the optical axis of the lens, that the level be accurately placed, and that the ground glass be set with its surfaces at right angles to the optical axis.

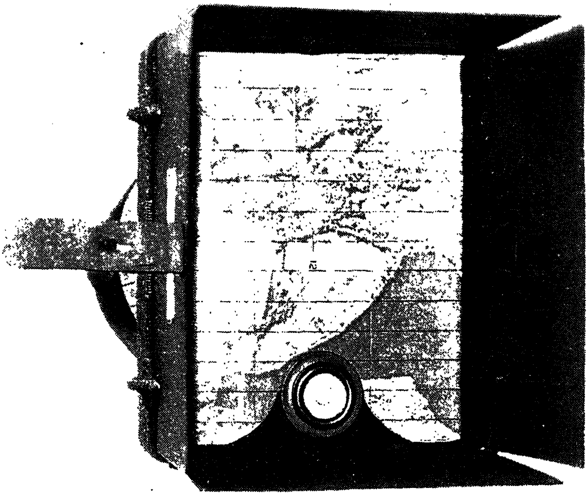


FIG. 16.—Top view of Fairchild vertical view finder, showing ground-glass screen. (*Courtesy of Fairchild Aviation Corporation.*)

There are several other forms of view finder, some of which are attached to the camera itself or to the mount. The multiple view finder built into the mount of the Mark Hurd camera is well displayed in Fig. 4. Its special features are described in Art. 38.

53. The Fairchild intervalometer.—The function of this instrument is the making of exposures at any predetermined interval of time. It is employed with automatic cameras equipped for electrical operation and having the required cable-connection sockets. When properly connected to the camera and a battery or other 12-volt electric circuit it makes the exposures at regular intervals of time in accordance with the setting of a dial of the instrument. The range of time intervals is from 6 to 75 seconds. It has a centrifugal governor which may be regulated with exactitude to give unvarying intervals between shutter tripping, regardless of change in the speed of the motor. The instrument is also provided with a button for manual operation, with signal lamps for pilot and observer which flash 3 seconds before an exposure is to be made, and with an exposure counter. Use of the intervalometer therefore also provides signal mechanism necessary for the best teamwork between pilot and photographer. Figure 17 shows the intervalometer with cables attached, the interval control dial, and the exposure counter.

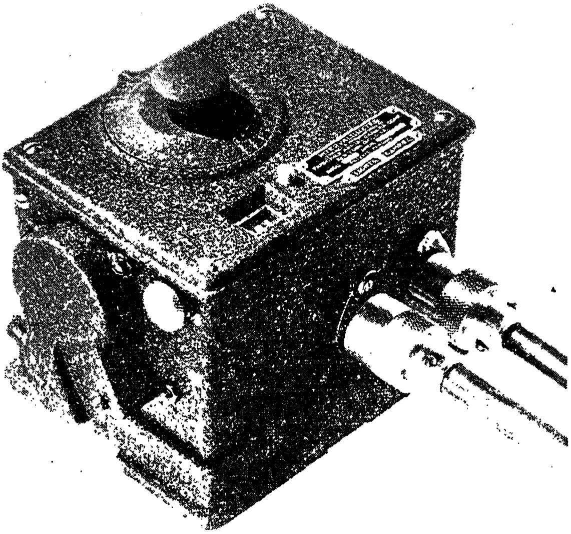


FIG. 17.—Fairchild intervalometer (automatic timer). (*Courtesy of Fairchild Aviation Corporation.*)

54. The altimeter.—Airplanes are ordinarily equipped with an altimeter which is placed in the instrument panel for the pilot's use. In systematic aerial photography it is desirable that one also be provided for the photographer, who often is placed where he cannot see the installed altimeter. Altimeters are made for use up to different heights. It is necessary that the height limit be equal to or greater than the ceiling of the airplane in which the instrument is to be used; otherwise it would fail to serve its purpose completely.

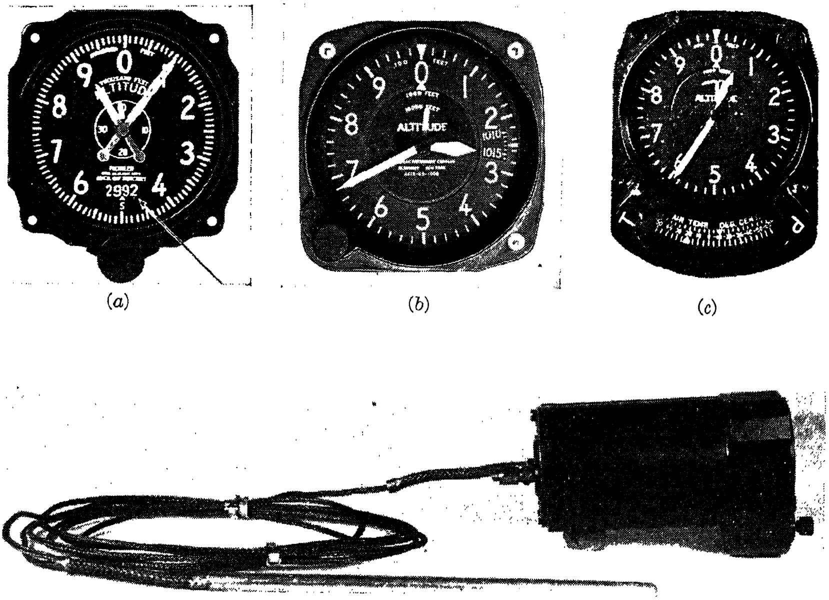
Beyond its value as an instrument in regular and general navigation, the altimeter has two specific uses in aerial photography. These are to indicate when an intended level has been reached and to guide the pilot along any desired level. A phase of the first use is that of returning to a previously used level in continuation of work on different flights and on different days. Another use relating to explorations is that of establishing a datum plane in a remote region when it is impracticable to obtain one by the usual methods of ground surveying.

In aerial photography the following qualities are of chief concern: sensitivity to slight changes of height, reliability in indicating those heights at which photography is to be done, and sufficient magnification of pointer movements and divisions of the dial to guide the pilot along a given level. The installation must be properly made to avoid artificial air pressures arising from flow of air about and through parts of the ship. This is accomplished by connecting the case of the altimeter with a Pitot tube. It is also important that the instrument be insulated against vibration.

The modern sensitive altimeter differs from the aneroid barometer by having incorporated in the linkages of its mechanism a device to change differential straight-line movements of the vacuum capsule into circular movements of the hand of the dial which are regularized in relation to height differences. This amounts to changing a logarithmic scale into an arithmetical scale. Graduations on the dial, therefore, are equal in spacing; and no matter at what level a given height difference is measured, the same number of divisions of the scale will be spanned. The scale conforms to average atmospheric conditions of the region in which the instrument is intended to be used or according to established standard altitude-pressure tables. In a strict mathematical sense, adjustment of the altimeter mechanism is possible only for two levels, but empirical calibration methods employed by manufacturers give satisfactory compensation spreading over a range from sea level to great heights.

Performance of all altimeters is affected by meteorological disturbances, by temperature and humidity changes, and by diurnal variations in atmospheric pressures. General disturbances and station pressure changes are provided for by a dial-setting mechanism (knob or ring). The effect of humidity is comparatively small and may be neglected during fair weather, the periods during which aerial photography is usually carried out. The daily atmospheric pressure variation also is small and quite regular in time and amplitude at a given place. At the usual levels of photography for mapping it is probable that the diurnal variations are absent or reduced to negligible quantities.

Temperature changes have a marked affect on atmospheric pressures. Inasmuch as changes as great as 100°F. are not uncommon in flights for photography, and because any temperature compensation incorporated in an altimeter must be done in accordance with a table of expected temperatures, it is necessary to reckon with temperatures actually encountered to obtain reliable altitude determinations; the readings must be corrected for variations from the calibrated compensation temperature basis. The corrections should be applied as recommended by the maker of the instrument or with full knowledge of the basic table on which the compensation was made.



(a) (b) (c)

(d)

FIG. 18.—American-made altimeters. (a) Altimeter dial. (*Courtesy of Pioneer Instrument Division of Bendix Aviation Corporation.*) (b) Altimeter dial. (*Courtesy of Kollsman Instrument Division of Square D Company.*) (c) Dial of temperature-pressure altimeter; (d) complete temperature-pressure altimeter. (*Courtesy of Material Division, U.S. Army Air Corps.*)

Figure 18*a* shows the dial of a sensitive altimeter made by the Pioneer Instrument Division of Bendix Aviation Corporation, Bendix, N.J., which is suitable for use up to altitudes as great as 35,000 feet above the standard sea level reference. It has three pointers which are geared together like the hands of a clock so that the shortest hand gives readings of 10,000 feet, the intermediate hand gives readings of thousands of feet, and the longest gives readings of hundreds of feet. Each graduation corresponds to 10 feet and readings can be estimated to 5 feet. There is also a window in the lower portion of the dial through which a counter number shows the air pressure in inches of mercury (or other units, as required) at which the instrument will read zero elevation. This is 29.92 in the figure. Pressure settings are made by turning the knob at the bottom of the instrument face. The altitude reading of the dial as photographed is 900 feet below the standard sea level pressure reference.

Figure 18*b* shows the dial of a sensitive altimeter for high-altitude flights which is made by the Kollsman Instrument Division of the Square D Company, Elmhurst, N.Y. It is graduated to 20 feet, and readings can be estimated to 5 feet. The unit of its barometric scale is millibars, and settings are controlled by the knob at the bottom left. Instruments with barometric scale in inches or millimeters of mercury are also made by this company. It has printed and issues technical notes concerning its instruments in which formulas for use in making temperature corrections of altimeter readings and examples are given.

Figure 18*c* is a photograph of the dial of a temperature-pressure altimeter invented[1] by F. G. Nesbitt of the Experimental Engineering Section of the Materiel Division of the Air Corps of the United States Army. This instrument is being developed at Wright Field, and experimental models have been built by the Kollsman Instrument Company. Figure 18*d* shows the complete instrument. The following account of this altimeter was furnished by the Materiel Division of the Air Corps:

> The temperature-pressure altimeter differs from the conventional pressure altimeter in that its indication is derived from both the mean atmospheric temperature and atmospheric pressure. The conventional altimeter is basically an absolute pressure gauge. Altitude is derived from both atmospheric temperature and pressure. In order to calibrate a pressure gauge in terms of altitude it is necessary to assume that certain conditions of temperature exist. This has been done by the establishment of a U.S. Standard Atmosphere.[2]
>
> The Standard Atmosphere assumes that the average air temperature at sea level is +15°C. the year around and that the air temperature decreases approximately 2°C. for every increase in altitude of 1,000 feet until a temperature of −55°C. is reached. The altitude at which this occurs is 35,332 feet in the Standard Atmosphere. Above this altitude the temperature is assumed to be constant at −55°C.
>
> In the conventional altimeter calibrated to the Standard Atmosphere, true altitude is indicated only when the actual temperatures throughout the air column coincide exactly with those assumed in the Standard Atmosphere. This condition seldom, if ever, actually exists, with the result that even though an altimeter may be mechanically perfect, it does not indicate true altitude. The error involved in using Standard temperatures may be plus or minus 10 per cent or more.

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[1] U.S. Patent No. 2081950.

[2] National Advisory Committee for Aeronautics *Report* 538.

The temperature-pressure altimeter incorporates a mechanism which is responsive to both atmospheric pressure and free-air temperature. The basic formula for obtaining true altitude is

$$H = 221.152 T_{ma} \log \frac{P_0}{P} \text{ feet}$$

where T_{ma} = mean temperature of the air column between the point of reference and the point of observation.
P_0 = atmospheric pressure at the point of reference.
P = atmospheric pressure at the point of observation.

For all practical purposes the mean temperature can be assumed to be the average of the temperature at the point of reference Tg and the temperature aloft Ta. The above formula may be written

$$H = 110.576(Ta + Tg) \log P_0/P$$

In the temperature-pressure altimeter, as illustrated in Figs. 18*c* and 18*d*, the zero setting system incorporates a ground-temperature setting in addition to the conventional ground pressure. A free-air temperature pointer and scale is also provided. The mechanism is so designed that the movement of the altitude pointers is a function of the average of the indication of the free-air temperature pointer and the adjacent ground-temperature reference mark in accordance with the above formula. The free-air temperature factor is completely automatic and continuous in operation, while the ground-temperature factor is manually set by the zero-temperature setting knob.

In use, there is only one operation required in addition to that required in the conventional pressure altimeter. This additional operation is the manual setting of the ground-temperature reference mark. If the aircraft is on the ground, the temperature knob is rotated until the temperature-reference mark coincides with the free-air temperature pointer. Then the pressure knob is set until the zero-altitude reference mark is set to the proper value. As the airplane climbs, the free-air temperature pointer moves to a lower reading, corresponding to the decrease in temperature. This movement automatically controls the temperature-aloft factor in the mechanism.

In flight when zero reference data are obtained from near-by ground stations by radio, the altimeter may be reset accordingly. In this case, however, the zero-temperature reference mark is set to the new ground temperature as read on the temperature scale. The reference mark is not set to the temperature pointer, because in flight this pointer will be indicating free-air temperature aloft.

A conservative estimate of the accuracy to be expected is 2 to 3 per cent based on results of tests made to date. This is in comparison to errors of 10 per cent or more in uncorrected pressure altimeters.

55. The Fairchild solar navigator.—The purpose of this instrument is to make it practicable to fly long straight courses for photography over all sorts of terrain and thereby to obtain a desired overlap between adjacent strips of vertical photographs taken for mapping. It has special value in photographing regions without culture, without maps, and without distinctive landmarks, such as tropical jungles and arid lands. Figure 19 shows the solar navigator as installed in an airplane for operation with a K-3C camera in a special mount. The following is an extract from a statement by Leon T. Eliel which was printed in *Photogrammetric Engineering,* No. 3, July-August-September, 1938, pages 202–203.

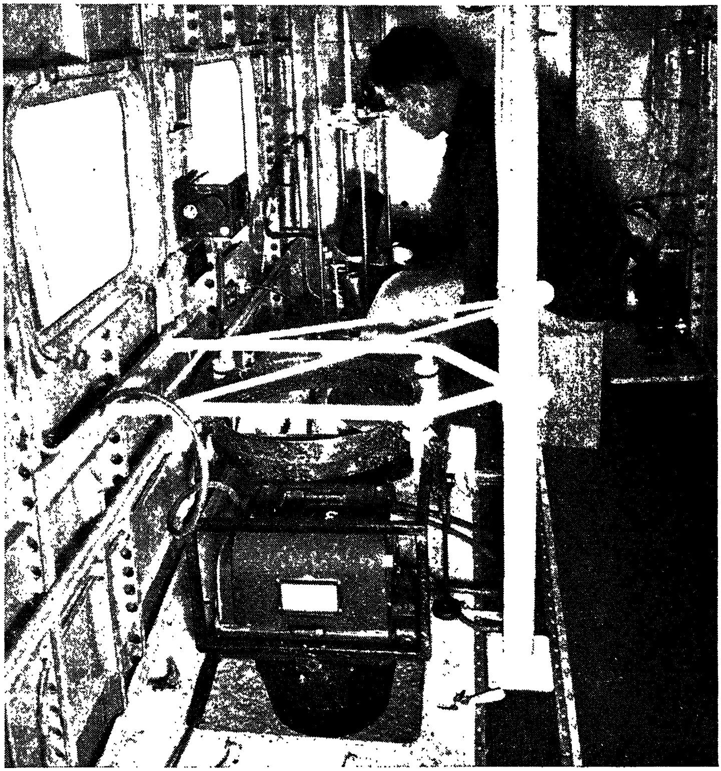

FIG. 19.—Fairchild solar navigator installed in an airplane with a K-3C camera in a special mount. (*Courtesy of Fairchild Aviation Corporation.*)

The instrument is mounted in a dome in the top of the ship and is set up exactly like an astronomical telescope, its axis pointing north in space so that a clock rotating a mirror on an axis parallel to the axis of the earth follows the sun. This mirror is adjustable for the varying declination of the sun from day to day and reflects the sunlight into a lens at the top of the instrument. After passing through the lens, sunlight is directed to two photoelectric cells. When the compass points exactly north, no light falls on the cells; but with the slightest deflection, sunlight falls on one cell or the other, creating a current registering the deflection on a galvanometer mounted on the instrument board. The head, or compass part, of the instrument is supported by forks, the base of which is extended in the form of a column connecting the compass to the view finder which is in the bottom of the fuselage.

The view finder is essentially a conventional instrument consisting of a lens and ground glass with drift lines thereon. When the desired course is north or south, the drift lines are arranged to be set parallel with the axis of the compass. When the course is in any other direction, such as 20°, the drift sight is rotated to an angle of 20° with the compass axis and clamped in this position. The drift sight is stabilized by a gyroscope against the roll of the airplane. The whole instrument is mounted pendulously at the base of the dome with provision on the base of the instrument for leveling it fore and aft. On the base of the instrument also is a slow-motion screw for causing images to follow exactly the drift lines.

It is the duty of the pilot to keep the ship on an even keel laterally, and it is the duty of the photographer to keep the navigator leveled up longitudinally. If the instrument is off level, a reading will appear on the galvanometer which is not truly indicative of a change in azimuth. For this purpose the pilot has a very sensitive level bubble mounted beneath the galvanometer by means of which he maintains the ship on an even keel. In smooth air the instrument works very satisfactorily. In rough air, however, its performance is somewhat short of perfection.

In operation the instrument is easy to set up and handle. The photographer can learn to make the necessary settings for latitude, longitude, equation of time, and declination in one lesson; and, in fact, if he will study his manual for an hour, he can make all these settings without any instruction. Two factors must be looked up in tables each day.

a. The declination of the sun

b. The equation of time

The operator carries in his kit a little pocket solar ephemeris table in which under the particular date the declination and equation of time are listed side by side.

Since the axis of the instrument points north in space (*i.e.*, toward the north star), the plane of the lens of the instrument is parallel to the equator of the earth. As the sun apparently moves through its seasonal changes from 23° north of the equator to 23° south of the equator, the declination mirror of the instrument must be set from day to day so that the sunlight is turned directly into the lens. That's all there is to the declination setting.

The equation of time setting is necessary because the sun day is not exactly 24 hours long. During part of the year the clock gets ahead of the sun by as much as 16 minutes, and during the balance of the year the clock lags behind the sun. Therefore, the equation of time must be added to and subtracted from the time carried on the watch to equal sun time. This factor is read directly out of the ephemeris table and merely has to be set on the clock.

Theoretically, as the airplane flies north or south, the latitude adjustment of the instrument should be changed to correspond to the latitude of the point over which the airplane is flying from instant to instant. Practically, however, this factor is usually negligible so long as the flights do not exceed 70 miles in length. For example, flying at latitude 37° north with a declination of 20° north (this means the sun is only 13° from the zenith at noon), the following maximum errors will result in the flight 70 miles long north and south with a latitude set for the mean of the strip. One hour and 15 minutes from noon a flight of this length will be deflected approximately 400 meters from a true north-and-south line by reason of failure to change the latitude setting. With a higher and lower sun the errors get less and less, being zero at noontime and negligible about 3 hours from noon. If the flight lines are only 35 miles long, the deflection from a straight line due to this cause is only one-quarter of the above, or 100 meters.

Theoretically and actually, the instrument breaks down completely when the sun is in the zenith, which means that when flying in the tropics there will be a brief time around noon when the instrument is inoperative. Ordinarily, however, in the tropics, photography must be done early in the day before the cumulus clouds commence to form. Thus, there is little likelihood that there would be any occasion to use the instrument during that period. Practically speaking, the instrument may be successfully used until the sun gets within about 10° of the zenith. This means that on one day a year in any tropical project there would be a period in the middle of the day of about an hour and a half when the instrument could not be used. On this day from soon after sunrise until about 11:20 in the morning the operation would be very satisfactory.

A mosaic that accompanied the article shows remarkably successful results in straight flying and coverage without gaps of a quadrangular area comprising about 300 square miles in arid western country.

56. Signal apparatus.—Forewarning that an exposure is about to be made is of material aid in vertical photography and in oblique photography, too, when exposures are being made at regular intervals. There are two points to this warning: The pilot can make sure that the ship is held steady at the instant of exposure, and the photographer can give particular attention to leveling the camera immediately before exposure. In automatic operation by intervalometer the signal is of more value to the photographer than when he is making the exposures by hand, for in the latter case it is necessary for him to keep track of time by the stop watch. The greatest value of preexposure signals comes from their aid to the pilot. It is difficult for a pilot to set the ship exactly right with crab to follow a long straight course. From time to time he finds it necessary to pull over because of drift. To get back on line it is better practice to jog over a little abruptly, then go on the original bearing again, doing this between two exposures in a few seconds, rather than to take a longer time and haul over with a gentle turn. If considerable adjustment to get back on course is necessary, several jogs may be made between exposures. The signal permits him to make such corrections in the flight line without interfering with proper leveling of the camera and without changing its orientation at exposure, thereby keeping the photographs squared to one another. The risks of exposure when the camera is greatly tilted and of losing the desired overlap between consecutive photographs because of change of orientation of the camera are avoided. Slight jogs with immediate return of the ship to its original bearing have only the effect of setting a photograph to one side a little with respect to the one preceding it. This takes little from the usefulness of the pair affected, whereas, if failure to obtain sufficient overlap on one side of a pair occurs, that pair will be unserviceable for most mapping purposes.

The simplest form of signal apparatus will usually suffice. This may be merely an electric lamp in a circuit with a switch placed conveniently for the photographer to throw on and off. He can then switch the lamp on a few seconds before an exposure is to be made and turn it off immediately after exposure. Should the pilot be righting the plane, a warning of 3 to 5 seconds before exposure will give him time to straighten out before the next exposure is to be made. A more elaborate arrangement of the apparatus consists of connecting the switch directly to the shutter trigger, which will turn the light off on being tripped, and having an automatic control to turn the light on after an interval of time equal to a few seconds less than the exposure interval. Such apparatus should have an interval regulator to permit settings to be made to accord with the exposure interval. The intervalometer (Art. 53) is equipped with automatic signal lamps.

57. The statoscope.—This instrument was devised to supplement the altimeter, and its particular function is to guide the pilot along a given level, making it possible for him to detect quickly and sensitively small variations in altitude. It has been made in various forms. In one form it consists of a manometer joined with a flask of air held at a constant temperature and having an interposed valve, which permits it to be opened (made ready) for operation at any height, whereupon it will indicate quite small changes from that level. Its sensitivity is such that a change in height of 20 feet at 10,000 feet above sea level causes a change in reading of about ¼ inch. It is therefore capable of being read to detect changes in height of 5 feet or less and to permit a pilot to fly a course with but small deviations from a starting level. The flask is surrounded with a packing of ice and water which is held in a vacuum container. The heights of two columns of colored and light antifreeze liquid contained in a U-shaped tube joined to the air in the flask form the indicator proper. The differential height range of the instrument for any starting level is necessarily limited by the mechanism. The valve provides for opening all branches of a T connection or for closing off any one of them. These arrangements are sufficient to set the instrument for ascending, descending, operation, or ordinary transportation. The instrument is made in two models: one as a visual indicator for the pilot, the other for recording. The recording statoscope should be used along with a visual one for the pilot.

Results[1] of the use by Brock and Weymouth of a recording statoscope designed by the National Advisory Committee for Aeronautics indicated a probable accuracy of 0.0005 of the air-station height in about 75 per cent of the tests, between 0.001 and 0.0005 for about 12 per cent, and more than 0.001 for about 13 per cent. The statoscope was calibrated for work at 10,000 feet, the flights being made at that level. The differences noted correspond to a range from 5 to about 18 feet. The experiments were made during 1928 and 1929, a sensitive altimeter being used by the pilot to keep at the chosen flight level.

58. Oxygen apparatus.—The maximum working altitude for men accustomed to living at ordinary land levels differs greatly with individuals. Even though it might be quite safe for young men in good physical condition to perform mild work at fairly high altitudes (15,000 to 18,000 feet), it is certain that after a time the mental processes will be slowed and precision of operation will fall off. Oxygen apparatus rigged for breathing the gas when needed provides for making flights at high altitudes without danger to health and with maintenance of high mental and physical efficiency. Oxygen is ordinarily taken in tanks equipped with hoses and valves. For moderately high altitudes (15,000 to 25,000 feet) it may be breathed at intervals as required through the mouth. For very high flying, masks should be available for continuous use. As a rule, sustained work should not be done at 15,000 feet and higher without oxygen for breathing. Long hours of work at 12,000 to 15,000 feet without oxygen may result in pulmonary disease if chilling is severe. Figure 9 shows an oxygen apparatus designed for comfort and natural breathing through the nose.

59. The surge unit which works the electric shutters of the T-3A camera is necessary for operating that camera. It requires an electric circuit of 12 to 14 volts giving 15 amperes at the unit. This is usually obtained with a standard 12-volt battery taken along for the purpose or from the ship's electric-supply line. The wiring for operation of the surge unit must have a diameter commensurate with its length to assure holding the voltage up to the essential value of 12 volts. If the insulated cable furnished with the unit is employed with a completely charged standard storage battery, there will be no danger of shutter failures from faulty installation. If the supply line of the airplane is to be used, its installation should be checked and its adequacy to operate the shutters along with any other duty tested to make certain that the surge unit will receive the amount of electricity required. A connected voltmeter should show 12 volts, and inspection should determine that all shutters are working when the trigger is thrown. The surge unit should always be placed with right side up (dial on top), and it should not be placed close to a large mass of metal. The box that incloses the unit carries the shutter-speed-regulating mechanism and a dial with the numbers 40, 60, and 80 on its surface. A knob provides for setting any one of these numbers at an index mark. The numbers mean $\frac{1}{40}$, $\frac{1}{60}$, and $\frac{1}{80}$ second, and the number opposite the index mark is the speed at which the shutter will be operated. The box is 6½ inches wide, 14 inches long, and 7¼ inches high and weighs 28 pounds, including the cables furnished with it.

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[1] Wood, Edward S., Jr., "Experiments with the Statoscope," *News Notes of the American Society of Photogrammetry*, April-May-June, 1936.

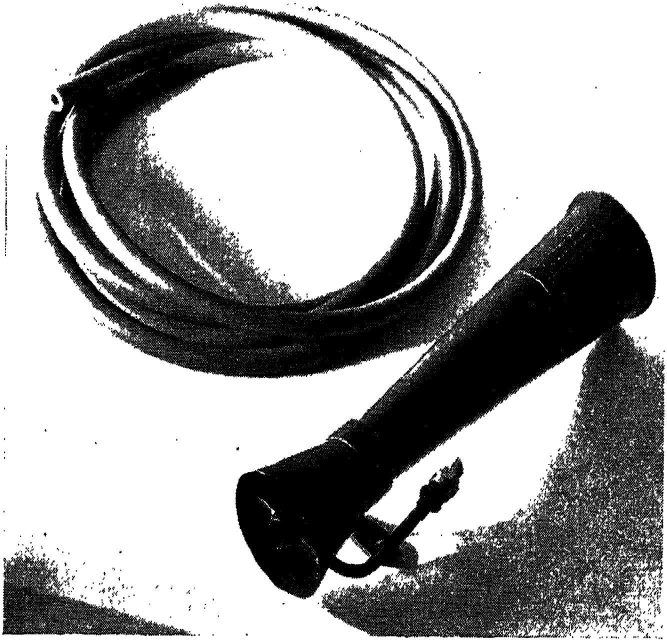


FIG. 20.—Venturi tube and hose. (*Courtesy of Fairchild Aviation Corporation.*)

60. Additional accessory equipment.—Following is a list of additional equipment commonly used in aerial photography: thermometer, temperature-altitude correction chart, Venturi tube, blower, electric battery, extra stop watch, extra loaded magazines (or in some cases extra rolls of film), cloth to cover camera at take-off and extra color filters. A horizon camera, or horizon mirror, and an auxiliary camera to record flight data are sometimes used. A Venturi tube with hose is shown in Fig. 20.

CHAPTER III

LENSES, COLOR FILTERS, AND SHUTTERS FOR AERIAL CAMERAS

61. Requirements of lenses for aerial photography.—To be satisfactory for aerial photography a lens must admit enough light to make a normal exposure under prevailing light conditions and must cover a reasonably large field (the larger the better); must be accurately and securely made up in its barrel; must give acceptably sharp definition throughout its field; must be highly corrected for distortion; must be well corrected for the several aberrations lenses are subject to; and must be reasonably free of physical defects, such as air bubbles and striae. In designing and making such lenses a maximum of skill is essential. Only manufacturers of the highest attainments in design and workmanship can produce lenses capable of meeting these requirements. That manufacturers in the United States do turn out lenses that qualify highly for aerial photography and photogrammetrical uses is fully confirmed by the results of tests at the National Bureau of Standards and by results obtained in actual surveying with them.

Before the present-day extremely rapid photographic emulsions became available commercially, it was necessary to employ the speediest obtainable lenses f/4 to f/5, which were sufficiently corrected to serve the purpose adequately. Since there are now available highly sensitive films and plates, some of which are six times as fast as the ordinary panchromatic, the speed requirements of lenses have been materially modified. Under present conditions slower lenses, and consequently lenses of established quality having wider fields, may be used with satisfaction.

62. Aberrations and defects of lenses.—The principal aberrations and defects in photographic lenses are

1. Loss of light from absorption and reflection
2. Chromatic aberration
3. Spherical aberration
4. Coma
5. Curvature of field
6. Astigmatism
7. Distortion
8. Nonuniform illumination

9. Flare
10. Incorrect assembly of the lens in its barrel (mount)

63. Absorption and reflection.—Loss of light from absorption and reflection from their surfaces is considerable in the complex lenses required for aerial cameras. In some lenses it may run as high as 50 per cent, and seldom is it less than 30 per cent. It is higher in lenses whose parts are not cemented (air spaced), because reflection accounts for greater loss than does absorption. However, transmission efficiency should not be considered alone. Under certain conditions other qualities may be more important.

64. Chromatic aberration.—White light is dispersed on passing through glass so that the rays of the separate colors, being of unequal wave length, do not hold together but pass along their several paths. Correction is accomplished by combining a positive (converging) and a negative (diverging) lens, the positive lens having a slightly higher refractive index and the negative lens higher dispersion quality. The two opposing elemental lenses are selected with respect to two primary colors that have the greatest effect on the sensitive surface of the plate or film. For panchromatic emulsions, which are sensitive to the longer rays of light (red end of spectrum) and which usually are to be employed with color screens cutting off all of the blue end of the spectrum, correction with respect to green and red would theoretically produce the desired results; for the orange and yellow between should not be too much dispersed to be objectionable. Yellow is the color to which normal eyes are most sensitive. It is therefore the most important color for visual focusing, but this is of little importance in cameras that are focused once for all use.

65. Spherical aberration.—No method appears yet to have been discovered whereby spherical aberration can be entirely eliminated from photographic lenses. Partial correction, which in lenses of the highest grade is satisfactory, can be accomplished by opposing positive and negative lens elements as is done to correct for chromatic aberration. It is customary for lens designers to correct precisely for spherical aberration in certain parts of the field, leaving the aberration in other parts as small as practicable. So even the best of lenses produce images on a flat surface which are more sharply defined in some parts of the field than in others.

66. Coma.—This is a phase of spherical aberration occurring in images formed by oblique rays. Here the blurring is not circular, as for axial rays, but pear shaped. In lenses consisting of a single piece of glass it is most pronounced in the outer parts of the field. If, in a complex lens, correction is made for coma along with that for spherical aberration in general, the result will give an image having residual aberration, which is termed *zonal aberration.* Thus there will be somewhere in the field a circular ring of incomplete correction wherein the image will appear less sharply defined than in the fully corrected parts.

67. Curvature of field.—Elemental lenses produce images not in a plane but in a curved field. A positive lens produces a field that is concave toward the lens; a negative lens, one that is convex toward the lens. In complex lenses compensation is made for curvature of field by use of both positive and negative primary elements. The correction for curvature of field goes along with those for chromatic and spherical aberrations, but it must be obtained through a compromise with the correction for astigmatism.

68. Astigmatism in a lens renders it incapable of producing sharp images by light passing obliquely through the lens to the marginal parts of the field. The effect is peculiar. It prevents sharp definition of lines in two directions at the same focal distance. This may be seen in the image of a slender cross or any object having two systems of lines at right angles to each other. When one system of lines is in focus, those of the other will appear to be poorly defined. By changing the focus of the lens, the effect may be reversed between the two systems of lines. Special glass has been the principal agent in the production of lenses that are well corrected for astigmatism.

69. Distortion.—The most important single correction in lenses for use in photogrammetry is that for distortion. It is imperative for accurate work, whenever measurements are to be made on photographs, that the displacements in the image be kept to inconsequential proportions. Lenses are said to be affected by "pincushion" distortion when the images of straight lines are curved so as to appear convex toward the center of the picture. When images of straight lines are concave toward the center, the effect is called *barrel distortion.* Distortion may be described otherwise as unequal magnification in the photograph. It may be kept to minor magnitudes by proper balancing of opposing elements, one in the front cell, the other in the rear. This is satisfactorily accomplished in high-grade lenses. In some photogrammetric uses, distortion brought into the image by the lens of the aerial camera is compensated by use of a projector lens which produces a like distortion in the opposite sense.

70. Nonuniform illumination.—It is inherent in photographic lenses, which are required to cover much greater fields than those in telescopes, that the intensity of illumination varies considerably from the center to the margins of the field, and the greater the field the more pronounced the difference in illumination. This is a serious defect in wide-angle lenses (field circa 90°) because the latitude of photographic emulsions is in no case great enough to take care of the great range of light intensity between axial and marginal rays. Moreover, the falling off is so great

lines parallel to the optical axis of the lens, is neither obstructed by the lens nor greatly diminished by reflection from the front surface.

The lens of Fig. 21 is of moderate field. It is quite probable that a wide-angle lens such as the Metrogon of Fig. 24 will transmit a somewhat higher percentage of the light to a point in its field corresponding to P (one in the 35° zone), for there would be little loss by obstruction, although a small part of the lens would be entirely ineffective and there would be high loss from reflection in a considerable part.

An example will be analyzed. Suppose the field of the lens to be 90°. The intensity at the margin of the plate because of the greater distance from the lens compared with that at the center would be 50 per cent, the distances being in the ratio of $\sqrt{2}:1$; and the intensity inversely as the squares, or 1:2. The circular aperture being in effect reduced to an ellipse for the oblique rays causes a further reduction of about 30 per cent, leaving 70 per cent of the remainder to pass through. The loss by reflection from the front surface and obstruction combined would probably be between 40 and 50 per cent. The amount getting through will be taken as 60 per cent. Finally there is the loss because of the obliquity of the rays, which amounts to about 30 per cent, leaving 70 (sine 45°) per cent effective. The product of the four effective percentages is the total percentage of illumination compared with that at the center of the plate. This is $0.50 \times 0.70 \times 0.60 \times 0.70 = 15$ per cent nearly. Although this is by no means an accurate determination of the relative intensities of light, it is a reasonable estimate. It may be several points in error, but it at least establishes the fact that the range in illumination greatly exceeds the latitude for normal exposure in films and plates.

71. Flare.—Unless the interior parts of a lens mount and those surfaces of the barrel immediately in front and back of the lens proper are completely blackened, light may be reflected from them to the plate, causing luminous spots which mar the picture. These defects can be readily corrected by proper application of black nonglossy lacquer.

Flare which results from reflections from surfaces of the lens elements is much more serious. Secondary images occur from this type of flare; so unless the lens has been designed to prevent objectionable reflections of this sort from reaching the plate, it will not be satisfactory. Because there are more reflecting surfaces in lenses having air-spaced elements than in those which are cemented, the former are more liable to this form of flare. In lenses of high quality, it is so subdued that it is seldom noticeable.

72. Incorrect assembly of the lens in its barrel.—Each piece of glass of the assembled lens should be brought accurately into that position which assures alignment of its center of curvature with the center line of the lens barrel. The test required to determine whether or not a lens is correctly assembled can be made only with proper laboratory equipment and by a competent physicist. The National Bureau of Standards is equipped to make such tests. In purchasing a lens it would be well to specify that it be so accurately mounted that the optical axis deviate not more than 1 minute of arc from the physical axis of the barrel and to require a report of the test substantiating the fact. Tests have shown that some photographic lenses are lacking in this respect, that instances have been found where the optical axis differs from the physical axis by as much as 3 or 4 minutes of arc. Such a derangement is not serious for ordinary photography, but in photogrammetric work errors admitted through such wide tolerance are appreciable and difficult to overcome.

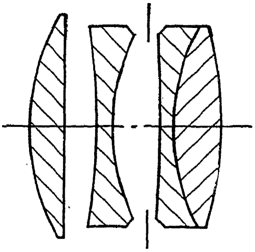


FIG. 22.—Section through the Bausch and Lomb Tessar f/6 lens. (*Courtesy of Bausch and Lomb Optical Company.*)

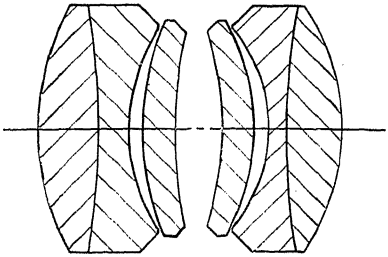


FIG. 23.—Section through the Bausch and Lomb Altimar lens. (*Courtesy of Bausch and Lomb Optical Company.*)

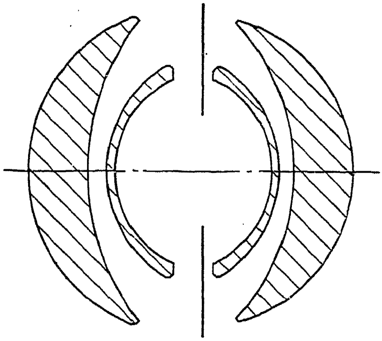


FIG. 24.—Section through the Bausch and Lomb wide-angle Metrogon lens. (*Courtesy of Bausch and Lomb Optical Company.*)

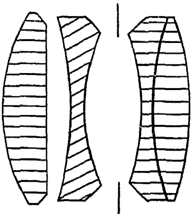


FIG. 25.—Section through an Eastman Anastigmat f/4.5 lens.

73. Characteristic forms of photographic lenses.—Because it is highly transparent and abundant, has the property of refracting light, and is a stable substance, glass is employed for making lenses. Moreover, its index of refraction may be controlled between certain limits. Each piece of glass (elemental lens) may be considered as made up of an infinite number of prisms, with all normals to the surfaces meeting at a single point.

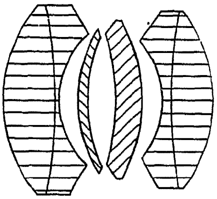


FIG. 26.—Section through a Ross 70° X-pres f/4 lens.

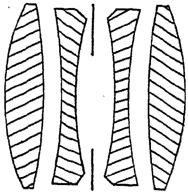


FIG. 27.—Section through Cooke Aviar lens.

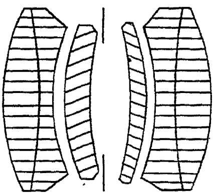


FIG. 28.—Section through Zeiss Aerial f/4.5 lens.

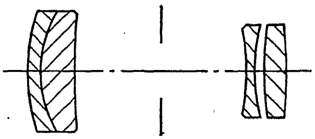


FIG. 29a.—Section through the Bausch and Lomb Telestigmat f/8 lens. (*Courtesy of Bausch and Lomb Optical Company.*)

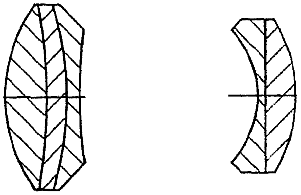


FIG. 29b.—Section through the Bausch and Lomb Telephoto f/5.6 lens. (*Courtesy of Bausch and Lomb Optical Company.*)

The common form of the modern photographic lens is that of two balancing components (cells) separated by an air space. This form not only permits the lens to be corrected for aberrations but provides for placing the shutter and the diaphragm at convenient and effective positions between the cells. There are two methods of bringing together the elements of a lens cell. One is to cement them with Canada balsam; the other is to leave an air space between. Loss of light is less with cemented surfaces, but for use in the tropics air spacing gives a more stable lens since, under prolonged exposure to humidity and heat, cement is liable to deteriorate sufficiently to disturb the original quality of the lens. Some lenses combine air spacing and cementing.

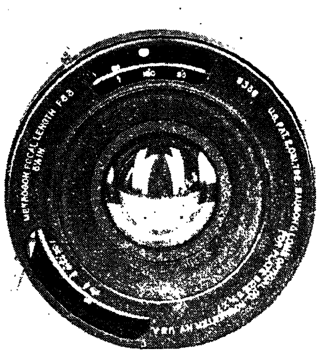


FIG. 30.—Bausch and Lomb Metrogon lens in heavy-duty Betax shutter. (*Courtesy of Bausch and Lomb Optical Company.*)

Both symmetrical and unsymmetrical lenses are employed in aerial cameras. Cells may be made up of one, two, three, or more pieces of glass. Figures 22 to 29*b* are sections of some typical lenses suitable for aerial cameras. Figure 30 shows the Bausch and Lomb f/6.3 Metrogon lens of 5⅛-inch focal length mounted in an automatic shutter, and Fig. 31 gives a front and rear view of the 6-inch Metrogon in a special mounting. In former years a high percentage of lenses of speed f/4 and f/4.5 and with focal lengths ranging from 8¼ to 12 inches were used in aerial photography, but the tendency now is toward wide-angle lenses with speeds from f/6 to f/7 and focal lengths from 5 to 6 inches. There is still a need for cameras of comparatively long focal lengths, but the great volume of aerial photography now lies in mapping and photogrammetry, which call for large coverage at moderate scale. The faster photographic emulsions of the last few years have made this trend feasible.

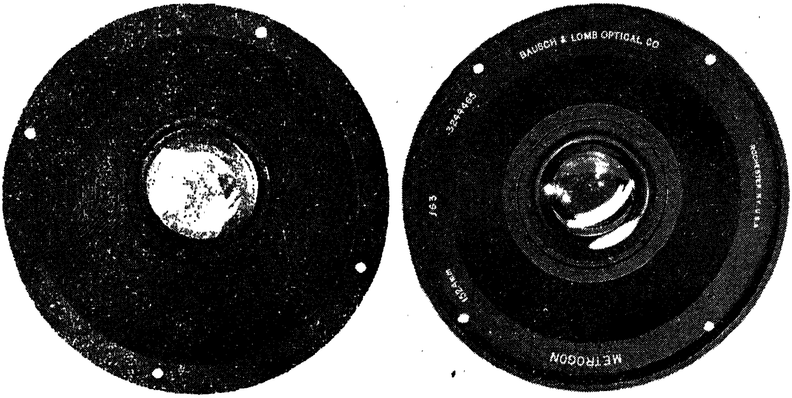


FIG. 31.—Bausch and Lomb Metrogon lens. (*Courtesy of Bausch and Lomb Optical Company.*)

74. Relation between color filter and film or plate.—The effect obtainable by the use of a particular color filter depends directly on the character and quality of the sensitive coating used in making the film or plate as well as on the properties of the filter itself. It is necessary, therefore, to state the photographic characteristics of this coating in order clearly to understand the function of the filter. For aerial photography, panchromatic emulsions, especially the supersensitive types, are so superior to the earlier orthochromatic and ordinary types that only film of this type will be considered here. It is intended to discuss this subject only so far as necessary to establish the relationships between film and color filter.

75. Panchromatic films.—As the name implies, panchromatic films are sensitive to all the visual colors of the spectrum, and the range of a particular type may extend into both the ultraviolet and the infrared regions, which are beyond the range of the human eye The most serviceable type for general use in aerial photography is that which has nearly equal sensitivity to all the visual colors from violet to red. Figure 32 shows the range and relative sensitivity of such a type of film compared with the range of sensitivity of the human eye to the colors of the spectrum. It is apparent from the diagram that this type of film will give a better registration of the various colors of a scene than can be obtained visually, except that in ordinary photography the colors do not appear in their several hues but only as tonal gradations between white and black.

Figure 32 is intended only to show the range of sensitivity of panchromatic film and of normal human vision along the spectrum, *i.e.*, horizontally. It is not intended to give a true comparison of intensities at any part of the spectrum (vertically). Commercial panchromatic films vary considerably in their range and receptivity to rays of light coming by reflection from objects differing in color. Some may be especially sensitive to red; others to yellow or green; but all are sensitive to blue and violet rays which generally have the effect of interfering with the longer rays of light. The chief function of the color filter in aerial photography is to act to screen these shorter rays wholly or in part from the film, thereby permitting the longer rays to register without objectionable interference.

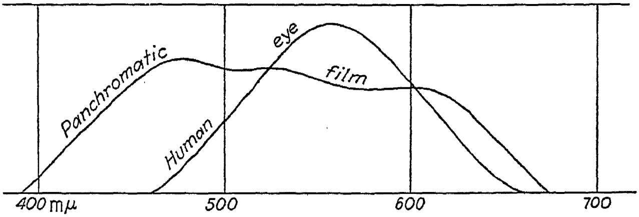


FIG. 32.—Range of sensitivity of the human eye and of a type of panchromatic film to colors of the spectrum.

76. Types of color filters.—There are several groups or types of color filter which are classified according to their purpose and power of selectivity to stop or transmit light. First are those intended to select and pass such rays of light as will give to the negative a range of tones comparable to the brilliance of colors seen by the eye when looking at the subject. Second are those which accentuate contrast and therefore aid in producing photographs with great range in tone between high lights and shadows or light and dark objects. Third are those whose purpose is the stopping of all but one color. These are useful for photography in color and if sufficiently effective would be valuable in photolithography to separate colors of a map in making printing plates. Finally there are those with which aerial photography is chiefly concerned: filters that permit clear photographs of terrain to be obtained through haze. The "smoked-glass" type of filter also has a place in special and unusual photography, as when photographing luminous objects.

77. Color filter required to obtain a clear picture.—In aerial photography, obtaining the clearest possible picture of the scene is usually of prime importance. Without a proper color filter this goal seldom can be realized, because

(*a*) Distant objects are partly veiled by moisture and dust particles (haze) which reflect and scatter the short rays of light. These rays have a strong effect on the film, acting like a veil between the camera and the scene to be photographed. The result is that the haze itself is caught in the picture at the expense of the scene desired.

(*b*) All available commercial films are highly sensitive to the shorter rays of light.

For mapping use and for other uses in which all details of the terrain are relatively important, photographs that show those details best are the most satisfactory. In pictorial aerial photography, where the purpose is to obtain arresting effects, emphasis is placed on contrast between high lights and shadows as well as on composition designed to set out and draw attention to the important part of the subject. Here, again, the color filter is necessary to obtain the desired result, but in this case a color filter that screens out not only the shorter rays but also some of those of intermediate lengths is generally used. In attempting to obtain attractive and unusual effects with certain subjects, such as snow-covered peaks, ridges, and their shadows, it is best to employ a deep-red filter with film sensitive to infrared rays.

78. Forms of color filter.—There are three general forms of filter: (1) dyed gelatin disks, (2) colored glass, (3) cells containing liquid dyes. The last mentioned form seldom is used in aerial photography, so consideration need be given only to the first two. In the United States the gelatin disk cemented between two flat pieces of optical glass is the form of filter that has been most generally used in the past. These cells are fastened on the front of the lens. They must be made with plane-parallel glass and mounted with the plane of the filter at right angles to the optical axis of the lens. A less expensive but less desirable method of employing the gelatin filter is to cut a disk of proper size for insertion somewhere in the lens itself. When this is done, care must be taken to prevent the disk from interfering with the proper spacing of the lens parts and to avoid finger marks becoming stamped on the gelatin in handling. Moisture penetrates the lens and eventually renders such disks unsatisfactory. When this occurs, the disks must be replaced. The second form (of colored glass) has not been so extensively used in the United

States as the first form. It avoids the danger of deterioration, which is a defect of the gelatin, but has not been quite so thoroughly standardized or made so readily available as the dyed gelatin. Figure 33 shows an excellent arrangement for attaching a filter in a cell on the front of a lens. Slots in a ring extension slip over lugs and permit the filter to be turned into position where a spring clip holds it securely in place. It may be removed or replaced in 2 or 3 seconds.

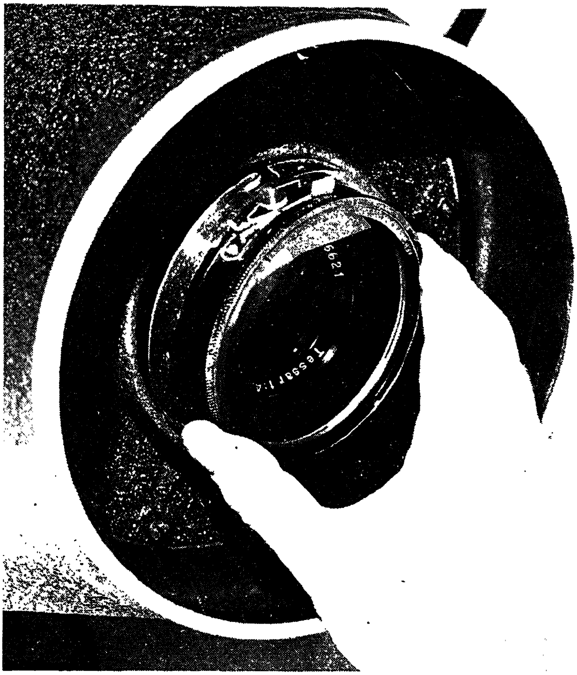


FIG. 33.—Attaching Fairchild bayonet-type filter to aerial camera. (*Courtesy of Fairchild Aviation Corporation.*)

79. Selection of the proper color filter.—Aerial photography often requires that exposures be made with fast shutters. Medium apertures give better definition than larger apertures. In order to keep within these desirable working limits, it is necessary to employ a color filter that has a low factor, *i.e.*, one that does not greatly increase the exposure time or does not require the use of a large aperture. Fortunately, with supersensitive panchromatic film suitable color filters for general use can be obtained with low factors. Means of testing filters are not readily available, so reliance should be put on the filters recommended by the manufacturer of the camera or film to be used. Some manufacturers offer color filters in sets. A set suitable for aerial photography as selected from the numerous Wratten filters so widely used is as follows:

WRATTEN-DYED GELATIN FILTERS IN GLASS CELLS

Designation	Color	Factor when used with supersensitive panchromatic film
Aero 1	Pale yellow	1.5
Aero 2	Yellow	2.0
Minus blue	Yellow	2.0
Red	Red	4.0
Red	Red	8.0

Colored glass having properties that make it satisfactory for use as color filters is made by the Corning Glass Works of Corning, N.Y. Spectrophotometric tests of various tints and color densities show that the short rays of the spectrum are cut off sharply and that a high percentage of the longer rays are passed. Selection may be made to eliminate rays shorter than about 450, 500, 550, or 600 millimicrons as desired, 85 to 90 per cent of the balance of the light rays being allowed to pass through.

80. Types of shutter.—Three types of shutter have been employed in aerial cameras: the focal plane, the between-the-lens, and the louver. In the ordinary sense, when used with a comparatively wide slit, the *focal-plane* shutter is the most efficient of the three types, and it may be made to give nominal exposures over a much wider range and embracing much higher speeds than is obtainable with the others. Efficiency of the focal-plane shutter depends on the distance of the curtain from the focal plane, the width of the light slit, and the aperture of the lens diaphragm. Under common practice in the construction and placement of focal-plane shutters the efficiency at stops ordinarily used in aerial photography ranges from 80 to 95 per cent. Nominal speeds ranging from $\frac{1}{5}$ to $\frac{1}{1000}$ second are possible with it. It is the only shutter capable of arresting the image when there is great relative motion between camera and subject and the distance separating them is comparatively short. However, it is not so desirable as the others for use in taking aerial photographs to be used for surveying purposes, because for a given nominal exposure the total time required for the light slit to pass across the focal-plane frame may be long enough for the travel of the airplane to produce a considerable movement of the image and too long to assure steady aiming of the camera. The length of time required for the slit to make its passage for a $\frac{1}{100}$-second exposure may be as much as $\frac{1}{10}$ second, during which at a speed of 150 miles an hour the airplane will travel about 22 feet. Therefore, although exposures made through a moving slit will give sharply defined photographs, they may be seriously deranged perspectively. There is no way to tell how much they are so deranged.

In the United States the focal-plane shutter has been abandoned for aerial photography except in cameras used to take photographs intended for illustrations, news purposes, or special study involving close-ups which require high-speed nominal exposures. Use of the focal-plane shutter is generally prohibited in specifications for aerial photographs intended for mapping purposes.

81. The louver shutter in its commonest form consists of a number of narrow and thin slats joined together, like those of a window blind, and made to open and close in rapid motion. Its form presents a troublesome problem in design to make it perfectly lighttight but does seem to alleviate the problem of wear and tear at high speeds and in the larger sizes. In a second form this type of shutter is made up of fanlike leaves which cover an entire circle and radiate from a center. In this case the leaves are pivoted along one edge and are made to turn on that edge for the exposure. This form requires shorter slats than the first and is used for the larger sizes. It is, however, less efficient because its center piece increases the obstruction to light. In neither form is the efficiency so high as that of the focal-plane shutter; but if used in a size large enough to avoid obstruction to oblique rays, it closely approximates that of the between-the-lens shutter. It is usually placed inside the camera and as close to the lens as practicable. This type of shutter is not used on aerial cameras made in the United States but is favored in some European countries.

82. The between-the-lens shutter.—This shutter for many years has been favored above all others in the United States. Having become established commercially long before the coming of the airplane, it was at once used in aerial cameras. In its common form it consists of a round body which contains the actuating mechanism, several flat leaves which open and close to make exposures, an iris diaphragm, a tripping lever, a speed-setting arm, and openings into which the front and rear cells of a lens are inserted. There is also an arm that serves to set the diaphragm at any aperture of the scale provided. The chief merits of the between-the-lens shutter are: (*a*) It permits light to go to all parts of the focal plane at the same time. This allows sharpness of definition to be the criterion for perspective quality as well as photographic quality. (*b*) Its efficiency is satisfactory throughout its range of speeds but is higher for slow than for fast operation. In the best models efficiency ranges from about 75 to a little over 90 per cent for operation at speeds from $\frac{1}{200}$ to $\frac{1}{30}$ second. (*c*) Its practical range of speed, being from about $\frac{1}{50}$ to $\frac{1}{300}$ second for lenses of short and moderate focal lengths and from about $\frac{1}{30}$ to $\frac{1}{100}$ second for lenses of long focal length, is adequate for mapping purposes and for most of the general purposes except for photographing near objects when rapid movement of the camera or subject is involved. (*d*) It can be arranged for setting at open position. This readily provides for making calibration measurements or tests.

When carefully designed and well constructed of suitable materials the between-the-lens shutter will perform surely and accurately through a long life under severe temperature ranges. Figure 34 shows this type of lens-and-shutter assembly as made by the Fairchild Aviation Corporation and used on the cameras manufactured by that organization. One of these shutters, on a Fairchild K-3B camera owned by the Institute of Geographical Exploration of Harvard University, which has a range of speed from 1/30 to 1/150 second, has been operated more than 7,500 times during a period of about 8 years without a single failure. Breakdown tests have shown this type of shutter to be capable of giving a much higher number of operations than the figure mentioned. Another model of the between-the-lens type of shutter is shown in Fig. 30.

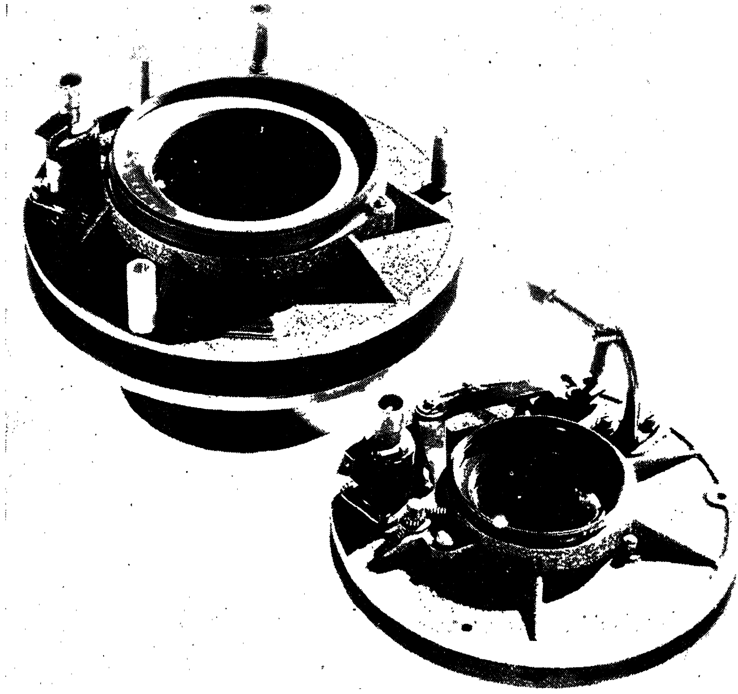


FIG. 34.—Fairchild Unit 1 between-the-lens shutter. (*Courtesy of Fairchild Aviation Corporation.*)

The between-the-lens shutters so far described are all operated mechanically. A departure from this form was taken in the design of the shutters of the T-3A camera (Art. 48) which are operated electrically to assure simultaneous tripping. They require the use of a 12-volt battery and a surge unit. Their quality is attested by the record over many years of service during which no shutter failure has been reported except in cases when the battery was allowed to run too low in charge on occasions of too great demand on it.

After considerable use the speeds of shutters are liable to change. It is therefore desirable to test shutters from time to time during their life so accurate information will be available of the actual speed for any setting. Tests may be obtained by sending the shutter to an accredited laboratory, or they may be made of between-the-lens shutters quickly at the locality of operation by use of a tester like that shown in Fig. 35, which is made by the Mark Hurd Manufacturing Corporation of Minneapolis, Minn. Its operation requires a standard 60-cycle 110-volt electric current, a piece of film or fast photographic paper about 3 by 15 inches in size, and facilities for developing. The speed is determined graphically from a scale provided with the instrument, and the efficiency can be calculated in a short time.

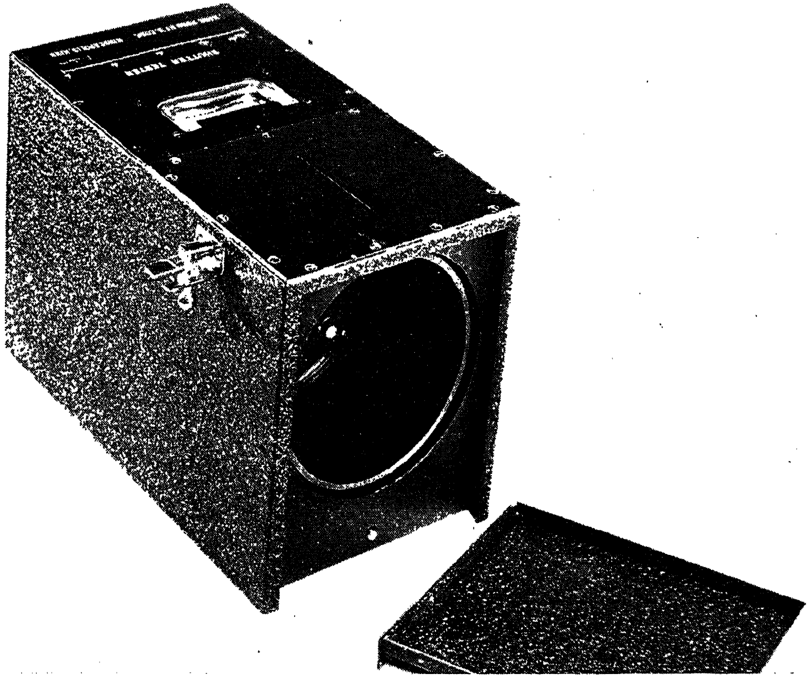


FIG. 35.—Mark Hurd shutter tester. (*Courtesy of Mark Hurd Manufacturing Corporation.*)

The shutter tester is 6¼ by 7⅛ by 11¼ inches in size and weighs 10 pounds.

CHAPTER IV

FLIGHTS FOR PHOTOGRAPHY—REQUIREMENTS AND PREPARATION

83. Requirements of the airplane.—(*a*) For taking obliques there must be a window or other opening through the side of the cabin and a clear view about 70 to 80° in angular scope laterally. If a window is used, it should be opened for making exposures. Exposures through a window glass should be avoided. Photographs so taken will not be suitable for mapping uses. An open cockpit ordinarily affords a satisfactory view between wing and tail on each side. Low or moderate travel speeds are preferable to high speed. Obliques of objects such as buildings, requiring shots to be made near by, impose no special performance capacity; but obliques taken of scenes remote from the airport or taken for mapping purposes in mountains may require special ceiling and cruising radius capacities as well as provision for installing a mount. Practically all air lines regularly operate aircraft that are suitable for oblique photography, so it is usually possible to engage an airplane for the purpose on short notice.

(*b*) For taking verticals, especially in moderate-scale mapping, the requirements of the airplane are somewhat exacting. An opening in the floor of the cockpit is essential, and it should be located to give convenient access to the camera for operation. The structure of the ship's framework should not interfere with cutting this opening at a suitable position. A vertical view finder, if separate from the camera, requires a second small opening close by the main opening. There also must be clear views ahead for the pilot. Vertical photography for mapping usually involves long flights at considerable altitudes; ceiling and fuel capacities should therefore be large. Most of the present-day flying for vertical photography is done between 10,000 and 20,000 feet above the ground, but the tendency is toward higher levels. A ceiling of 25,000 feet qualifies an airplane well for vertical photography in standard and exploratory mapping except of the higher mountain ranges—Andes, Alaska Range, and Himalayas. Longer exposures are permitted by moderate flying speed than by high speed. The use of proper shutter speeds, extra-rapid film or plates, and suitable color filters obviates to a large extent the disadvantage of high speed in the airplane. This factor is of only secondary importance except at 200 miles per hour and over. There should be ample room in the cabin for installations and freedom to operate. A space 4 feet wide and 6 feet long is enough for any of the standard cameras and accessories. Because of the particular features and modifications of airplanes required for installing the camera and accessories, special arrangements must be made at airports for vertical photography.

(*c*) Multiple-lens cameras require airplanes with retractable landing gear to avoid obstruction to views or the use of hydroplanes in which an opening is made in the hull of the boat. Otherwise the requirements are the same as for vertical photography.

84. The flight map.—The most serviceable type of map for use in making photographic flights for verticals and obliques at regular and short intervals along routes is the standard U.S. Geological Survey or Army mile-to-inch (1:62,500) or 1:125,000 scale topographic map printed in colors. These are available of much of the United States and of some of its possessions. Even those which are old and lacking in cultural details of recent years may be highly serviceable for making photographic flights. Maps at scales smaller than 1:125,000 are as a rule not satisfactory for photographic guide purposes. Standard maps are not likely to be available of remote regions. In this case it is necessary to utilize a less satisfactory map, and it should be the best one that can be procured from those available at the offices of organizations engaged in making maps or from agencies selling them. The Map Information Office of the U.S. Department of Interior can furnish information about maps of any area in the United States. The Library of Congress has a great collection of maps which is world wide in scope. The Institute of Geographical Exploration has a large collection of maps and information about the character of those available and the sources. Libraries in all large cities can furnish map information.

85. Map substitute.—In the absence of a suitable map it becomes necessary to prepare a substitute. This can be done satisfactorily by aerial photography, but it takes time and is expensive. In general the procedure consists of taking overlapping verticals along lines across the direction of flight for the photographs that are to be used for mapping. Such photographs should be taken at intervals of 10 to 20 miles, depending on the prominence of features or landmarks. The lines should be close enough together to permit landmarks appearing in the photographs of one line to be visible to the pilot when flying over the next line. The strips of photographs so placed are tied together by another strip extending at right angles from one to the other. These are sufficient to prepare map sketches of the areas covered, using the altimeter reading to give the scale, and to join these strip sketches together well enough to lay down the flight lines and to permit the pilot to follow them closely enough to assure getting the necessary side lap between photographs of adjacent flights. The sketches may be made well enough by laying the photographs in matched position on a large table, tracing off the prominent features, then reducing the tracings to the desired scale by pantograph or by photography.

86. Preparing the flight map for vertical photography.—If the area is large, the map should be subdivided into sections 15 to 30 miles broad. Boundaries of the sections should be placed at rivers or other lines that are plainly visible from a distance. It is important that those boundaries to be crossed by the photographic strips be well defined by landmarks that are to serve as references in bringing the ship on to the flight lines and as check references at the ends of flights. The sections may be 20, 30, or even 50 miles long as may be convenient, depending on the disposition of features that are suitable for use as border lines. To continue with this explanation it will be assumed that a subdivision of the map, which will be treated as a unit, is 20 miles broad and 40 miles long and that the photographs will be taken on lines laid directly across the area; *i.e.*, the lines will be 20 miles long. (The 15-minute quadrangle is a unit of area frequently used in making photographic flights. It is about 17 miles from the south to north border, the direction in which flights are usually made. In this instance a standard map of the quadrangle should be supplemented by parts of the maps of adjacent quadrangles, to the north and south, if available, the three pieces being pasted together to form a map extending across the borders of the area.)

87. Number of flight lines with given side lap of photographs.—Before the flight lines can be placed on the map, it is necessary to know the side lap to be given to the photographs, the height of the flight level aboveground, and the camera dimensions. The side lap is ordinarily made as small as the conditions imposed by the terrain, the map, and the skill of the pilot permit, keeping in view that it is objectionable to have gaps between strips and expensive to close them. A reasonably safe side lap is 25 per cent of the length of the photograph (the camera is usually set to bring the long dimension of photographs across the line of flight). It will be assumed that the flight level is to be 12,000 feet above the ground, that the focal length of the single-lens camera is 8¼ inches, and that negatives are 7 by 9 inches. Each photograph will therefore cover terrain that is 7/8.25 × 12,000 by 9/8.25 × 12,000 feet in extent, or about 10,180 by 13,100 feet. The net width of terrain covered by each strip will therefore be 75 per cent of 13,100 feet = 9,825 feet. To assure complete coverage it is necessary to allow the photographs of the outer lines to overhang the borders. This also will be taken as 25 per cent of the length of the photograph; adding that amount (3,275 feet) to the length of the terrain takes care of the overhang at both borders. The terrain to be considered is therefore 5,280 × 40 + 3,275 = 214,475 feet long; this divided by 9,825 feet gives 22 strips, nearly. It is safe practice to take the next higher number if the fraction of the figure resulting from this computation is greater than half a unit; to take the next lower number when the fraction is less than half a unit.

88. Method of placing flight lines on a map.—Figure 36 is given to show how the lines are now placed on the map. Starting at the right-hand border, the first line is drawn at a distance equal to 3,275 feet inside that border; then the last line is drawn at a like distance inside the left-hand border. The space between these two lines is now divided into 21 equal parts to give the positions of the remaining flight lines.

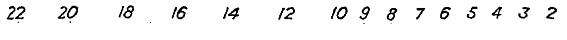


Fig. 36.—Placement of flight lines on a map.

If, as often is the case, the photographs are to be taken at a given approximate scale, the height level above ground for the flights is obtained from the equation Height = f/scale. Assuming the desired scale to be 1,450 feet to the inch, which is the same as 1/17,400,

$$\text{Height} = \qquad\qquad = 11{,}950 \text{ feet (approximately)}$$

89. Preparing the map for oblique photography.—This has to do with obtaining obliques of terrain, not obliques of particular objects. Obliques to be employed for mapping or for terrain studies usually require that a series of photographs be taken in orderly sequence or at regular intervals along a selected line or route and often with the camera held as close as practicable at a chosen angle to the horizon or to a vertical line, the purpose being to photograph a region or a considerable area according to a plan. In this case lines are drawn on the map to be followed and photographs are taken at regular intervals along the route. Additional photographs may be taken along parallel routes and along other courses across the first system of routes. The use of the map is then quite similar to the use for vertical photography. Instances of this use occur in the method employed in Canada werein obliques serve for making small-scale reconnaissance maps.

90. Preparation for taking special obliques.—Another use of the map for oblique photography arises when it is intended to obtain an oblique at a proper height with the camera held at a certain inclination to the horizon and it is desired to have the photograph include a given defined area. It may then be necessary to select an object shown on the map which the pilot will fly over on a certain bearing so that when that position is reached the required clear view will be available for taking the photograph. In this case it is necessary to allow for crab of the ship because of wind, the course being altered accordingly, and the photographer must have a line by which he will orient the camera for exposure the instant the position is reached. The preparation of the map requires that the height of the airplane, the scope of the camera, and the angle of obliquity be correlated to give the picture desired. Templets made for a particular camera held at various angles for exposures at various heights enable the proper position, orientation, and height to be selected and indicated on the map. This use of the map requires thorough cooperation between pilot and photographer.

91. Estimate of film, flying time, and magazines.—Photographing large areas systematically requires the making of careful estimates of film and flying time; for without the information thereby obtained there could be no assurance of having enough film on hand to complete the undertaking or to take advantage of a spell of favorable weather. Even for small areas there is need for an estimate, if solely to avoid unnecessary expense in the purchase of film. An estimate should be based on utilizing entire working days or in the case of small missions the completion of the job in a single flight. It will therefore be assumed that the airplane to be used has sufficient fuel capacity and cruising radius to stay aloft long enough to employ the hours from 9 A.M. to 3 P.M. actually on photography, or a total flying time of 8 hours, and that its cruising speed is 150 miles per hour. Such an airplane may not be the most economical to use under all circumstances; for example, if operation is from a near-by field, it might be better to employ a less expensive airplane of small fuel capacity, making two flights per day when weather and light are favorable.

92. Overlap between photographs of a flight line.—Referring again to Fig. 36 and the example employed wherein 22 flight lines were found to be necessary to photograph an area 20 by 40 miles in extent, it will be further assumed that the overlap between consecutive photographs is to be 60 per cent of the width of the photograph. The net gain at each exposure will then be 40 per cent of 10,180 feet, or 4,070 feet in round numbers. Each flight line is 105,600 feet (20 miles) long. The number of photographs required to stretch from border to border is 105,600/4,070 or 26; but to assure having the entire strip covered with stereo pairs of photographs and to allow for slight misjudgment in starting and ending photography, two extra exposures ought reasonably to be added at each end of a line. The number of exposures required for each flight line is therefore 30. As there are 22 lines, the total number of exposures required for the area is 660. At 100 exposures per roll of film, each roll will serve for 3 lines with enough film for 10 exposures left. The remnants cannot be used on other flight lines because a magazine cannot be taken off and another put on to replace it without producing a gap in the string of photographs. Either the remnants may be saved for oblique shots on another occasion, or the extra exposures reckoned to allow still another extra exposure at each end of every flight line. The total number of rolls of film for the area is 8. The last roll will be required for only 1 line, leaving 70 exposures available for rerunning any lines where gaps may have been left.

93. Flying time required.—The flight lines total 440 miles, but mileage must be added to each one to provide for bringing the ship into position before the point is reached for making the first exposure. Let this be 5 miles, or a total of 110 miles. Assuming that photographs will be taken in both directions, a preliminary run should be made over the area and back to determine the proper orientation of the camera and the exposure interval in each direction. This adds about 100 miles more. The grand total for photography is therefore 650 miles. At 150 miles per hour the time consumed will be nearly 4½ hours. To this, however, must be added about 10 minutes for changing magazines. With an airplane capable of cruising for 8 hours there would be left nearly 3 hours for travel to and from the area, allowing about ½ hour as a safety margin. The job could be completed in one day under the assumed conditions, even though the area were as far as 200 miles from the airport or landing field, but at that distance no time would be left to make extra runs to close possible gaps. At a distance of 150 miles there would be time to make two extra runs across the area.

The allowance of 10 minutes for changing magazines is based on having one magazine for each roll of film—eight in all—and making seven changes. Five magazines giving 150 exposures each would take care of the photography, leaving enough for three extra runs.

It should be stated that the results of these estimates leave no margin for delays in connection with finding the flight lines, in determining the crab of the ship, in setting the camera in proper orientation, and in determining the interval between exposures. The estimate is based on the job being done by experienced personnel and the terrain presenting no difficulty for finding landmarks by which the flight lines can be identified and followed. Areas that have been sectionized with land lines and have straight roads along section lines would offer the least difficulty. Arid and tropical jungle lands are the most difficult to photograph without gaps.

94. Comparative estimate with the T-3A camera.—Estimates for photography with multiple-lens cameras are made in the same way as for single-lens cameras. The five-lens (T-3A) camera photographs a strip whose width is about five times the flying height aboveground. At the 12,000-foot level, the width of the strip will be a little over 11 miles. An area 20 by 40 miles would require but five lines across its width, the side lap and overhang being nearly 2¾ miles (in terms of ground span), or three lines (two lines would give too little overlap) across its length, giving a side lap of 3 miles and an overhang of 3¼ miles. This arrangement was achieved graphically by trial and error. The number of exposures would be 150 for the five short lines and 180 for the three long lines. It would therefore be more economical to take the photographs along the shorter lines. In comparing the number of exposures here required with the 660 required for the 8¼-inch, 7-by-9, single-lens camera it should be understood that the scale of the photographs would be 1/24,000 for the five-lens camera and about 1/18,000 for the single-lens camera.

95. Estimate with two T-3A cameras together.—Two T-3A cameras mounted together (tandem) would require the same number of flight lines as for a single T-3A camera, but the number of exposures would be greatly reduced. By this arrangement of two multiple-lens cameras, a greater distance between exposures is traveled, yet the effective overlap may still be kept at 60 per cent or more. To obtain a 60 per cent overlap in the case being considered, the distance between exposures would be about 4.8 miles. However, it is better practice to have these photographs overlap more than enough for each to embrace the centers of four other photographs. This requires an overlap of about 80 per cent which, at a height of 12,000 feet, gives a distance between exposures of 2.4 miles (roughly). Each of the 20-mile flight lines would therefore require 12 exposures (allowing two extra at each end), and the area a total of 60 exposures.

96. Estimate with the nine-lens camera.—The nine-lens camera would require about the same number of exposures as the two T-3A cameras together, *viz.*, about 60 at 12,000 feet above ground, to photograph a rectangular area 20 by 40 miles in size. With this camera the scale of the photographs would be the same as the scale of the single-lens photographs of the example—about 1/18,000.

97. Estimate with a wide-angle single-lens camera.—A wide-angle single-lens camera, having a focal length of 6 inches and giving a 9 by 9-inch negative, used at 12,000 feet above the ground, the side lap being

25 per cent and the end lap 60 per cent, would require 304 exposures to cover a 20- by 40-mile area, allowing 2 extra at each end of every flight line. Each photograph would cover a square 18,000 feet on a side. The number of flight lines required is 211,200 + 4,500 divided by 13,500, which is 16 (nearly). The number of exposures per line is $^{105600}/_{7200}$ + 4, or 19.

98. Summary.—In the example used less than half as many wide-angle single-lens photographs would be required to cover the area adequately for mapping as of the 7 by 9 photographs of $8\frac{1}{4}$ focal length, which have been much used for mapping in the past. This advantage would not be so great in every instance. It is reduced somewhat for smaller areas; and if the scales of the photographs are to be the same with each camera in the example used, the relative number of photographs would be approximately as $^{406}/_{660}$ in favor of the wide-angle camera, instead of $^{304}/_{660}$.

The maximum advantage by reduction of the number of exposures is obtained with the nine-lens camera; but leaving out consideration of scale, this advantage is closely approximated by use of a pair of T-3A cameras together.

Other items besides the number of photographs must usually be considered. These are of concern in connection with the use to be made of the photographs. At present, for photogrammetric uses, including the contouring of standard maps, the wide-angle single-lens photograph is preferred above all the others; but for planimetric mapping, exploratory purposes, and the extension of control by means of the photographs themselves, the multiple-lens photograph has advantages that cannot be overtaken by the single-lens photograph. For making high-class controlled mosaics the single-lens limited-field photograph of rather large scale is best.

99. Loading magazines and installing camera in airplane.—It should be constantly kept in mind that photographic flights are expensive and that there is little chance to correct mistakes during flight. Therefore extreme importance attaches to having the camera in good condition, properly assembled, and properly installed. No matter what the type of camera, its functioning must be thoroughly understood. Inspection before loading should include testing and setting the shutter at the chosen speed, setting the diaphragm at the right stop, testing the film winding and shutter-tripping mechanism, and examining the focal planes and all rollers with which the film comes in contact. It is important that there be no burs or grit to score the film or cause unnecessary friction when it is wound into place. The front and back surfaces of the lens must be clean and bright; this is necessary for the color filter also. All interior parts and surfaces of the camera should be scrupulously clean and free

successfully done. No attempt will be made to describe the best procedure to do this beyond stating that the film should first be cut diagonally across and pushed under the backing plate while being held on its track. The magazines of the four oblique chambers have removable covers; loading them is easy. Even after sufficient practice in loading the camera in daylight, it may take all of an hour to complete the job in the darkroom. If there should be any doubt about having enough time to load this camera on the day when it is to be used, it should be loaded the day before.

101. Important cautions about loading any magazine or camera are: (*a*) Be sure that the roll of film and the empty spool are placed in their proper spindles. The empty spool must always be put in the spindle that is geared to the winding mechanism. (*b*) Be sure that the emulsion side of the film is brought toward the lens. In spooling film the emulsion (sensitized) side is always turned toward the core of the spool. The film should also be wound on the take-up spool with the emulsion side turned in.

102. Important points about installing a camera in an airplane for vertical photography are: (*a*) The camera must be securely held in place and be convenient for operation (if operated by hand). (*b*) It must be thoroughly insulated from vibration by shock absorbers. (*c*) Provision must be made for orienting the camera to compensate for crab of the airplane and for leveling it. (*d*) The opening in the floor must be large enough, and extraneous parts of the airplane, such as landing gear, must be arranged to assure an unobstructed view for the camera. The fourth point is almost insignificant with respect to single-lens cameras, because seldom does a problem of this kind arise with them. With respect to multiple-lens cameras it is insurmountable in some airplanes that otherwise would be highly satisfactory. These cameras require a larger opening than do single-lens cameras (an exception might arise when the single-lens camera carries its own view finder or is mounted with a view finder); and, having fields that range from 130 to 140°, they require a very wide space, free of obstructions to the view, below the ship proper. Retractable landing gear is usually necessary to make the land plane wholly satisfactory; the hull of the hydroplane provides the necessary view below, but it requires a special watertight door to close fast the opening for landings. Single-lens cameras of moderate field usually may be mounted an inch or more above the floor of the cabin. In this case it is unnecessary to remove the cover of the opening until after the ship has left the ground. If the arrangement is such as to require the lenses and front of the camera to be exposed to the opening, a cloth or other cover should be placed over the camera front to protect it from mud or flying sand and gravel during take-off. If work is being done where dust or sand is stirred up from the ground in considerable quantity, it is advisable to have the camera completely covered to protect it during take-off and for a few minutes afterward to give time for the dust to be cleared away from the cabin. Grit penetrates to the working parts of any camera, causing wear and interfering with smooth operation; lodged in the focal plane or rollers over which the film passes, it may seriously mar negatives or cause a jam.

103. Installation for oblique photography consists merely in mounting the swivel support unless automatic operation is to be provided; in this case a battery and cables are also taken along and placed where secure and convenient. The swivel mount may require that supporting framework be installed, to which the mount is fastened. Otherwise the mount may be fastened directly to the wall of the ship below the opening or to a window sill. The turret mount, which permits the camera to be swung from one side of the ship to the other, requires an open cockpit or a special design of a cabin not seen except in military airplanes.

104. Installation of accessory equipment.—The *vertical view finder* is placed immediately in front of the photographer's seat, between it and the camera, if practicable. It may be placed to one side or even behind the photographer provided it is within his reach for proper utilization. Its base piece is screwed to the floor of the cockpit, the view finder proper being joined to the base by lugs inserted in elongated recesses which hold it fast after it is given a short turning movement. Installation consists in cutting a 4½-inch circular hole in the floor and screwing the base round it. The opening in the skin of the ship opposite the hole in the floor should be a little larger, its character depending on the distance between floor and skin.

The *framework* that supports one form of camera mount is anchored directly to the floor of the cabin or attached to the walls. When attached to the framework of unlined cockpits it is fastened with special clamps.

The *intervalometer* can be mounted on the wall of the cabin in a position within reach and full view of the photographer. The *battery* must be placed where the cables can connect it with the intervalometer and camera. The *surge unit* used in operating the T-3A camera must also be placed for connection with the battery and camera. The *altimeter* may be strapped or hung in front of the photographer or slung in a case within his reach. *Extra magazines* are placed in compartments or recesses which prevent them from dislodgment when the ship is banked for a turn. *Signal apparatus* is mounted according to its arrangement, the pilot's lamp being in or near the instrument panel. The *Venturi tube* is usually attached to a strut, a wing, or the fuselage in the slipstream and where convenient to lead a hose from it to the camera. The inside *thermometer* should be placed near the altimeter so as to register the temperature round it; the outside thermometer is placed far enough from the fuselage to get it completely away from the heat of the engine. The *map* should be arranged for folding and unfolding readily. A convenient place to carry it is on the upper part of the leg where it can be held by a large rubber band round the leg. Here it is in position for easy reading and ready reach for adjustment. The *oxygen tank* must be securely attached in position for use and regulation of its valve.. The *altitude-correction chart* is carried in a pocket or compartment of the cabin.

105. Procedure for taking oblique photographs.—In taking an oblique photograph of a particular object the pilot's task is to fly past the object with the ship steady on a course that will permit the photographer to get an unobstructed view from a position to give proper lighting and an effective composition aspect of the subject and surroundings. The photographer's duty is to get the picture at the right moment. To take advantage of the momentary opportunity he must time the shot accurately, have the camera set to get a normal exposure, and hold the camera steady. The pilot should avoid making a turn or bank at the time the exposure is to be made.

106. Taking vertical photographs of an area for mapping.—On arrival at the area and after having reached the chosen height as indicated by the altimeter, the work consists of three parts: (*a*) computing the correction to the altitude reading and proceding to the proper level according to the corrected reading; (*b*) making a preliminary run; (*c*) making the photographic runs and exposures. In the last two parts there are several precautions that must receive particular consideration.

107. Precautions necessary on the preliminary run.—In making the preliminary run the tendency is to hurry through so photography may be begun and the job finished as soon as possible. Unwarranted haste is liable to result in poorly arranged photographs and gaps between the photographs of adjacent runs and in the end increase the flying time required to complete the work. Ample time (without waste) should be taken for the preliminary run to obtain the data required for making the photographic runs as accurately as practicable. Half an hour in some instances may be required to obtain satisfactory data. If photographic exposures are to be made running in both directions across the area, flight data should be obtained on preliminary runs made both ways. Experienced pilots can obtain on a preliminary run in one direction a bearing that will be sufficiently accurate for determining the required bearing in the opposite direction. However, this procedure should not be approved by the photographer unless he is confident of the pilot's ability to fly parallel photographic runs in both directions from data obtained in one direction only. Both pilot and photographer are responsible for success in the flight work. Each should keep this in mind and endeavor to cooperate with the other, having the utmost of success in view. Until a team becomes proficient in working together, it is best to take photographs in one direction only. If then a poor bearing is used, the actual run made may be askew to desired lines, but they probably will be parallel to one another.

108. Starting flights on photographic runs.—In making the photographic runs there is a tendency on the part of the pilot to get on a flight line too close to the border of the area so that the photographer does not have an opportunity to check the position of the ship with respect to the line to be followed, level the camera, and make the first exposure before reaching the border. The runs should actually begin 4 or 5 miles outside the border of the area and be extended 2 or 3 miles beyond at the end of the run. The photographer should insist on having enough time to check the ship's position with the flight line on the map before making an exposure.

109. Work on the preliminary run.—The duties of the pilot on the preliminary run consist in crabbing the ship toward the wind enough to make it follow the direction of flight lines and determining the compass bearing when the proper crab has been found. This involves use of the map and ranging the ship in with two or more landmarks ahead. In many airplanes the view ahead for the pilot is obstructed by parts of the fuselage and engine. A crab gives the necessary forward view and therefore in this respect facilitates the task. The process is necessarily one of trial and correction. The photographer assists by determining the drift in the view finder and telling the pilot its direction, whereupon a new bearing is tried, to be followed by another test and correction if necessary. Once the correct bearing has been found, the pilot flies on that bearing with a steady keel and at the corrected altimeter reading while the photographer determines the orientation of the camera and the interval between exposure. It is good practice to determine the exposure interval three or four times and to use the mean of the determinations. An extra stop watch or the stop watch of the camera and the view finder are used for this purpose. When flying over terrain flecked with many distinct details it is practicable to pick up at the leading crossline a second object as the first one crosses the trailing line of the view finder, a third object as the second crosses, and so on, taking the total time and dividing it by the number of objects used. If done well, this method gives an accurate determination of exposure interval in a minimum of time. It usually takes only 3 or 4 minutes to determine the interval, but over terrain having few details distinctive enough to catch one at the leading line when needed it may take much longer.

In Fig. 37, if the speed of the airplane is correctly represented by the line *AB*, and *BC* accurately indicates the direction and speed of the wind, it is necessary to head the ship toward B to fly along the line AC. If AC is a north line, the compass will show north to be east of the index mark of the compass, *i.e.*, the bearing will be west of north. To fly southward along the same flight line under the same speed and set of the wind the ship would have to be crabbed an equal amount to the west of south. Its speed relative to the ground would then be much greater than when flying northward.

110. Safe heights for flights without oxygen.—It is generally agreed that 15,000 feet above sea level is as high as men in good physical condition can work for several hours at a time without having to use oxygen. Flying for photography and operation of a camera do not involve strenuous exercise but do require sustained effort and the carrying out of the work with speed and precision. Individuals in subnormal condition run a risk in working at 12,000 to 15,000 feet above sea level, especially if chilled, of contracting pneumonia. Even though they escape serious physical consequences, efficiency in both mental and physical work is reduced by too rare air. It is probably a safe rule to use oxygen at 15,000 feet and higher for all work and from 12,000 feet upward on work lasting more than 3 hours. Although it is not strictly necessary to so safeguard the health at 12,000 feet, use of oxygen will materially assist in execution of the work by keeping (or reviving) one from sluggishness. It is dangerous for even a very hardy person to go as high as 20,000 feet without oxygen.

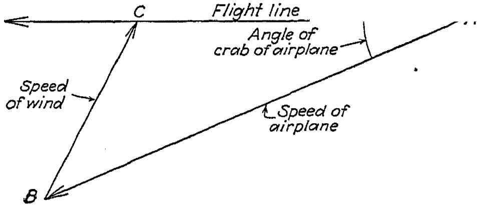


Fig. 37.—Diagram showing relationships between speed of airplane, speed of wind, and angle of crab of an airplane to travel a given course.

111. Basic normal exposure for aerial photographs.—It is well to have a practical basis established by experience to start from in dealing with the question of normal exposure. From this base, modifications can be made in accordance with knowledge of the intensity of sunlight, color filters, and photographic materials. The field of the lens, the aperture, the shutter speed, the color filter, the sensitivity of the film or plate, the character of the subject, and the intensity of the lighting—all these items affect the exposure. The basic normal exposure for this purpose is as follows: field of lens 60°; aperture f/5.6; shutter speed 1/100 second; minus blue color filter whose factor is 2; panchromatic film; general terrain scene with cultivated fields, woodlands, bodies of water, meadows, houses, roads, railroads, and other culture; midsummer at latitude 42°, the sun being between 60 and 70° high. This is for a vertical or low oblique on a cloudless day between 10 A.M. and 2 P.M.

Variations from this basic case will now be considered. There is no need to discuss changes of aperture or shutter speed, because their effects are given by a scale. If it had been desired to have the shutter speed $\frac{1}{200}$ second, the only change required would be to open the aperture to f/4; $\frac{1}{50}$ second would permit the aperture f/8 to be used. It might be well to state that neither of these changes would be desirable, for there is no assurance that either would have improved the photograph. The image might have been softened in either case: from lack of definition with the f/4 stop and from image movement with a shutter speed of $\frac{1}{50}$ second.

112. Variation in lighting.—First to be reckoned with is a change of lighting. The intensity of sunlight varies directly as the sine of the angle of elevation of the sun or as the cosine of the angle of incidence. This may be noted visually by looking at a steep slope in the early morning or late afternoon when the sun shines on it, the light coming from behind. That slope will be brilliantly lighted while gentler slopes and level ground also in the sunlight appear in comparison as though under a cloud. At 60° altitude the light is about 86 per cent as effective as it is at 90° (when directly overhead). There will be so little change in intensity between 60 and 90° that it may be considered that in this range of altitude the sun's factor is unity. At 30° the intensity is half that at 90°. A rule concerning lighting may therefore be stated: For altitude between 50 and 90°, make no change from the basic exposure; from 20 to 50°, use next larger stop, or half shutter speed; from 10 to 20°, use second larger stop, or one-quarter shutter speed. Below 10° the light fails so rapidly and shadows are so long that it is not advisable to undertake aerial photography unless necessary or to get an oblique shot of an object presenting a front to the sun.

In fact aerial photography for mapping should be avoided when the sun is lower than 30°. This rule must be broken in high latitudes during the winter seasons if photography must be carried out then. At latitude 42°N (vicinity of Boston) the sun is but 24½° high at midday on the shortest day of the year. Aerial photography should be avoided on cloudy days, but it may sometimes be necessary to make exposures on such days. Completely overcast skies reduce the light to about one-fourth its intensity on cloudless days. Masses of cumulus clouds prevent getting satisfactory aerial photographs, for in vertical photography they may completely obstruct some of the landscape and will shadow other parts.

113. Effect of the color filter in reduction of light intensity.—Second to be considered is the color filter. On very bright days when there is apparently no haze whatever, a lighter filter might give just as good results in verticals and low obliques as the minus blue. By its use either a smaller stop (f/8) or a faster shutter speed could be substituted. If the factor of the lighter color filter is 1.5 (compared with 2), the best use of this gain would be to substitute a shutter speed of 1/150 second for the 1/100 second. The basic setup would not permit the use of a denser color filter (factor of 4) without sacrificing definition.

114. Effect of the high-speed panchromatic film.—Third for consideration is the high-speed supersensitive panchromatic film. Since it gives two to six times the speed of the earlier panchromatic emulsions, there is an enormous gain to be utilized. It permits both shutter speed and aperture to be changed to improve definition; or if there is a top speed limit of 1/100 second to the shutter (as is the case in some of the cameras in present use), the stop may be changed to f/11, the definition being improved about as much as possible. The gain also may be utilized to employ slower lenses which are both less expensive and give greater fields.

115. Field of the lens.—Next for consideration is the lens of wider field. Unfortunately, as has been shown, the wider the field the greater the range of illumination from the center to the margins. Photographic materials do have considerable flexibility with respect to their reaction to light; but unless each negative is treated individually during development, it is not practicable to equalize the effect of exposure over the different parts. There is no satisfactory method of developing a long roll of film except to immerse it in a solution and let all parts of it be acted upon to the same degree during the process of development. The result is negatives that are normal in density in some parts, subnormal or more dense in other parts. If the range in density of the negative is not too great, printing may be accomplished by manipulation to weaken the light or increase it as required or by allowing the developer to act more on some parts than on others. Although it is fortunate that photographic plates and films do have this quality of flexibility (latitude) and it is possible to print successfully from negatives varying much in density, the difficulties that have to be coped with cause much labor and uncertainty in attempting to standardize printing. It is the price that has to be paid for the gain realized from the use of wide-angle lenses; the gain usually outbalances the labor and difficulties. In using extremely wide-angle lenses the best that can be done is to let enough light go through for the marginal parts of the film to receive the minimum amount of light necessary to get within the limit for printing and let the center be overexposed, in view of the fundamental fact that if insufficient light is given, no amount of developing will bring out a suitable image but an overexposure can be printed although it may take more time.

116. The subject.—In the last place there is the question of the subject. Farming land with its cultivated fields, farmhouses, and woodlands may be considered as the basic subject, for it is the commonest encountered. Unsettled forested areas as found in the tropics and northlands may be considered as requiring 1½ to 2 times as much light as the basic subject. In fact in nearly all vertical photography except of barren lands (deserts and arid regions) the subject may be considered basic. Deserts and arid regions may be considered as having twice or three times the brightness of the basic subject so that higher shutter speeds or smaller apertures may be used. It is in oblique photography that the great variety of subjects and great range of lighting are found. As a rule it may be considered that a given subject requires more exposure the nearer it is to the camera; but the corresponding exposure difference is not great. For example, there would be little difference between a distance of 2 miles and a distance of 5 miles, but the difference between 2 miles and 20 miles might be equivalent to one stop in the diaphragm scale. Between 2 miles and ¼ mile there would also be a difference of one stop. Snow scenes require much shorter exposures than are required by the basic subject. This is exemplified strikingly by photographs of high mountains consisting of snow-capped peaks, névés, and glaciers which are obtained by exposures with one of the new supersensitive films, using f/11 stop, 1/150 second, and a color filter having a factor of 4.

117. Lighting conditions required.—Vertical photography for mapping requires that the light be spread evenly over the terrain. This uniform lighting is obtained only when there are no clouds or when the sky is completely overcast and the clouds are above the level at which the photographs are to be taken. It sometimes happens that a thin layer of clouds in the levels round 20,000 feet above sea level do provide a possible opportunity for taking verticals of considerable areas, but the occasions are comparatively rare. Photographs taken when the sky is overcast are lacking in contrast because of the absence of shadows. They have a somewhat dull aspect but may have excellent quality for study of details and for mapping use. Ground fog and mist interfere with aerial photography; but haze, if not constituted of too great a percentage of smoke and dust, can be successfully penetrated by use of a proper color filter. Photographs taken through a thick layer of haze will not be so clear or so brilliant as those taken when there is little haze. Oblique photographs for pleasing effects should always be taken with the sun shining on the subject. An important point is to have the light coming from behind the camera (over the shoulder) or from a side. Shadows may be employed to heighten the effect. Competence on choosing the best position and angle for an oblique is attained through study of the value and use of shadows as items of composition. Photographs should never be taken with the sun's rays coming toward the front of the lens.

CHAPTER V

LABORATORY EQUIPMENT AND ITS USE

118. Film-developing equipment.—There are three types of film-developing apparatus that have been used in the United States: the *Eastman*, the *Fairchild-Smith*, and the *Stinneman*. Each has advantages over the others under certain conditions. Each is here described separately, its advantages and limitations being indicated in comparison with the others.

119. Eastman developing outfit.—This apparatus consists of four cylindrical, nesting, monel-metal tanks, one of which has a watertight cover, two large reels, a special celluloid apron with which the film is wound onto one of the reels, a changing stand with three spindles which serves for winding the film and apron onto the reel, a second stand on which the developing tank is placed for turning and holding upright or upside down, a special hook for lifting and placing the loaded reel in and removing it from the tanks, and a spring girdle which is used to bind the loaded reel. The developing tank is placed on the turning stand and locked in upright position before being filled with the solution. When wound up together, the film, apron, and bound reel are immersed in the developing tank which has been filled with the developing solution, the special hook being used to lower the reel into the tank. This is done slowly and carefully to avoid air bubbles clinging to the film. The watertight and lighttight cover is screwed on the tank; and after this is attached, the room may be lighted. At intervals of two or three minutes during the period of development the tank is turned over, locked in position, and left until turned over again. This assures uniform action of the developer on all parts of the film.

In getting ready for development, the roll of film is put in one spindle of the stand, the apron wound up in one of the reels in another, and the second reel (empty) put in the third spindle. The film goes in with its back against the apron, and to keep it in place its inner end is fastened to the apron with adhesive tape. The apron has a narrow corrugated strip along each edge. These strips separate the layers of the apron when it is wound up and keep the emulsion face of the film from touching anything but the solution. The apron is a few feet longer than the standard 75-foot roll of film, and it is about 10 inches wide. Film of any width from 9½ inches down and not more than 75 feet long can be developed in the outfit.

At the end of the development period the room is again darkened, and the reel with film and apron is taken out, rinsed in one of the other tanks, and then placed in the tank with the fixing solution. After the film has been in the fixing tank for about 10 minutes, the room may be lighted again and the rest of the work completed with light. When fixation is complete, the reel is taken out and given a thorough washing in the rinsing tank. At completion of the washing the changing stand is again used to unwind the film and apron, the film going to the dryer, the apron being wound on the empty reel. While the film is in the fixing tank, the hook is used to lift the reel up and let it down repeatedly to stir the solution and assure immediate and uniform action over the film. Fixing should be allowed to go on for 30 minutes.

The length of time that the film is to remain in the developing tank depends on the strength and temperature of the solution. This is given in connection with the formulas in Art. 140. It is important to rinse the film thoroughly between development and fixation to avoid rapid deterioration of the fixing solution and to assure complete fixation in the period of time the film is immersed in it. Final washing of the film should be with not less than 12 complete changes of water or in running water giving the equivalent.

The tanks nest together, and the largest is about 18 inches in diameter and 12 inches high. It takes 20 gallons of developing solution and about an equal quantity of fixing solution to fill the tanks sufficiently for operation.

120. The Fairchild-Smith developing outfit consists of three nesting stainless-steel tanks, a film-handling framework with two reels of the same metal, a film-transfer stand, and a small electric motor which winds the film from one reel to the other automatically. The outfit is available in two sizes: One accommodates film 120 feet long of any width up to 9½ inches; the other, film 250 feet long within the 9½-inch width. The tanks of the smaller outfit are 12 inches deep, 14 inches long, and 6 inches wide. They hold 3 gallons of solution. Figure 38 shows the 3-gallon outfit. The tanks of the larger outfit require 5 gallons to fill. To prepare for development the film is unwound slowly from its ordinary spool on to one of the reels of the handling framework, being taken up on the reel quite snugly. With the film entirely wound on to this reel, the loose end is fastened to the core of the other reel and a turn or two taken on it. The core of each developing spool has a special clamp which holds the end of the film in a tight grip. The handling framework with the spool held by hand firmly in place without slack is now immersed in one of the tanks filled with the developing solution, and immediately and rapidly the film is wound by hand from the first to the second spool, a knob on the top of each spool being provided for the purpose, care being taken to pay out the film without permitting slack to back up on the supply reel at any time. Unless care is taken to hold the film snugly on the supply reel, sticking may occur from penetration of the solution too soon into the inner layers of the film during the first change from reel to reel. As soon as all the film is transferred to the second reel, the winding is reversed to take the film back onto the first reel, the film now being allowed to slacken a bit on the supply reel. This winding back and forth is continued until development is completed. The purpose of the motor is to relieve the tedium of hand winding. In place on top of the framework it will carry on the winding, the action being automatically reversed from one reel to the other at completion of a cycle. It is not essential to use the motor, for the process of developing can be completed by hand, but operation necessitates continuous attention. Some users of the outfit recommend making the first transfer of film from one reel to the other in a tank filled with cold water, then placement in the developing solution. This is a

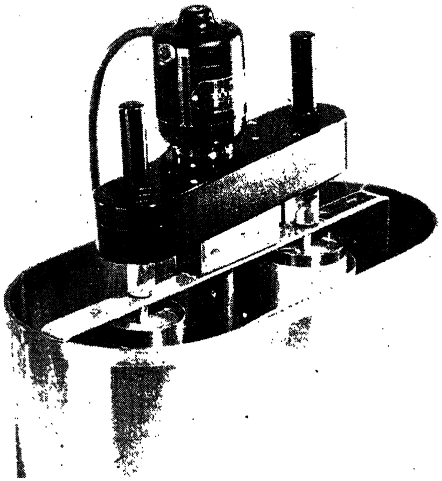


FIG. 38.—Fairchild-Smith roll-film developing outfit (F-47). (*Courtesy of Fairchild Aviation Corporation.*)

safer procedure than placing the dry film directly in the developer, for it permits rapid transfer from one reel to the other immediately on immersion of the film in the developing solution.

Precautions must be observed in the use of this outfit. (*a*) The film must be started on the roller properly to have it roll up smoothly. The emulsion side may be turned either in or out. (*b*) Extreme care must be taken in rolling up the film in the solutions in order not to bind it too tight. (*c*) The film must be kept moving to assure uniform development action on all parts. (*d*) The ends of the film for several inches are quite liable to be marked by pressure against the core. This is difficult to avoid; hence it is best to allow for it by having two or three blank frames at each end of the roll.

When development has been completed, the framework with film is transferred to another tank in which the film is thoroughly washed by putting it again through the rolling and unrolling process, the water being changed several times. After rinsing, the film is placed in the third tank filled with the fixing solution. There, too, the film is wound from reel to reel until fixation is completed. Fixing requires at least 15 minutes of manipulation. Final washing is done in the rinsing tank by rolling the film from one reel to the other in at least 12 complete changes of water. On completion of washing, the film is transferred directly from the framework reel to the dryer.

Developing solutions are ordinarily used only once in this outfit, but the action capacity of standard solutions is sufficient to develop two rolls in the 3-gallon tanks. In the 5-gallon tank the capacity of the developer is almost completely used up by 250 feet of 9½-inch film, so it is risky to use the solution more than once for that much film or its equivalent in two or more immediate runs.

121. The Stinneman outfit consists of two or more tanks, a special metal tray which has a spiral fin projecting upward from its base, a hook handle for moving and depositing the tray, a film-transfer stand which has a stud on which the tray is mounted for turning, and a bracket to hold a full spool so the film may be readily drawn for winding round the fin of the tray, clamps for holding the film fast in the tray after it has been wound in the spiral, and a special spool with one flange which is used to transfer the film from the tray to the drying reel at the end of washing. The film is fastened with one of the clamps to the spiral at its inner end, then wound on slowly, being drawn taut with its back against the spiral until entirely taken up. The second clamp is then attached at the outer end to hold the film in place. When properly placed on the tray, the film stands edge up with its emulsion side entirely exposed to the solution except for about an inch at each end where clamped. The tray is then immersed in the developing tank, being let down gently to avoid dragging along air bubbles. Development with this outfit is carried out entirely in a darkroom, for there is no cover for the tank. To assure thorough and uniform action on the film, the tray is raised and lowered several times during the period of development. On completion of the development the tray is removed, thoroughly rinsed, then placed in the tank with the fixing solution. As in all development methods, it is necessary to give the film a thorough final washing before putting it on the dryer.

Tanks for this outfit are made in several sizes with trays to match, the sizes to be used depending on the width and length of film to be developed. One tank is about 26 inches in diameter and 12 inches deep; another is slightly wider and 14 inches deep. Two other tanks are about 18½ inches wide and 14 inches high. The spiral fins on the trays are also made in different heights such as 2½ inches and 7 inches. The 12- by 26-inch tank is filled to within about 2 inches of its top for developing

122. Comparison of developing outfits.—It will be noted that there is no opportunity with any of these outfits to examine the film during treatment. Because of the sensitivity of panchromatic film to all light, viewing it before fixation is well advanced is prohibited; thus practically the entire process has to be carried out with the film shielded from light. It is therefore necessary to develop solely by time with given quantities of solutions under temperature control. On this point there is no advantage between them. With respect to bulk and weight, the Fairchild-Smith apparatus gives a decided advantage over the others. It also requires much smaller quantities of solutions and is therefore more economical to operate for a single roll of film, because solutions that are used but once and thrown away do not require large quantities of chemicals. Its chief handicap is the defect that causes damage or marring of negatives close to the ends of the film. To avoid this it is necessary to waste about 3 feet of film at each development.

Both the Eastman and the Stinneman outfits require large quantities of solutions. Therefore, except when several rolls are to be developed and the solutions used repeatedly, they are expensive to operate. The Eastman outfit is the most elaborate and the heaviest of the three. It also requires more space for operation than the others. The Fairchild-Smith outfit is the most compact and handiest to operate. Between the Eastman and the Stinneman outfits the advantage is with the latter because of the absence of parts that are subject to rather rapid deterioration (a defect of the celluloid apron) and because it requires somewhat less space. The apron of the Eastman outfit not only must be carefully handled to avoid damage but also must be dried after each use. However, putting safety from marring or damage to the film as the foremost point, there is no better outfit than the Eastman.

Giving chief consideration to long life, the Stinneman outfit is first. All points being considered, however, for both laboratory and field use the Fairchild-Smith outfit is preferred over the others. It is the one now used much more extensively than the others.

123. The Fairchild-Smith portable roll-film dryer.—This is a compact film dryer which may be transported with ease and quickly set up for use in a small space. It consists of a cylindrical aluminum drum 28 inches in diameter and 10 inches wide, mounted on a metal stand. A 1-horsepower electric air blower inside forces jets of air against the film, which is passed at a regulated speed round the periphery of the drum in contact with numerous wooden rollers. There is a bracket on the base of the stand for seating the film-supply spool (or the film-handling framework of the Fairchild-Smith developing outfit) and a spindle near the top of the drum for the take-up spool. The take-up spool spindle and many of the wooden rollers are rotated by an endless sprocket chain geared to the motor, the geared-in rollers feeding the film along practically without friction to the take-up spool in step with its rotation.

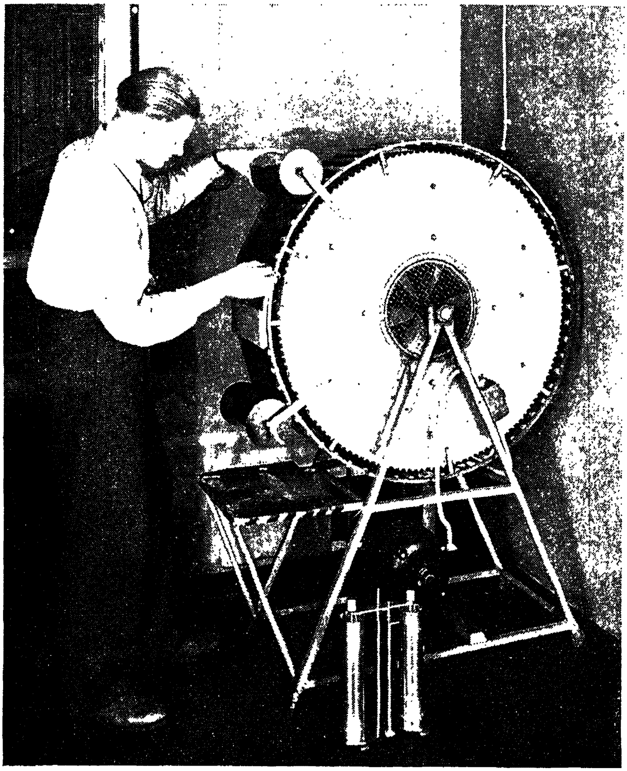


FIG. 39.—Film drying machine. (*Courtesy of Aero Service Corporation.*)

In passing along the rollers the film is held with its back against them by the continuous blast of air from the blower so that except at two water-removing rollers there is no contact with the emulsion side of the film. The blower operates at a constant speed, but the rate of movement of the film may be varied in speed from 1 to 10 feet per minute under differing conditions of humidity so as to provide sufficient time for the film to dry before reaching the take-up spool. Threading of the film is accomplished by means of a special metal bobbin which, on being dropped into a gate in the drum, feeds the film round the drum. On emerging from the drum the end of the film is dry, and after attachment to the take-up spool the remainder of the operation is automatic.

The dryer will accommodate film of any width up to 9½ inches and any length up to 200 feet. Under average humidity it will dry film at the rate of 100 feet in 16 to 20 minutes. It occupies a space only 22 by 34 inches and weighs 136 pounds, including the carrying case which serves as a table on which it is placed for operation. It is prepared especially to withstand dampness. Figure 39 shows a dryer of this general type which is used by the Aero Service Corporation of Philadelphia.

124. Air Corps film-drying outfit.—The type of film-drying equipment that has been most generally used in past years consists of old-fashioned buggy wheels (lacking rims) joined together with long, narrow, cloth-covered wooden slats attached to the felloes to make up a reel. An axle of steel rod or pipe extends through the hubs and protrudes several inches at each end to rest in half-cylindrical bearings. The bore of the hub is filled around the axle to give a snug fit, and a bolt is passed through the hub and the axle to join them rigidly together. The reel therefore may be rotated in the bearings either by hand with a crank attached to the axle or by an electric motor through a belt or by pushing on the slats. The design is simple and may be arranged for dismantling for shipment, although the outfit is heavy.

The film (roll of negatives) is wound onto the reel with its back against the slats and is held with spring clamps attached to short pieces of elastic cord which in turn are hooked to a slat. Having attached one end of the film to a slat it is fed on as the reel is slowly turned by hand. Starting at one end of the reel, the film goes on spiral fashion like the thread of a screw, so that no part overlies another part. A special spool holder is provided for handling a roll of negatives in transferring it to the reel, or the film-handling framework of the Fairchild-Smith developing outfit may be used for this purpose as may also the film-changing stand of the Eastman developing outfit or the special film-transfer spool of the Stinneman outfit. When the film has been entirely wound on the reel, a second clamp is attached to hold the loose end. Immediately afterward a soaked-and-wrung sponge or a damp, soft cloth is held against the film while the reel is turned by hand to mop excess water from both sides. This hastens the drying and avoids spotting, which may come from chemicals or mineral matter dissolved in the water. Mopping is necessary to obtain first-class negatives. It must be carefully and gently done to avoid scratching the easily torn saturated emulsion, the film having slack at the time to permit application to its back. Fingernails must not touch the film during the wiping. After wiping, the elastic of the clamps should be moderately stretched to assure holding the film in place. Drying may be accomplished by leaving the negatives on the reel at rest, but it is greatly expedited by rotating the reel. If it is rotated by motor, care must be taken to fasten the film securely, or it is liable to be thrown off and badly damaged. For security, rotation should be stopped at intervals of about 10 minutes to test and adjust the tension, which should not be allowed to increase or decrease too much. A fan placed to circulate the air round the reel also aids in speeding up the drying. The fan, however, must be used with caution to avoid stirring up dust which will lodge on the negatives and mar them. When completely dry, the negatives are wound again on a film spool for filing.

The drying reel at the Institute of Geographical Exploration is made of three buggy wheels about 4 feet in diameter and slats 87 inches long of ¾-inch-square stock slightly rounded on top and covered with soft cloth. Its journal is a steel rod 1⅛ inches in diameter and about 8½ feet long. The support is a rather massive framework of channel-steel bents joined together with lighter pieces of steel across the base and lower back. A diagonal piece of steel braces the base framework. Open bearings rest on top of the bents about 5 feet above the floor. The wheels are fastened to the axle with slender bolts which are let into holes drilled through the hubs and axle. The bottom of the reel stands about 3 feet above the floor; the top, a little over 7 feet. It occupies a floor space close to a wall of about 56 by 105 inches.

A ⅛-horsepower motor rests on a steel plate attached to the base of the bent at the right end, and an electric switch for the motor is placed on the upright face of that bent at the front. A plug-in cord extends from the switch, and the switch is connected to the motor by a metal-incased cable. A long belt extends completely round the reel and engages a 2-inch pulley at the motor. The motor is rated at 865 revolutions per minute; and as the gear ratio is about 1:25, the reel rotates about thirty-five times per minute. This is somewhat faster than desirable. It would be better to have the reel turn at between ten and twenty times per minute, thereby reducing the centrifugal force and lessening the danger of the film being dislodged. Four electric lamps mounted in a row on the wall behind the reel 7 feet above the floor provide for examining the negatives while they are being dried.

This is a highly satisfactory film-drying outfit, but it occupies considerable space and is too heavy for easy moving. It was designed particularly for stationary use in a laboratory with ample space. The arrangement is such that short and tall persons alike can transfer the film to the reel with ease. When there is no need to expedite the drying process, the film is merely put on and left to dry without revolving the reel. A good time is at night when no one is in the laboratory to stir up dust. No sweeping should be done when wet negatives are on the reel.

125. Table for examining negatives.—For the purpose of examining negatives and labeling them, a table with a glass surface lighted from below is used. This may be simply a drafting table with a section cut out and a plate of glass fitted into the opening. A lamp may be placed below the glass to shine upward; or if the table can be placed near a window, light may be reflected upward from a white board or a mirror inclined to reflect the light. Negatives are most conveniently filed by rolling them back on the spool furnished with the film. This requires that spindles be attached to the table for ready handling of the negatives. They are placed one at the back and one in front opposite the ends of the plate of glass, so that in being wound from one spool to another the negatives pass over the glass. It greatly facilitates labeling (India ink is best to use for this purpose) if the glass is long enough to permit four or five negatives to be seen at once. By the time that the last negative of those in position has been labeled, the first, or far one, is dry and may be wound onto the take-up spool. Under this arrangement no time is lost waiting for ink to dry.

A special cabinet for examining and labeling negatives, used at the Institute of Geographical Exploration, is about 3 feet long, 12 inches wide, 10 inches high at one end, and 5 inches high at the other. Its sloping top is fitted with an opal-glass plate which extends practically from end to end. A battery of electric lamps illuminates the glass plate with great brilliancy. A bracket spindle with a crank is attached at each end and a little below the top. These spindles are of the proper size to hold a 9½-inch spool. Adapters are used to accommodate shorter spools. The cabinet is supported on a table 30 inches high which has a top whose width and length equal the width and length of the cabinet. Drawers in the table provide convenient storage space for accessories and extra articles. A long plug-in cord provides for placement in available positions. On this table negatives may be examined, with the examiner either standing or sitting on a stool. The cabinet is made of light wood, the lamps being housed in a manner to prevent the light from shining in any direction except toward the top. Ventilation is provided by holes along the top of the sides. The weight of the cabinet complete with spindles, lamps, and electric cord is 18½ pounds.

126. Contact printer.—By far the largest amount of photoprinting is done with contact printers. This is the speediest and simplest method. It is accomplished by lighting the back of a negative while it is pressed firmly against the sensitive surface of photographic paper. Sensitive films and plates too may be used instead of paper to obtain positive transparencies. The essential features of a contact printer are a strong and smooth diffusing glass plate to support the negative; intense light; and a pressure back which, when brought against the sensitive paper, film, or plate, assures perfect contact throughout the negative. The printing of negatives in rolls also requires a spindle at each side to handle the negatives, move then expeditiously into position, and hold them there. It is also important to have provision for regulating the amount of light and for distributing it with differing intensities over the negative.

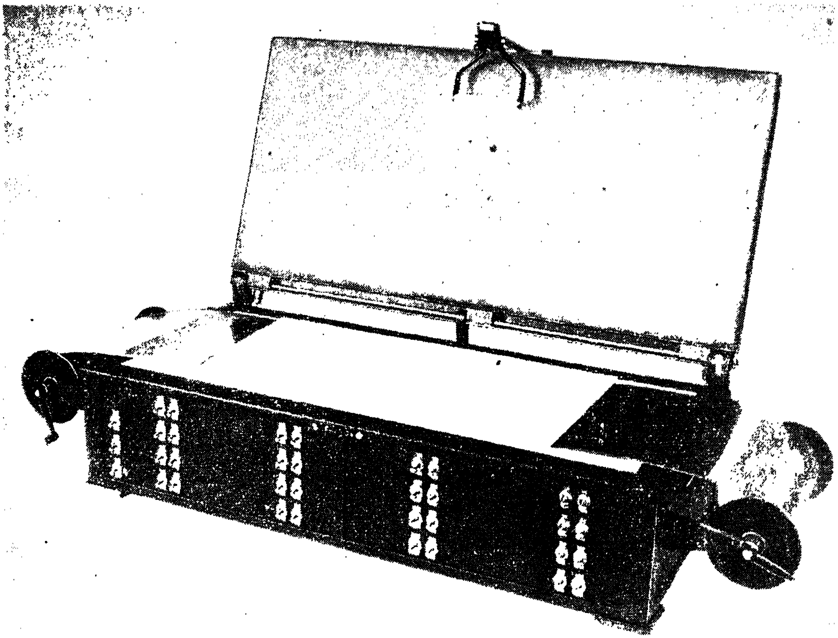


FIG. 40.- -Fairchild type F-58-B aerial roll contact printer. (*Courtesy of Fairchild Aviation Corporation.*)

Figure 40 shows a recent model of contact printer, made by the Fairchild Aviation Corporation. This is arranged for use on a table or stand whose top is at a convenient height above the floor while the operator is standing or is sitting on a stool. Spindles hold the negative in place, and guide rollers permit them to be brought smoothly into position. A pressure back is hinged and has a convenient handle for lifting and lowering it into place against the paper and negative. When the back is let down, it automatically turns on the printing lamps as contact is made and turns off the lights when raised. The back may swing up to rest against a stop where it is out of the way for setting a negative and paper in place. A large battery of 2½-watt argon (blue light) lamps provides for regulating the intensity of light and for distributing it as desired. The back has a pneumatic-rubber cushion which gives positive contact all over the negative. This printer is designed for convenient and rapid use, being constructed of first-class materials. Opal glass used to support the negative affords adequate diffusion of the light. The pneumatic cushion has been proved by many years of use to be the most satisfactory means of obtaining uniform pressure over the area and of maintaining it without difficulty.

The printer of Fig. 40 is 29¾ inches long, 8¼ inches high, and 13¾ inches wide, and it accommodates negatives up to 9 by 18 inches in size. Its weight is 64 pounds.

There are 96 sockets for the argon printing lamps; 32 for 7-watt, white, viewing lamps; and 8 for 7-watt amber lamps. The distribution of the sockets for printing lamps affords a wide variation of arrangement to distribute the light as needed. Switches on the front of the printer provide for rapid selection of the lamps. Although there are many lamps, their low power makes it unnecessary to employ a blower for ventilation. The spindles accommodate rolls of film up to 200 feet in length.

This type of printer is made in two other sizes, one for 9- by 9-inch negatives on spools and another for 20- by 24-inch cut negatives. The printer for 9- by 9-inch negatives weighs 32 pounds and is equipped with 48 argon printing lamps, 16 white viewing lamps, and 4 amber lamps. Its size is 15¾ by 8¼ by 13¾ inches. The printer for 20- by 24-inch negatives weighs 128 pounds and is equipped with 192 argon printing lamps, 64 white viewing lamps, and 16 amber lamps. Its size is 29¾ by 8¼ by 27½ inches.

An earlier contact printer for roll film which has been used by the Air Corps of the Army is equipped with an electric blower to permit continuous use without getting too hot. It and the blower are mounted on a base plate 16 inches wide and 28 inches long. The cabinet rises 18 inches above the base plate, and it is 14 by 15 inches in horizontal section. There are nine sockets for printing lamps, and the contact plate which is of clear glass is about 11 by 11 inches in size. The opening below the contact plate is about 10½ by 10½ inches, so prints from cut negatives up to 10 by 10 inches in size may be made. The spindles for roll film accommodate spools up to 9½ inches long.

This printer is provided with two plates of diffusing glass which are held in cleats attached to the side walls about an inch below the contact glass. The diffusing glasses may therefore be drawn out to permit dodging paper to be arranged on them. A pneumatic cushion is mounted on its lid, and a clasp holds the lid tight against the photographic paper to give uniform contact. When the clasp is pressed into place, contact is made with the exposure lamps. Switches on the front of the cabinet afford selection of any or all lamps. Negatives pass across rollers that prevent friction, and the lid comes to rest against a back stop when raised. The outfit weighs 78½ pounds.

127. Printing by projection.—Contact printing can be done only at the size of the negative; there is no opportunity to regulate or change the scale of the print. To make enlargements or reductions it is necessary to employ a lens and a camera made especially for the purpose. This is printing by projection. Ordinary enlargements and reductions are made by placing the negative and the photographic paper or plate perpendicular to the optical axis of the lens. In this type of printing it is necessary to consider four items: (*a*) lighting the subject, (*b*) perpendicularity to the optical axis, (*c*) conformance to the fundamental law of optics for sharp definition, and (*d*) methods whereby the negative and paper may be held flat.

Lighting requires the employment of special lamps such as arc or vapor lamps. There is no particular difficulty in this provided the proper equipment is available. Perpendicularity is obtained by physical tests such as reflecting images or by measurements of images on a ground glass to detect distortions. Scales or other means of making measurements coupled with the use of a ground glass provide for obtaining sharp definition and keeping within the requirements of optics. The negative is usually held flat by placing it between two glass plates. Other methods used to flatten it against a glass plate involve the use of blown air or vacuum. If both negative and sensitive paper are maintained in planes without the necessity of imposing glass plates between them and the lens, no problem of refraction arises. But glass plates are ordinarily used for this purpose, and they require that proper consideration be given to their effects in distorting the image. In straight printing the problem may be resolved by use in front of the negative of a glass plate of good quality which has the same thickness and index of fraction as the glass plate used in front of the sensitive paper. Under this condition the distortion of one glass plate will be compensated by the other glass plate. Some enlarging cameras are provided with a mechanism that automatically keeps the subject in focus on the image plane. Projection printing is greatly facilitated by the use of autofocusing cameras.

128. Transforming printers.—The mathematics involved in this type of printing is explained in Art. 184. Fixed transforming printers merely require that the negative be placed accurately in position in order to obtain a print correct with respect to perspective properties. Figure 41 shows such a printer which is employed to make prints from oblique negatives of the T-3A camera. To obtain prints uniform in tone, it is necessary to have the lamp placed to permit dodging and to observe precautions in developing the prints. Practice with this printer is necessary to obtain satisfactory photographs.

Transforming printers more general in character provide for printing from negatives by projection from one plane to another differing in inclination. These require that advance information of tilt and other factors be available and that settings be made accordingly.

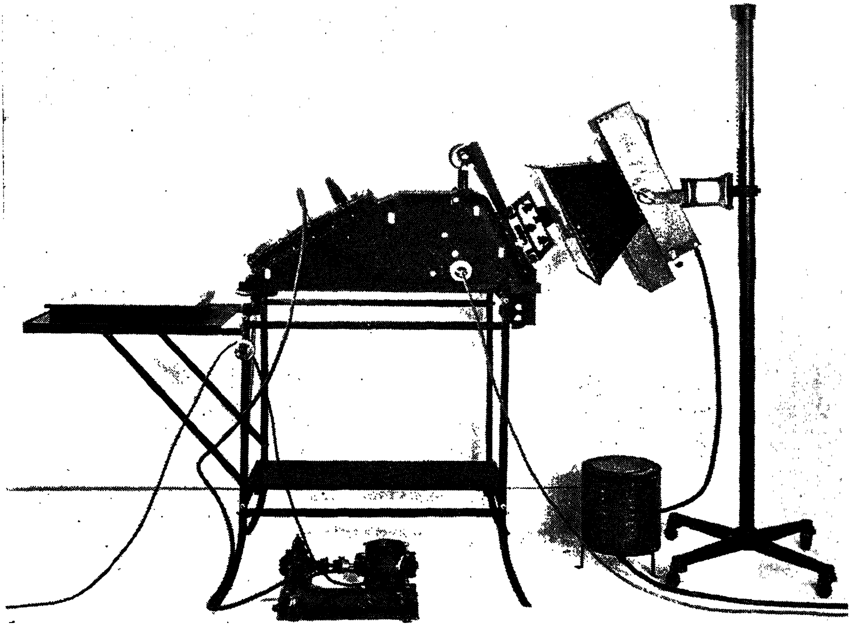


FIG. 41.—A B-7 transforming printer as furnished with each T-3A (5-lens) camera. The printer shown above is connected and ready for operation with the arc lamp and vacuum pump in place. (*Courtesy of Fairchild Aviation Corporation.*)

129. Drying racks for photographic prints.—Photographic prints to be used in mapping should be allowed to dry without hindrance in expansion or contraction. It is inadmissible to roll them on metal plates, as is commonly done in commercial laboratories. The most effective method is to place them on trays of cheesecloth stretched on frames. A stand to hold several trays is simple and inexpensive to construct; it is the type of rack most generally used. It should be placed in a heated room where the humidity is low, and there should be ventilation to expedite the drying. The rack should be large enough to accommodate the maximum number of photographs to be dried at one time. Dimensions of such a rack, which holds about 750 8- by 10-inch prints, are as follows: width, 54 inches; length, 78 inches; height, 7½ feet. It has 18 cloth-covered trays built of wooden members ⅞ inch thick. Each tray is provided with three wooden stretchers which prevent the cloth from sagging objectionably. Each tray is also braced at the corners with short diagonal members. The stand is also braced with diagonal members at the top. A cover of heavy manila paper protects the trays from dust.

130. Print-drying machines are available in the market. In general they take the form of a cylindrical drum containing a heating unit and a wide endless belt of canvas or other tough cloth which is made to travel round the drum at slow speed, carrying the wet prints into contact with the drum to effect the drying. An electric motor is employed to operate the machine.

131. Copying camera.—An excellent setup for a copying camera is to place it in a darkroom with its lens projecting through a wall into another room. A large easel supported by overhead beams and connected to tracks on the beams by running gear is placed directly opposite the lens in the outer room. This arrangement is good in that it does not require use of powerful lamps in the darkroom; but, as commonly set up, it does not provide for projection printing. However, a combination copying and enlarging camera could be arranged in this fashion by providing for lighting at either end and having both rooms equipped for darkening.

The copying camera of the Institute of Geographical Exploration is equipped with two lenses, one of which has a focal length of 24 inches, the other of 20 inches, and is large enough to take plates or cut film from 8 by 10 inches up to 20 by 24 inches in size. Its lens projects through a wall, and the easel, which is 52 inches high and 96 inches long, may be placed at any distance from the lens ranging from about 18 inches to 18 feet. Additional gearing of cogs, rollers, and weights provide for moving the easel laterally and up and down to center a subject with respect to the optical axis of the lens.

132. Projection cameras.—For making enlargements on bromide paper, projection cameras are rigged with the optical axis .placed either horizontally or vertically. A common arrangement is to place the camera on a carriage that runs along rails in the floor and to project to a wall or board attached to a wall. Placed overhead for projection downward to a table is a suitable arrangement for an autofocusing camera if there is sufficient headroom. This arrangement requires less floor space than horizontal cameras, but it is not practical for cameras that have to be got at for focusing and projection to close dimensions by scale measurements. Many excellent cameras of the two types are manufactured in the United States.

133. Lenses for copying and projection cameras.—Many suitable lenses for these cameras are available. Process lenses or those of other designations that are well corrected for distortion and give sharp definition are most suitable. Speed is of minor importance. The focal length should be great enough to assure coverage within the corrected field of the lens of all subjects to be copied. Excellent lenses of these types ranging in focal lengths up to 36 inches and working at stops of f/8 to f/16 are available in the stocks of reliable manufacturers.

134. Sinks with water and waste connections.—All photographic laboratories should be provided with commodious sinks well supplied with hot- and cold-water faucets, placed especially for convenience in washing both negatives and prints. Sinks from 24 to 36 inches wide, about 9 inches deep, and as long as practicable in the darkroom give good service. The sink should be fitted throughout with panels of wooden slats made in sections 2 or 3 feet long, which may be removed as desired for cleaning and for use of any part of the sink where its full depth is required. The slats should be at a level of about 36 inches above the floor to afford convenient use of trays without fatigue. Water faucets should be placed at a level of about 18 inches above the bottom of the sink and be supplied from pipes on the wall behind.

135. Print washers.—Several types of washers are available. One of the best for double-weight paper prints is a round metal tank about 5 feet in diameter and about 10 inches deep, provided with a round perforated metal tray 5 inches deep which revolves within the tank on a center pivot. Many vertical fins attached to the outer wall of the tray are acted on by a jet of water passed through the wall of the tank to cause the tray to revolve. A water pipe led down from overhead is connected both to the jet and to four dead-end pipes which form a cross above the tray. These dead-end pipes are drilled with many small holes in line to give sprays of water into the tray. Two curtains of paraffined lightweight canvas are attached to wooden bars resting on two of the dead-end pipes. These curtains dip into the water when the tank is filled to operating stage and assist in keeping the prints from sticking together, the motion of the tray and of the water causing them to move about in a generally continuous circular whirl. The tank has an automatic waste-regulating attachment which keeps the water constantly at a desired level. A cleanout waste connection at the bottom of the tank and one near the top to prevent overflow completes the equipment.

One washer of this type, constructed of copper and fitted with brass water pipes, has given excellent service for many years. It is not so satisfactory, however, for single-weight prints, which tend to fold in it and are liable to be injured. Washing is rapid and effective. Its capacity is about 30 16- by 20-inch prints at a time, which will receive thorough washing in 30 minutes to an hour, depending on the temperature of the water.

136. Other equipment required in the laboratory.—Numerous additional articles of equipment should be provided for the photographic laboratory. The more important of these are given in the list below:

Two sets of scales (avoirdupois or metric), one to weigh small quantities of chemicals, the other for large quantities and heavy articles.

A set of developing and washing trays ranging in size from 11 by 12½ to 18 by 24 inches. A deep tray 12½ by 15 inches is highly serviceable for developing many moderate-size prints at a stretch. If larger prints than can be readily handled in the 18 by 24-inch tray are to be made, selected trays for the purpose should be obtained rather than to try to put up with one not wholly suitable

Lamps, arc, vapor, and safe (amber or green)
Alarm clocks or clocks with luminous dials
Thermometers
Pails of enamelware or of stainless metal
Graduated beakers and measuring cups
Funnels of glass or enamel ware
Bottles, ½-gallon, 1-gallon, 2-gallon and 5-gallon
Soft flat sponge
Waste cans of galvanized steel
Extra film spools for the cameras to be used
Paper cutter
Shears
Linear scale of stainless steel, 24 or 36 inches long graduated to hundredths or fiftieths of an inch or graduated in centimeters and millimeters
Beam compass of stainless steel
Straightedge of stainless steel
Triangles of plastic material, 6-inch and 12-inch
Dry mounting press and iron
Blotters
Paperweights
Squeegee
Ferrotype plates (for occasional nonmapping photography)

Ample shelf room should be provided for chemicals, other supplies, and vessels. Tables, stools, and stands are also needed. Stock supplies should include adhesive tape (cloth), Scotch tape of paper or cellulose, dry mounting tissue, and gum arabic.

137. Labeling and numbering negatives.—Some cameras are equipped with a recording mechanism that prints on each negative its number of a series and all other information required to identify it with respect to others of the lot and for indexing. Many aerial cameras, however, are not so equipped; it therefore becomes necessary to label the negatives and number them consecutively. The negative-examining table is used for this purpose. Labeling usually consists in printing with India ink in small characters on the back of the negative in a corner where least obstructive a letter to designate the area, the last two ciphers of the year, and a number, *e.g.*, H-38-21. This is sufficient with additional notes placed on the filing container to identify each photograph when printed. It is well to place the label in the corresponding corner of each negative and to have it south side to you for reading when the photograph is oriented on a map as customarily drawn for reading—south edge down. The label is ordinarily written on the back surface of the negative; it then prints without reversal. In time, though, and from repeated handling the ink may flake off, for this surface does not permit the ink to penetrate. Permanent labels are obtained by drawing on the emulsion surface. Placed on this surface the label must be drawn in reverse. It is good practice first to label the negatives on the back surface, then at a later time to trace them over on the emulsion side.

138. Dodging.—This term is employed to signify the measures taken to distribute the light to the various parts of a negative to equalize the effect on the print. It is made necessary because nearly all photonegatives are not uniformly dense throughout their area. Dodging may be accomplished by (*a*) arranging the lamps of the printer to give more intense light at the denser parts of the negative than at the lighter parts, (*b*) partly shading with layers of paper or other translucent material, (*c*) moving a shield about between the light and the negative, or (*d*) painting the back of the negative with stains. The reasons for unequal densities between the central and outer edges of negatives have been explained in Art. 70. Unless a form of dodging is employed in printing, the photographs will be darker in tone at their margins than in the center. A small amount of compensation may be obtained by manipulation during development of the print, but tonal quality can be achieved only by using all practical means to distribute the light proportionately to densities and to compensate as far as practicable during development. Dodging necessarily slows the progress of printing; it is less deterrent with a fixed setup than when a movable shield is used, but in some cases the fixed setup is not practicable. Staining the backs of negatives to prepare for printing is hard to do successfully. It requires experience, for it must be graduated in density smoothly and inversely in proportion to the densities of the negative.

139. Standard U.S. equivalents in avoirdupois and metric weights, in U.S. liquid and metric measures, and in Fahrenheit and centigrade temperature scales are as follows:

1 pound = 16 ounces = 7,000 grains = 453.5924277 grams (National Bureau of Standards)
1 ounce = 1/16 pound = 437.5 grains = 28.3495 grams
1 gram = 0.035273 ounce = 15.4324 grains = 0.001 kilogram
1 gallon = 4 quarts = 128 liquid ounces = 1,024 drams (U.S. Apothecaries)
= 231 cubic inches = 3,785.43 cubic centimeters = 3.78543 liters.
1 liter = 1.05668 quarts = 33.814 liquid ounces = 270.51 drams (U.S. Apothecaries)
= 1,000 cubic centimeters = 1 cubic decimeter.
1 liter of water weighs 1 kilogram (as a practical conversion equivalent at ordinary temperatures)

The equation to convert a temperature reading from degrees Fahrenheit to degrees centigrade is C.° = 5/9 (F.° − 32);

32°F. = 0°C.
60°F. = 15.5°C.
65°F. = 18.3°C.
70°F. = 21°C.
125°F. = 51.7°C.
160°F. = 71°C.
212°F. = 100°C.

140. Developing solutions for film.—Formulas 1 and 2 given below have been used for many years, give satisfactory results, and are recommended for developing roll film with the equipment described herein. The chemicals and water are listed in the quantities required to charge the tanks of standard developing outfits. If pure chemicals are used, normal exposures with reliable film have been made, and the instructions have been followed, not only should there be no failure, but first-class negatives should be obtained. Inasmuch as there is no ready way for the user to apply tests to determine the purity of chemicals, the safe thing to do is to purchase only from reliable concerns.

Formula 1. Pyro Solution for Developing Panchromatic Film
Quantities Required to Charge a 3-gallon Tank

	Avoirdupois		Metric	
Potassium metabisulphite	1¾ oz.	68 grains	54	grams
Metol	½ oz.	59 grains	18	grams
Sodium sulphite	7½ oz.	53 grains	216	grams
Sodium carbonate	1 lb.		432	grams
Potassium bromide	½ oz.	59 grains	18	grams
Pyrogallic acid	5 oz.	35 grains	144	grams
Water to make	3 gal.		11.36	liters

Special Notes. Metol is sold under various other trade names as Elon, Pictol, Photol, and Rhodol. Care must be taken in handling this chemical, for if not quite pure it may injure the skin.

The length of time the film should be developed will depend on the following factors: (*a*) type of film, (*b*) character of the subject, (*c*) season of the year, (*d*) altitude at which the exposures were made, (*e*) method of development, (*f*) temperature of the solution, and (*g*) amount of contrast desired.

For Eastman Super XX Aero Topographic Base and Agfa Triple S Aero Acetate panchromatic film the following may be used as a guide:

For verticals of average terrain with woodlands and cultivated areas or of urban areas taken when the foliage is full and green at altitudes ranging up to 10,000 feet, with the Fairchild-Smith developing outfit, to get normal contrast develop for 10 minutes at 65°F. or for 7 minutes at 70°F.

For verticals at 15,000 to 20,000 feet above the terrain add 2 minutes to the period of development.

For photographs taken in the late fall or early spring when there are no leaves and little green, add 2 to 3 minutes to the period of development.

When development is by tray or with an outfit that keeps the solution at full action on the film during the entire time, shorten the periods by one-fourth to one-third.

If more contrast is desired, lengthen the period of development; if less contrast is desired, shorten the period.

With normal exposures and using a standard developer at the proper temperature, the contrast in negatives will depend on the length of time that the film is kept in the developing solution, increasing with the time. It is therefore just as important to stop development before contrast has gone too far as it is to give enough time for development. Brilliant negatives are obtained by regulating to the utmost exactitude every action involved.

Ordinary panchromatic film and other types of film having less sensitivity and less range through the spectrum require longer periods of development.

As a rule, the nearer the subject to the camera the greater will be the contrast between objects of the scene; therefore to avoid excessive contrast in the negatives give less time in development for shorter distances, more for greater distances.

The pyro formula, as used in many laboratories during the past several years, is given below in quantities sufficient for a 20-gallon tank.

Potassium metabisulphite	12 oz.	340.2	grams
Metol	3 oz.	85.05	grams
Sodium sulphite	3 lb.	1.36078	kg.
Sodium carbonate	6 lb.	2.72155	kg.
Potassium bromide	3 oz.	85.05	grams
Pyrogallic acid	2 lb.	907.18	grams
Water to make	20 gal.	75.71	liters

In some laboratories the solution is prepared by adding a small part of the sodium sulphite after the potassium metabisulphite has been dissolved, letting the metol follow this, then adding the balance of the sodium sulphite. The order of listing is observed thereafter.

As here given, the solution is not so potent as formula 1; therefore more time must be allowed for development. As a guide, add from 3 to 5 minutes to the periods of time stated for formula 1.

Stained negatives may be obtained by using less sodium sulphite. One-half the quantity stated will give a satisfactory stain when the solution is fresh. If this modified solution is kept a few days before use, it, too, should have sodium sulphite added to prevent excessive staining. Special care must be taken to maintain the temperature (keep it from going too high) when employing a solution to give stain. Moderately stained negatives give better results in projection printing than unstained negatives. Staining should not be attempted until proficiency at developing has been acquired. Staining gives little if any advantage for contact printing.

Formula 2. Metol-hydroquinone (M-Q) Solution for Developing Panchromatic Film

Quantities Required to Charge a 3-gallon Tank

Water at 125°F.	2½ gal.		8 liters
Metol	¾ oz.	54 grains	24.76 grams
Sodium sulphite	38¼ oz.		1,084.37 grams
Hydroquinone	3½ oz.	5 grains	99.55 grams
Sodium carbonate	19 oz.	100 grains	545.12 grams
Potassium bromide	2 oz.		56.7 grams
Water to make at full strength	3 gal.		11.36 liters

This is the most popular and generally used solution for film. It is a good keeper, so is satisfactory for making concentrated stock solutions for quick use as needed. It is given with many variations as to proportions of chemicals for various types of emulsions. As listed here the proportions accord with the practice followed in some of the laboratories of the Air Corps of the United States Army. The formula is practically the equivalent of the Eastman D19 formula.

This formula gives somewhat higher contrast than formula 1. However, the rules given for formula 1 may be used as a guide, keeping in mind that shorter periods of development may give more satisfactory results.

In the tropics, when it is impossible to maintain desirable temperatures (60 to 70°F.) for developing solutions, it becomes necessary to do the best practicable. It is possible by evaporation alone to lower temperatures of water sufficiently to develop film at night by use of a special formula known as *kodak developer DK*-15, which is given as formula 3.

FORMULA 3. FOR DEVELOPING FILM IN THE TROPICS

Quantities Required to Charge a 3-gallon Tank

Water (at about 125°F.)	1½ gal.	5.5 liters
Elon	2¼ oz.	63.79 grams
Sodium sulphite, desiccated	36 oz.	1,020.58 grams
Kodalk	9 oz.	255.15 grams
Potassium bromide	½ oz. 96 grains	20.40 grams
Sodium sulphate	18 oz.	510.29 grams
Cold water to make	3 gal.	11.36 liters

At 75°F. develop for 5 minutes, or at 70°F. develop for 6½ minutes.

After the solution has cooled to air temperature, it may be further cooled by covering the vessel with burlap and keeping this wet for several hours. Immerse the film in a tank of water at a temperature not higher than 85°F. to rinse; then transfer it quickly to the following hardening bath:

Water (at a temperature as much below 85°F. as can be obtained)	3 gal.	11.36 liters
Potassium chrome alum	12 oz.	340.19 grams
Sodium sulphite, desiccated	24 oz.	680.38 grams

The film must be agitated for about a minute after going into the hardening bath, or else streaking is liable to result. Leave the film in this bath 3 or 4 minutes; then place it in the standard fixing and hardening solution.

In using the Fairchild-Smith outfit, special care must be taken with the film to avoid pressure in reeling it back and forth. In using this tropical developer it is safer not to employ the motor but to carry out the operation entirely by hand.

141. Keeping qualities of developing solutions.—Metol-hydroquinone solutions are better keepers than metol-pyro solutions. It is safer practice, however, not to trust to solutions that have been prepared some time ahead of use but to prepare fresh developing solutions as needed. If solutions are to be kept, they should be tightly closed in full containers and put into a cool place. Properly stored metol-hydroquinone solutions will keep for several days; metol-pyro, for a few days.

142. Capacities of developing solutions.—Three gallons of a standard developing solution will develop 125 feet of 9½-inch film; a 20-gallon tankful, ten 75-foot rolls of film of the same width (or ten 125-foot rolls of 6-inch film) if used within a period of a few days. One gallon of a standard print solution diluted for use will develop 30 or more prints 8 by 10 inches in size, or about 65 5- by 7-inch prints.

143. General rules for mixing developers.—(*a*) Dissolve the chemicals in the order listed in the formula or as specifically directed.

(*b*) Make certain that each chemical is *completely* dissolved before adding the next.

(*c*) Mix by adding the *chemical* to water (not water to the chemical) at 125°F. or lower. When the water available is of questionable purity, it is good practice to dissolve each chemical separately in a small quantity of hot water and add the solutions to one another in accordance with the listing of the chemicals.

(*d*) Stir thoroughly. It is good practice to use a cheesecloth strainer immersed in the water to catch the chemical as it is added (but pour it in slowly to avoid caking of powder), the strainer being bobbed up and down until the chemical has passed through completely. Continue stirring until no trace of the chemical is left.

(*e*) Prepare the solution with half to two-thirds as much hot water as required for the final solution. At the end add enough cold water to make the quantity of solution needed. The temperature of the cold water should be low enough to bring the final solution to the desired temperature.

144. Chemicals essential for developing solutions.—A developing solution must have a *reducing* (developing) agent, such as metol, hydroquinone, and pyro; a *preservative*, such as sodium sulphite; an *accelerator*, such as sodium carbonate; and a *restrainer*, such as potassium bromide. A combination of metol and hydroquinone or metol and pyro makes a better reducing agent than any one of them alone; therefore most formulas have two reducing chemicals. Without a preservative the solution soon oxidizes, becomes discolored, and is liable to stain the film.

145. Functions of the chemicals used in developing solutions.—The preservative acts to reduce oxidization from the air, prolongs the life of the developing solution, and keeps it powerful to do its intended work. Reducing chemicals alone are not active enough to develop the latent image on exposed film, plate, or paper. They must be aided by an accelerator. The restrainer makes the solution selective; *i.e.*, it prevents action on the grains of silver salts which have not been affected by light. It therefore not only makes the solution do its intended work but prevents it from doing objectionable work and prolongs its usefulness.

146. Developing solution for photoprints.—This is the metol-hydroquinone formula commonly used for developing film in trays. It is here listed to make a stock solution that is to be diluted as directed below.

Metol	¼ oz. 71 grains	3.0876 grams
Sodium sulphite, desiccated	6 oz.	44.935 grams
Hydroquinone	1½ oz. 46 grains	12.021 grams
Sodium carbonate, desiccated	9 oz.	67.40 grams
Potassium bromide	¼ oz.	1.873 grams
Water to make	1 gal.	1 liter

For slow papers dilute with an equal quantity of water. For medium-speed papers use one part of stock solution to 2 parts of water. For bromide (fast) papers dilute with 4 measures of water to 1 of the stock solution. Use at temperatures between 60 and 70°F.

147. Standard fixing solution.—The following acid-hardening fixing-solution formula is widely used and is recommended for both film and prints. It is made in two parts; after all chemicals are completely dissolved and the second (stock) solution has cooled, *part of it* is poured into the first solution and completely mixed. Do not pour the hypo solution into the hardener.

FIRST SOLUTION			
Sodium thiosulphate (hypo)	2 lb.	239.65	grams
Water	1 gal.	1	liter

SECOND (STOCK-HARDENER) SOLUTION			
Water at 125°F	2 qt.	½	liter
Sodium sulphite, desiccated	8 oz.	59.913	grams
Acetic acid, 99 per cent pure glacial	8 oz.	59.913	grams
Potassium alum	8 oz.	59.913	grams
Add water to make	1 gal.	1	liter

For fixing film use 1 part of the hardener to 4 of the first solution; for prints, 1 part of hardener to 10 of the first solution.

Never allow the temperature of the fixing bath to rise above 70°F. Let film remain in the bath for 30 minutes; prints, for 15 minutes.

148. Capacity of fixing solutions.—Three gallons of a standard fixing solution will safely take care of two rolls of 9½-inch film 125 feet long or three rolls of 6-inch film 125 feet long or three rolls of 9½-inch film 75 feet long, if an acid stop rinser is employed, or one roll without it. One quart of the standard solution will fix thirty 8- by 10-inch prints, or forty-five 5- by 7-inch prints, provided the acid stop rinser is used, or half as many without it.

149. Acid stop rinsing bath.—The following stop bath serves to check development immediately and to lengthen the life of the fixing bath. It is more commonly used for prints than for film but may be used for either.

Water	1 qt.	1 liter
Acetic acid, 28 per cent	1½ fluid oz.	35 cc.

150. Contact printing.—In addition to the printer, three trays and a washing tank are required. The trays, which are to hold the developing solution, the acid stop bath, and the fixing bath, respectively, should be arranged in that order with the washing tank adjacent to the fixing bath. Trial printings with the dodging paraphenalia are made first to test its effectiveness and to determine the length of the exposure. Having completed this preliminary work the printing may be carried on in either of two ways: (*a*) by making all the exposures and boxing the exposed sheets, then passing them through the solutions, or (*b*) by exposing and taking each print through the solutions to the fixing bath, there to remain the allotted time before transfer to the washing tank. The first procedure is the more orderly one and is usually followed by experienced workers. The second method is, however, better for the beginner to follow until he acquires confidence with experience.

Two points in particular require emphasis. First, it is necessary to slide the print into the developing solution with a quick movement in order that all parts may get an even start and to avoid air bubbles which may leave spots. It is well to have an expert laboratory instructor illustrate this step. The second point is to keep the prints from overlying one another in a manner to prevent the solutions from having full effect. This applies to both developing and fixing. Prints have to remain in the developing solution only about 1 to 2 minutes; some even less than a minute. They should remain in the fixing bath about 15 minutes and be washed in running water until the potassium permanganate test shows a negative amount of hypo remaining in the water. During the washing it is necessary that each print be moved about much of the time.

151. Test for hypo in washed prints or negatives.—The certain way to determine whether or not a print or negative has been thoroughly washed is to apply the potassium permanganate test. It is necessary to prepare a solution for this purpose, and it is customary to keep on hand a stock solution prepared as follows:

Potassium permanganate	8 grains	1.0955 grams
Sodium hydroxide (caustic soda)	16 grains	2.1910 grams
Distilled water	1 pt.	1 liter

The test is made by adding 1 cubic centimeter of this stock solution to ½ pint (0.2366 liter) of water and then allowing the wash water to drip from a few prints or the film into this vessel. If the color of the solution changes to orange or yellow in half a minute or less, the prints or film should be washed some more. If there is little or no change in the violet color of the test solution, the washing is complete.

152. Projection printing.—Making enlargements, reductions, or transformed prints requires the use of a laboratory camera. The work is necessarily much slower than contact printing. It is moderately rapid in transformation printing, for the camera is fixed, requiring no adjustments whatever. In making enlargements and reductions, considerable time must be spent on setting the camera. The camera once set, printing may proceed with fair speed. If the enlargement is considerable, condensers should be used to focus the light upon the negative. Ordinarily the light is spread by diffusing (ground or opal) glass. The autofocus enlarger minimizes the preliminary work of enlargements (or reductions). The autofocus camera is especially valuable in saving time. In photolithography and other enlargement accurately to dimensions, however, this type of camera, being somewhat deficient in accuracy, is not commonly employed.

In common photography the dodging necessary in making enlargements or reductions may be done with a shield (cardboard with a 2- or 3-inch hole) moved about between the lens and the photograph or between the light and the negative. Enlargements require longer exposures than reductions, for the paper is then farther from the lens. Bromide printing papers are used almost exclusively in projection printing. They are faster than chloride papers but not so sensitive as film; hence proper safelights may be used. After test exposures have been made to determine the amount of dodging and the proper exposure time and to check sharpness of definition, the procedure is to make a lot of exposures, boxing the exposed prints, and then, after completion of this step, to proceed with the development, fixing, and washing. Note that a weaker developing solution is used for bromide paper than for chloride paper (contact prints).

153. Types of photographic paper.—Photographic papers are classified in several respects, according to their photographic and physical properties. In speed they are termed *slow*, *medium*, or *fast;* in surface quality, as *glossy*, *semimatte* (velvet), *matte* (no sheen), or *rough;* in grades, as *contrasty*, *normal*, or *soft;* in thickness, as *single weight*, *medium*, or *double weight;* in colors, as *white*, *cream*, or *ivory;* with respect to stability of dimensions or resistance to differential change, as *ordinary*, *good*, or *exceptional* (specially prepared for mapping use).

Slow-speed (chloride) papers are generally used in contact printing; fast (bromide) papers, in projection printing; medium papers may be used for either contact or projection printing. Single-weight glossy prints are commonly used for making mosaics and ordinary photographs; double-weight matte or semimatte papers are almost entirely used in mapping, although in special cases mapping may require papers that have high resistance to differential change in dimensions. Soft papers are used for printing from contrasty negatives; contrasty papers, from flat negatives; and normal papers, from normal negatives. White papers alone are used in aerosurveying. Illustrative obliques may sometimes be printed on ivory or cream-colored papers or papers with rough surface to obtain unusual effects. Sepia prints are obtained by the use of a chemical toner.

Special photographic papers for use in mapping are now made in two ways: (*a*) by waterproofing with impervious coatings and (*b*) by lamination, a thin sheet of metal being cemented between two layers of paper.

Both methods are successful in reducing objectionable differential change in dimensions, but these specially prepared papers are several times more expensive than ordinary photographic papers.

154. Classification of negatives.—Photonegatives are classified in several respects: as underexposed, normal, or overexposed; as contrasty, normal, or flat; as brilliant or dull; as dense or thin; as poor, good, or exceptional with respect to differential change in dimensions. Negatives may also be stained purposely or unintentionally during development. A moderate stain improves negatives for use in projection printing; an excessive amount of stain greatly increases the time required in printing. Underexposed negatives are thin and highly translucent; overexposed negatives are dark (dense) and check the passage of light. Brilliant negatives have the quality of crisp definition, clarity of detail, and moderate but ample range in contrast. Dull negatives are lacking in contrast; they may also be soft or deficient in definition. The quality to resist differential change in dimensions depends on the base that carries the emulsion. Contrasty negatives have great range in density from detail to detail, from the high lights to shadows; the image of an object may be dark; its shadow, very light. Flat negatives have too little range in density from detail to detail. In general a negative is considered dense if a strong light is needed to distinguish details; contrasty, if some details appear quite bright, others dark, when viewed against a light. There is one exception to this, which is particularly noticeable in negatives made with wide-angle lenses: such negatives may be dense (overexposed) in the middle but thin (underexposed) at their margins, *i.e.*, graded in density, because at exposure more intense light reached the center of the field than the borders.

CHAPTER VI

READING AND INTERPRETATION OF AERIAL PHOTOGRAPHS

155. Position for the photograph.—A vertical photograph of terrain lighted by the sun near the zenith may be examined in any orientation. An oblique should be placed with the nearer part of the terrain next to you in order to simulate a view obtained when looking over the side of an airplane. A stereo pair of verticals must be placed with the right one to the right, the left on the left side, and properly oriented with respect to each other and the axis of the stereoscope. There will be no confusion in this regard if one photograph is laid over the other with details matched, then the two are drawn apart, keeping the overlapped parts adjacent to each other. After having been placed for stereo examination, a pair of verticals as a unit may be turned 180° in azimuth and viewed with the same degree of satisfaction as in the original position. If there is little tilt in the photographs, the stereo effect will be obtained over the entire zone of overlap when the orientation is correct. If either photograph is considerably tilted, it may be necessary to shift one of them a little to get the stereo effect when looking at the images near the borders. When examining wide-angle vertical photographs with the stereoscope, it is best to view that part of the overlap zone from the middle to the farther border, then turn the photographs together (or the stereoscope) 180° in orientation and view the remaining part from the middle outward. It gives a more natural effect to look downward and outward than to look downward and "between the legs." This is especially important when looking at composite photographs with a stereoscope, for the borders show objects that were much farther from the camera at exposure than those pictured near the center.

156. Position when shadows are dominant.—In the study of large-scale photographs individually, shadows play an important part. When pronounced, as over deeply dissected terrain, shadows also will considerably affect the reading of physiographic forms in general views. The rule is to place the photograph so that the shadows are cast toward you. The natural effect of relief will then be obtained. Placed in other positions, hills may appear to be valleys and valleys to be hills, or structures may be confused with their shadows. It is necessary to become familiar with the influence of shadows in order to become proficient in reading aerial photographs correctly and expeditiously. Shadows often reveal objects. Towers, spires, elevated water tanks, chimneys, other tall structures, cuts, and fills are given away by their shadows.

157. Aerial photographs taken for mapping purposes are in general too small in scale to distinguish many objects by characteristic details; to a large extent, objects must be determined by their general shapes and surroundings. Features in the form of lines (roads, railroads, trails, streams, aqueducts) are easier to determine than those which merely appear as spots (dwellings, barns, lodges, etc.). Details having characteristic patterns such as race tracks, sewage-settling tanks, transmission lines through woodlands in cleared strips, orchards, cemeteries, and many others are readily recognizable. In some instances objects may be recognized by their tone in contrast to surroundings; open swamps usually appear darker than adjacent meadows, and woodland swamps likewise appear darker than adjacent areas of dry woodlands. Newly plowed land will appear lighter in tone if of sandy soil than fields with crops. The appearance of water depends on the angle of incidence of the light, the color of the bottoms below, and the amount of stain or suspended matter carried; bodies of water appear in tones ranging from very bright to almost black. The use to which water in ponds is put may be determined in many instances by the character of the shores and structures along shore. Those used for boating and swimming will likely have boat houses and diving platforms or piers. Reservoirs for water supply will have neatly cleared shores and probably a small gatehouse at the dam; there will be an absence of structures indicating recreational use. The approximate stage of water in a pond, reservoir, lake, or sea can often be obtained by the appearance of the shore, for at low stages beaches appear as wider bands than at high stage. An artificial pond may be distinguished from a natural pond by its dam.

158. Influence of the scale of the photograph.—In general the larger the scale of the image the easier it is to recognize an object. Some objects appear exactly as we expect to find them; others, quite different. Experience must be gained with the appearances of details in aerial photographs at different scales. Ease of discernment depends directly on the size of the image as well as the character of an object, its color, and the lighting of it. With a little magnification the eye can detect a line on the photograph even if its width is but one thousandth of an inch (three or four hundredths of a millimeter) wide. If that line is bright in tone against a dark background, even lesser widths may be detected. Spots must be from five to ten thousandths of an inch in size to be detected, and they must present a contrast with their surroundings to make identification certain. In a photograph whose scale is 1:10,000, lines on the ground must therefore be about 10,000 times a thousandth of an inch (10 inches) wide to afford ready identification. Exceptional cases have been noted; *e.g.*, in a photograph whose scale was 1:18,000 the image of the steel rails of a railroad could be clearly seen where sunlight was reflected directly from them.

The scale of vertical photographs required to distinguish objects of various sizes and types taken with the sun shining directly on the objects is indicated below.

Scale of Photograph	Objects That Can Be Discerned
1:5,000	Individual small shrubs, chimneys, individual persons on walks, copings around house tops, skylights, railroad tracks in freight yards, automobiles on streets, concrete walks
1:10,000	Individual persons on sidewalks, copings around house tops, skylights, railroad tracks in freight yards (uncertain unless light is favorably reflected), automobiles on streets, concrete walks
1:15,000	Skylights, copings around house tops, railroad tracks (number parallel with difficulty), automobiles on streets, concrete walks
1:20,000	Skylights, bright and large copings around house tops, automobiles on streets, concrete walks.

159. Importance of stereo observation.—The stereoscope is a great aid in reading and interpreting aerial photographs. It is so important that verticals are practically always taken with more than a 50 per cent overlap in order to provide for stereo examination and to give them a maximum value for mapping. Moderate magnification is also an important aid. Magnification cannot be carried very far, however, because too high power will so restrict the field of view that relationships with surrounding objects are lost to view and enlargement of the grains of the emulsion will interfere with getting a true idea of the object under examination. It has been found by experience that magnification between two and four diameters gives the best results. Magnification may be obtained either by enlarging the photograph or by incorporation of magnifiers in the optical system of the stereoscope.

160. Field trips to get experience.—It is important to become acquainted with the appearances of objects by experience. Study of photographs should be accompanied by field trips with the photographs in hand and direct identification of objects pictured. As many different types of terrain as practicable should be studied, and familiarity gained with all types such as tropical jungles, deserts, arid lands, New England scenes, farm lands of the Middle West, mountains, and plains. The character of suburban areas and of sections of cities can be determined accurately after proper study of types.

161. Miscellaneous details.—Some detailed characteristics should receive attention. The modern highway is laid out with regular curves details purposely hidden from the photographer is attained by careful study of examples of camouflage and by notice of small telltale items either overlooked in preparing the camouflage or unintentionally or unavoidably left exposed. Proficiency can be obtained in this highly specialized field only by continued study under conditions which permit verifications to be made and which offer examples of the subject with which to gain the necessary experience. Deduction may here be brought to a high degree of usefulness by the individual who is clever in analysis. Repeated taking of photographs of areas of interest provide for making comparisons to detect significant changes. A recently taken aerial photograph compared with a fairly large scale map of good quality will usually be sufficient to determine new works of an enemy in military operations. Collections of large quantities of materials unless well concealed will betray their character in one way or another. Movements of troops and trucks are caught by photography at night using flash bombs. In detection work, sharpness of definition in the photograph is the most valuable quality. Printing for study of details in shadows requires special consideration.

CHAPTER VII

GEOMETRICAL CHARACTERISTICS OF AERIAL PHOTOGRAPHS

164. Distortions caused by tilt in vertical photographs.—With facilities now available it is never possible to obtain a truly vertical photograph, *i.e.*, one whose plane is strictly parallel to the plane of the ground (horizontal). Under good air conditions and by skillful operation of both airplane and camera it has been demonstrated that a set of photographs covering large areas can be taken with but small amounts of tilt. Seldom is a vertical tilted more than 3°; usually, of a lot taken for mapping, more than half will be tilted less than 1°. In letting contracts for aerial photographs to be used for mapping it is common practice to limit the allowable tilt to 2°. For mosaic making the tilt should be kept below 1°, or else those photographs having more than 1° of tilt should be partially corrected for tilt when printed. It contributes to both speed and economy to have the photographs well taken with a minimum of tilt.

For topographic mapping of comparatively large areas or at large scales, it is necessary to overcome the distortions of tilt to avoid inadmissible discrepancies and to maintain a proper standard of accuracy in the resulting map. There is therefore prime need to know the character of the distortions that tilt causes. Perhaps the best way to visualize the effects of tilt in a vertical photograph is to see how a regular-shaped figure like a square laid out on flat ground will be distorted on a tilted photograph of it. In Fig. 43 it is assumed that the camera was tilted 15° when the photograph was made of such a square. The upper part of the diagram presents a vertical section through the air (camera) station S, and it includes a line to represent an accurate map of the area containing a plot of the square true to scale. The photograph is placed at its focal distance SC from the air station, and it is assumed to have been made with its borders parallel to the sides of the square and its center C in line with the air station S and the center C_1 of the square. The plumb line strikes the photograph at V; the map, at V_1. The angle of tilt is the angle CSV.

In the lower part of this figure the center of the photograph is placed at the center of the square. In the upper part, Q_1R_1 represents the square in the plane of the map corresponding to QR in the photograph; and inasmuch as QR is parallel to two sides of the square, it is the distance apart of the remaining two sides of the square. In the lower part of the diagram, therefore, $C'R'$ is laid off equal to CR, and $C'Q'$ equal to CQ. It is evident from the relationships existing in the upper part of the diagram that the side of the square passing through R will appear on the photograph equal to its map length multiplied by SR/SR_1 (similar triangles). In a like manner the side passing through Q = map length multiplied by SQ/SQ_1. Construction of the square as it will appear on the photograph can then be completed, and it is found to take the form of a trapezoid. A study of the figure now discloses interesting and useful facts:

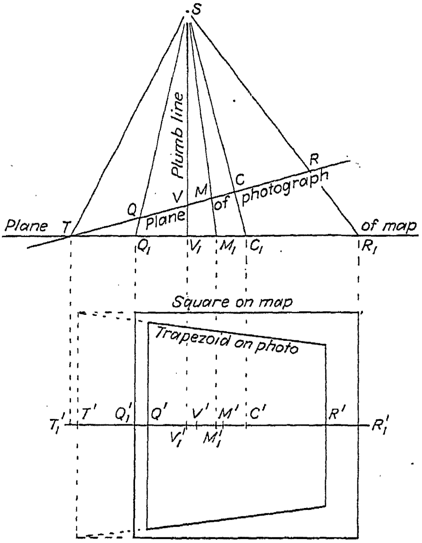


FIG. 43.—Diagram illustrating the effect of tilt on an aerial photograph.

(*a*) In the direction of tilt (along the line TR) the scale of the photograph is changing not uniformly but in a geometrical progression.

(*b*) Parallel to the axis of tilt (at right angles to the direction of tilt) the scale differs from part to part of the photograph, but along any line in this direction a proper uniform scale could be used.

(*c*) The center of tilt distortion is at M. This point is located at the intersection of the principal plane and the plane of the photograph at a distance from the center of the photograph equal to the focal length of the photograph multiplied by the tangent of one-half the angle of tilt. (The distance $CM = SC \tan 7\frac{1}{2}°$.)

(*d*) Along the intersection of the plane of the photograph and the plane of the map (through T in the upper part and T' in the lower part), the scale of the photograph equals the scale of the map. (This is evident from the fact that both photograph and map are here at the same distance from the air station S.)

Proof of the statement in (*c*) above can be readily had graphically by placing a piece of tracing paper over the lower part of the diagram and using M' as center, drawing radial lines through the corners of the trapezoid, prolonging them out beyond the corners of the square. Now shift this tracing until each radial line cuts through its corresponding corner of the square. M will now be found to fall at M_1', which is its true plotted position.

From the foregoing study it is evident that, although satisfactory scale measurements cannot be made on a greatly tilted aerial photograph of flat terrain, true directions to objects pictured may be obtained if the center of distortion can be found (Art. 165). Further examination along the lines here followed indicates that in slightly tilted photographs, directions accurate enough for plotting on maps can be obtained by drawing radial lines from the center of the photograph (for *M* is then quite close to *C*) or from a point of any object near the center. Even a moderate amount of relief over the terrain pictured does not seriously interfere with this procedure. The radial-line method of plotting is based on this characteristic of tilted vertical photographs.

165. Angles turned about the center of distortion of a tilted aerial photograph are equal to the corresponding horizontal angles of flat terrain included in the photograph.—Inasmuch as flat terrain lies virtually in a horizontal plane, use may be made of any horizontal plane to demonstrate the truth of the statement, because angles cut from parallel planes by any two intersecting planes are equal. It simplifies the demonstration to make use of that horizontal plane which intersects the tilted photograph at the center of distortion; so this plane will be employed. The line of intersection of this plane with the photograph is perpendicular to the principal plane.

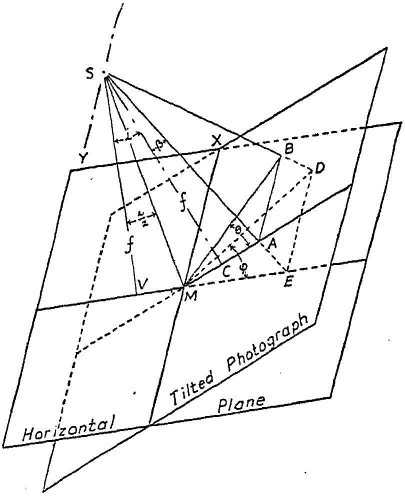


FIG. 44.—Diagram used to show that angles turned about the center of distortion of an aerial photograph are equal to the corresponding horizontal angles.

In Fig. 44 let *S* be the perspective center, *C* the optical center, and *M* the center of distortion of the photograph. *SY* is the axis of tilt; and *MX*, which is parallel to *SY*, is the line of intersection of the photograph and the horizontal plane. From *S* drop a vertical line that meets the horizontal plane at *V*. Then *SCV* is the principal plane of the photograph, and both *CM* and *VM* are perpendicular to *MX*. The principal plane cuts the horizontal plane along the line *VME*; so *ME* is likewise perpendicular to *MX*.

Let *D* be the position of any point of the terrain as projected into the horizontal plane, *S* being the center of projection. The corresponding position (image) of the point in the photograph is at *B*. Draw *DE* in the horizontal plane perpendicular to the principal plane to meet it at *E*. Connect *E* with *S*. The plane *SDE*, being perpendicular to the principal plane, intersects the photograph in the line *AB*, which is parallel to *ED*;

for the triangle *SED* is similar to the triangle *SAB*, *A* necessarily being on the line *SE*.

Proof will be accomplished by showing that the angle *BMA* (designated by θ) equals the angle *DME* (designated by φ).

Let i designate the angle of tilt *CSV*, f the focal length *SC* of the photograph, and β the angle *ASC* in the principal plane. Then

$$AM \tag{2}$$

and

$$ED$$

(3)

In the similar triangles *SBA* and *SDE*

$$\frac{AB}{SA} = \frac{ED}{SE}$$

but

$$SA = \frac{f}{\cos\beta}$$

and

$$SE = \frac{f}{\cos(\beta + i)}$$

for $SV = SC = f$; therefore

$$AB = \begin{matrix}\cos(\beta + i)\\ \cos\end{matrix}$$

In the triangle *MAE* the angle at *M* equals i, the angle at *A* equals $90° + \beta$, and the angle at *E* equals $90° - (\beta + i)$. Then

$$= \sin[90° - (\beta + i)] : \sin(90° + \beta)$$

so

$$AM = \frac{EM \sin[90° - (\beta + i)]}{\sin(90° + \beta)}$$

Substituting in equation (2) the preceding derived values of *AB* and *AM*, gives

$$\tan\theta = \frac{ED}{EM} \times \frac{\cos(\beta + i)\sin(90° + \beta)}{\cos\beta \sin[90° - (\beta + i)]}$$

but, since $\sin(90° + \beta) = \cos\beta$ and $\sin[90° - (\beta + i)] = \cos(\beta + i)$, the angle functions in the numerator cancel those in the denominator, leaving

$$\tan\theta =$$

which compared with equation (3) shows that $\theta = \varphi$.

166. Effect of tilt on points of a photograph.—Tilt has the effect of reducing the scale on the upper side of the photograph and of increasing it on the lower side. In Fig. 45 a tilted photograph is shown in position with respect to its equivalent vertical. The common perspective center of both is at S; the principal point of the photograph at C; its center of distortion at M; and its vertical point at V. The equivalent vertical lies in the horizontal plane through M.

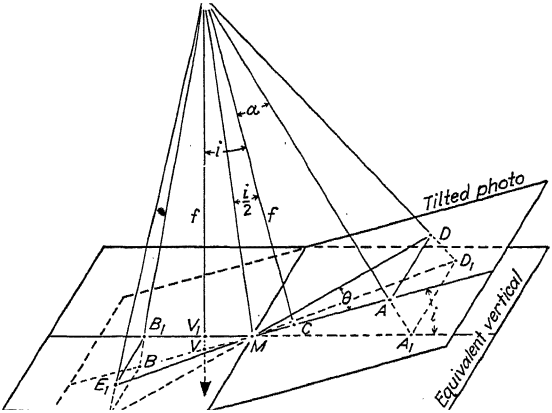


FIG. 45.—Diagram used to obtain equations for determining the amount an image point is thrown out of position by tilt.

It is evident that distortion along the principal line AB can be stated in terms of f, i, and α as follows:

On the upper side

$$d = MA_1 - MA$$

or

$$d = f\left[\tan(\alpha + i) - \tan\alpha - 2\tan\right. \tag{4}$$

On the lower side

$$d = MB -$$

or

$$= f\left[\tan\alpha - \tan(\alpha - i) - 2\tan\frac{i}{2}\right] \tag{5}$$

In these equations α is used to designate the angular distance from the principal point to the foot of the perpendicular dropped from any point to the principal line.

On the upper side of the photograph let D be any point on the line DA, which is perpendicular to the principal line MA. Let the rays SD and SA be drawn and prolonged to meet the equivalent vertical at D_1

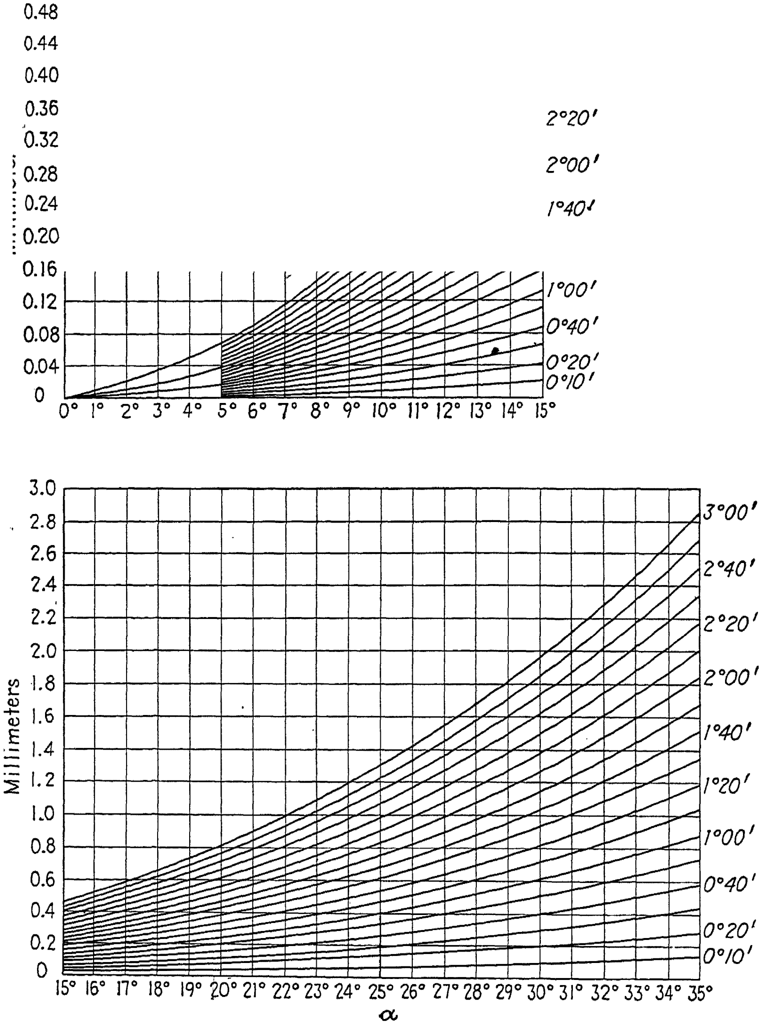


FIG. 46.—Graph to obtain the distortion of image points caused by tilt on the upper side of a photograph.

and A_1, respectively, and let the angle AMD be designated by θ. In Art. 165 it was shown that angle AMD equals angle A_1MD_1. Moreover, AD and A_1D_1 are both perpendicular to the principal plane; therefore they are mutually parallel, and the triangles AMD and A_1MD_1 are similar. Hence

$$AM:A_1M = DM:D_1M$$

In a similar manner it can be shown that

$$BM:B_1M = EM:E_1M$$

Equations (4) and (5) are therefore applicable to all points of the photograph.

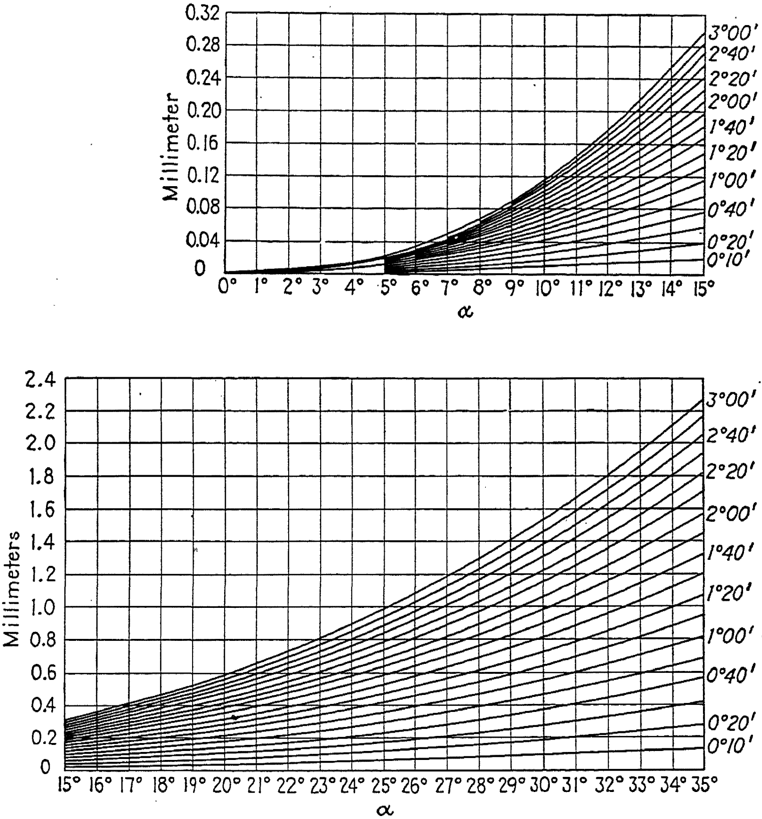


FIG. 47.—Graph to obtain the distortion of image points caused by tilt on the lower side of a photograph.

To determine the amount of tilt distortion at any point of an aerial photograph, the procedure is to drop a perpendicular from the point to the principal line, measure the angular distance α of this point from the principal point, and apply the values of α, i, and f to equation (4) or (5) depending on whether the point is on the upper or lower side of the photograph; then divide the value of d so found by the cosine of the angle θ.

For the purpose of expediting the determination of tilt distortions, Tables 2 and 3 have been computed, and the graphs of Figs. 46 and 47

Table 2.—Tilt Distortions with Reference to the Center of Distortion, in Millimeters, for f = 100 millimeters
Upper side of photograph

Tilt	Angular distance from the principal point of photograph								
	5°	10°	15°	20°	25°	30°	35°	40°	45°
10′	0.003	0.009	0.021	0.038	0.064	0.097	0.143	0.201	0.293
20	0.005	0.018	0.042	0.077	0.129	0.196	0.288	0.414	0.588
30	0.008	0.029	0.065	0.118	0.195	0.297	0.436	0.625	0.888
40	0.010	0.038	0.087	0.160	0.262	0.399	0.585	0.840	1.191
50	0.013	0.049	0.111	0.201	0.329	0.502	0.734	1.056	1.496
60	0.016	0.060	0.132	0.244	0.398	0.605	0.886	1.273	1.807
1°10′	0.020	0.071	0.158	0.286	0.467	0.712	1.042	1.494	2.121
1 20	0.023	0.082	0.184	0.331	0.538	0.819	1.198	1.717	2.439
1 30	0.027	0.096	0.208	0.375	0.611	0.927	1.357	1.944	2.760
1 40	0.030	0.107	0.234	0.421	0.682	1.037	1.517	2.173	3.084
1 50	0.035	0.119	0.260	0.467	0.758	1.148	1.679	2.406	3.412
2 00	0.039	0.132	0.287	0.514	0.832	1.261	1.843	2.639	3.746
2 10	0.043	0.145	0.314	0.562	0.907	1.376	2.009	2.878	4.083
2 20	0.048	0.159	0.343	0.610	0.985	1.492	2.177	3.117	4.422
2 30	0.052	0.173	0.371	0.659	1.063	1.608	2.346	3.358	4.768
2 40	0.058	1.187	0.400	0.711	1.141	1.727	2.519	3.605	5.114
2 50	0.063	0.202	0.430	0.761	1.223	1.847	2.692	3.852	5.467
3 00	0.068	0.217	0.460	0.812	1.303	1.969	2.870	4.105	5.869

Table 3.—Tilt Distortions with Reference to the Center of Distortion, in Millimeters, for f = 100 millimeters
Lower side of photograph

Tilt	Angular distance from the principal point of photograph								
	5°	10°	15°	20°	25°	30°	35°	40°	45°
10′	0.002	0.009	0.021	0.039	0.061	0.096	0.140	0.203	0.289
20	0.004	0.018	0.041	0.076	0.124	0.192	0.280	0.404	0.574
30	0.006	0.026	0.060	0.113	0.185	0.285	0.421	0.605	0.857
40	0.008	0.034	0.079	0.149	0.244	0.377	0.558	0.800	1.136
50	0.009	0.041	0.099	0.185	0.305	0.468	0.692	0.994	1.412
60	0.011	0.049	0.117	0.220	0.362	0.558	0.824	1.186	1.685
1°10′	0.012	0.056	0.135	0.254	0.421	0.648	0.955	1.377	1.956
1 20	0.013	0.063	0.152	0.286	0.475	0.735	1.087	1.563	2.223
1 30	0.014	0.070	0.169	0.320	0.531	0.822	1.215	1.748	2.484
1 40	0.015	0.076	0.185	0.352	0.585	0.906	1.342	1.931	2.746
1 50	0.016	0.082	0.202	0.384	0.639	0.990	1.466	2.112	3.003
2 00	0.017	0.088	0.217	0.414	0.692	1.074	1.589	2.291	3.257
2 10	0.018	0.093	0.232	0.445	0.743	1.154	1.711	2.468	3.510
2 20	0.018	0.099	0.247	0.475	0.794	1.236	1.832	2.642	3.757
2 30	0.019	0.104	0.261	0.503	0.845	1.313	1.950	2.814	4.004
2 40	0.019	0.108	0.276	0.532	0.894	1.392	2.066	2.983	4.245
2 50	0.019	0.113	0.289	0.560	0.942	1.470	2.183	3.152	4.484
3 00	0.020	0.117	0.302	0.587	0.990	1.545	2.297	3.317	4.723

have been prepared from those tables. Figure 48 illustrates a device that may be prepared to read values of α from a photograph. The radii of the arcs are equal to $f \tan \alpha$, so that when drawn on a transparent sheet the figure may be placed over a photograph with its center at the principal point and the angular distance α read at once.

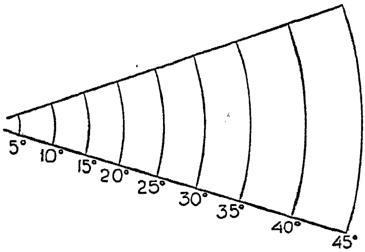


Fig. 48.—Device for obtaining values of angles at the perspective center of a photograph.

The curves in Figs. 46 and 47 having been drawn from the tables for a focal length of 100 millimeters, values of distortion obtained from them must be multiplied by the quotient of the focal length of the photograph in millimeters divided by 100.

167. Displacements in vertical photographs caused by ground relief.—In a vertical photograph whose plane is strictly parallel to the plane of flat ground (or of the shores of a body of water) that it embraces, there is complete similarity in relationships between the images in the photograph and the actual objects in the ground plane. If, however, the objects are not all in a single plane but some stand at higher elevations than others, the photograph will not present images of the objects in their proper arrangement with respect to one another. The objects considered may all stand on level ground, some rising higher than others, as do buildings; or they may lie in the surface of the terrain which extends over hills, mountains, and valleys. Figure 49 is drawn in a vertical plane through an air station S to include a truly vertical photograph and the profile of rough terrain that the photograph has registered in that vertical

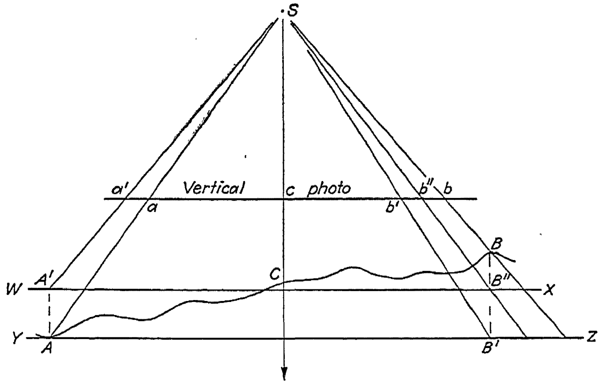


Fig. 49.—Diagram showing how displacements caused by ground relief are registered in a vertical photograph.

plane. *WX* and *YZ* are two assumed horizontal (datum) planes of reference, such as sea level (*YZ*) and a plane parallel to it but much higher (*WX*).

It is evident from the diagram that

(*a*) The displacement of the image of any object as *A* and *B* occurs along the line drawn from the center of the photograph to the image (*ca*, *cb*).

(*b*) The amount of displacement depends on the height of the object above the datum plane of reference, its angular distance (angles *aSc*, *bSc*) from the center of the photograph, and the scale of the photograph.

(*c*) The displacement will be outward from the center of the photograph if the object is above the datum plane and inward toward the center if the object is below the datum plane. With respect to the datum plane *WX*, *a* is displaced inward from *a'* to *a*; *b*, outward from *b''* to *b*. With respect to the datum plane *YZ*, *a* is not displaced, but *b* is displaced outward much more (*b'b*) than it is with respect to *WX*.

(*d*) An object appearing at the center of the photograph is not displaced, regardless of the relief of the ground or what datum plane is used.

The significance of the datum plane is of concern only when we undertake to measure the height of objects or to determine differences in elevation between objects pictured in the photograph. It is not necessary to consider datum planes in the work of making mosaics or planimetric line maps further than a reference to the general level of the ground to obtain approximate scales of photographs and mosaics.

It is important to note that in the present case the center of the photograph coincides with the vertical point. In tilted photographs the displacements caused by relief radiate from the vertical point, not the center.

168. Effect of tilt distortion combined with relief displacement.—The combined effect of tilt distortion and relief displacements will be considered next. For this purpose the square and trapezoid comparison will again be made. Suppose that in the previous instance instead of being on flat ground the square had been laid out on rolling terrain with no two corners at the same elevation; then its image on the tilted photograph will be a trapezium without parallelism between any two of its sides. Figure 50 represents the condition between the plotted square on the map and its corresponding image on the photograph. The angle of tilt of the photograph is taken as before at 15°. *V* is the vertical point; *M*, the center of distortion; and *C*, the center of the photograph. The center of the square on the map is also at *C*. Compared with the image of a square on flat ground (trapezoid drawn in dotted lines) displacements of the corners have taken place radially from the vertical point, as has every point of the sides.

If from *M* radial lines are drawn through the corners of the trapezoid on a piece of tracing paper placed over the drawing and an attempt is made to locate the position of *M* on the map with this tracing, either the lines cannot be made to go through the corners of the square altogether so no location can be made; or if the lines do pass through the respective corners, a false location of *M* will be obtained. But even in this case of large tilt and large relief displacements, if the tracing is shifted to have each radial line fall as close as possible to its corner, opposite lines passing on opposite sides of their respective corners when coincidence is impossible, an approximate location of *M* will be obtained that is not greatly in error.

As the amount of tilt becomes less, the vertical point and the center of distortion tend to approach the center of the photograph and the amount of distortion grows less. Also, the smaller the relief the less is the displacement. In actual cases encountered in practical work, tilt as great as 3° sometimes occurs, but by careful flying and careful operation of the camera it is possible to have all photographs within 2° of level. On a photograph taken with an 8¼-inch lens and having a tilt of 2° the vertical point will fall at a distance of 8¼ tan 2° from the center. This amounts to about $\frac{3}{10}$ inch. The center of distortion falls about midway between the center and the vertical point. Taken at 14,000 feet above the ground, the photograph will have a scale of about 1:20,000. Objects standing 500 feet above the general level of the ground and appearing 3 inches from the center of the photograph will be displaced in image about 2.9 millimeters; at 4 inches from the center, about 3.8 millimeters. In order to visualize the effects of distortions and displacements of the magnitude commonly met with in aerial photographs, Fig. 51 is drawn at a reduced scale under the conditions stated above. It represents a 7- by 9-inch photograph at half size. It will be seen that the greatest derangement, caused by relief, of direction lines drawn from the center of the photograph is somewhere in the zone off to the right and to the left of the principal line and that the greatest derangement of the same direction lines caused by tilt is approximately at 45° to the principal line but that the derangements of direction lines are much smaller than the derangements in scale. In every case it will be noted that directions are changed only slightly more than the width of a line.

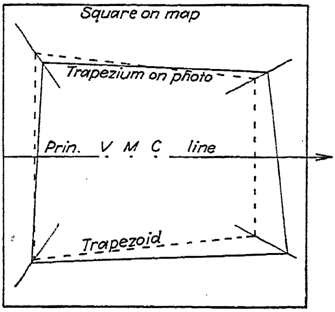


FIG. 50.—Effect of tilt and relief in changing the shape of an image of a square.

It has been pointed out that, although aerial cameras are equipped with spirit bubbles for leveling, these do not make it possible to keep the camera accurately leveled. Even though in some cameras interior bubbles are arranged to form an image at the edge of the photograph as exposures are made, there is no assurance that the bubbles will indicate the tilt truly. In fact, no ready means exists of finding out the amount and direction of tilt. Moreover, it is of considerable practical value to have a ready means of obviating the effect of tilt in determining a good average value for the scale of a photograph, a figure better than can be got from the altimeter reading. Figure 51 shows that a satisfactory determination of scale can be arrived at if the distance is known between two objects whose images appear in the photograph, each about equidistant from the center, one on one side, the other on the opposite side and where a line connecting the two will pass near the center. This is the most favorable situation for the two images, because, there, one will

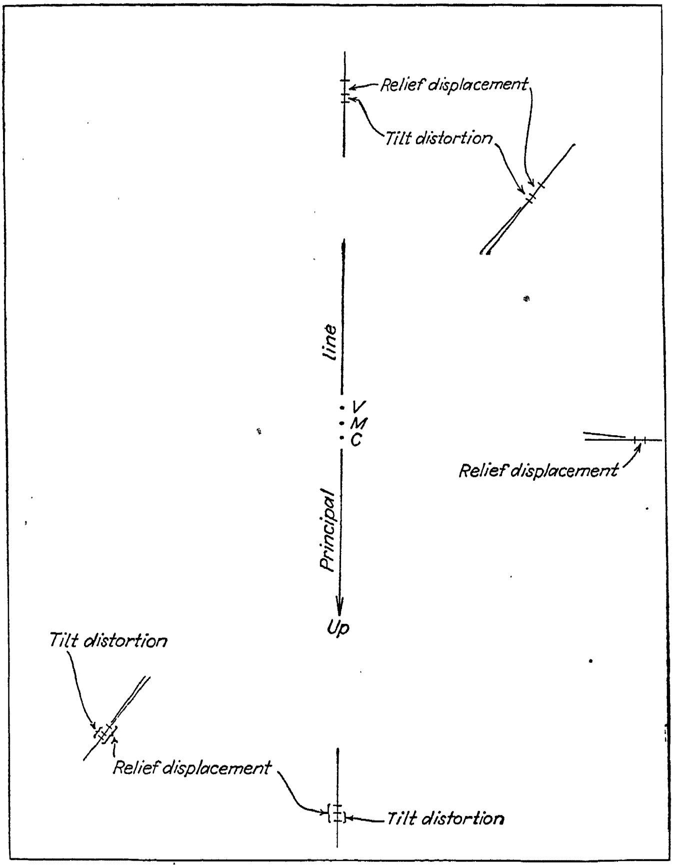


FIG. 51.—Actual amounts of relief displacements and tilt distortions on an aerial photograph, whose scale is about 1:40,000, when the tilt is 2° and the relief is 500 feet.

be drawn toward the center about as much as the other is thrown out from the center. The points chosen for making the measurement should be at about the same ground level. For this purpose use should be made of a distance between two objects in a valley or on a plateau at the general level of the terrain. Examination under a simple stereoscope gives comparative levels of objects in the zone of overlap, and this test is sufficient for the present purpose.

As there is no rapid method of finding the elevations of objects appearing in a photograph, when orienting a photograph or obtaining directions to objects, the best that can be done quickly is to avoid the use of points on hills if the picture covers valley land in general, or of points in ravines or low places if the picture is largely of plateau land.

169. Effect of tilt and relief in composite vertical photographs.—Composite verticals made with multilens cameras are subject to the same kind of relief displacements and tilt distortions as are single-lens verticals. The effect of tilt distortions in deranging angles at the center of a photograph is the same for a given degree of tilt no matter how great the angular field of the photograph. Hence the maximum derangement of angles between lines drawn radially from the center of the photograph, because of tilt, can occur in either type of vertical. For a tilt of 2° (the greatest tilt commonly encountered) a line bisecting the tilt line (direction of tilt) and the axis of tilt will give a direction to any object in that quarter which is in error by slightly more than 1 minute of arc. In a certain part of the photograph each of two such radial lines may be out of true direction so that the angle between them will be in error by a little more than 2 minutes. This amount (about 3 feet in a mile) is so small that it may be entirely disregarded in all plotting work except of maps at quite large scale—1:5,000 and larger.

The magnitude of relief displacements is much greater on the outer parts of composite verticals than anywhere on single-lens verticals. Near the borders of composite verticals, relief as small as 100 feet will cause objects to be displaced a quite appreciable amount. Of special interest, however, is the fact that these displacements have less effect in deranging angles between lines drawn from the center of tilted photographs than do displacements of images of objects that appear nearer the center. The following comparison is made between single-lens verticals having a focal length of 8¼ inches and composite verticals of 6-inch focal length. Relief of 500 feet and tilt of 2° are assumed for each type of photograph. Images (of objects) lying 4 inches from the centers of these single-lens verticals and images lying 15 inches from the centers of composite verticals are ordinarily those farthest from the center which are used in mapping. Points at these respective distances are therefore taken for the comparison.

On the single-lens vertical the derangement of direction lines drawn from the center to those images lying in the most unfavorable part may be as great as 9 *minutes* of arc.

On the composite vertical, the corresponding derangement of direction lines will be only about 2½ minutes of arc.

This advantage in favor of the composite vertical is chiefly because of its much greater angular field.

This shows that, although single-lens verticals of small angular field are best for mosaic making, the composite vertical or single-lens vertical of large angular field is more suitable for plotting, and the larger the field the better.

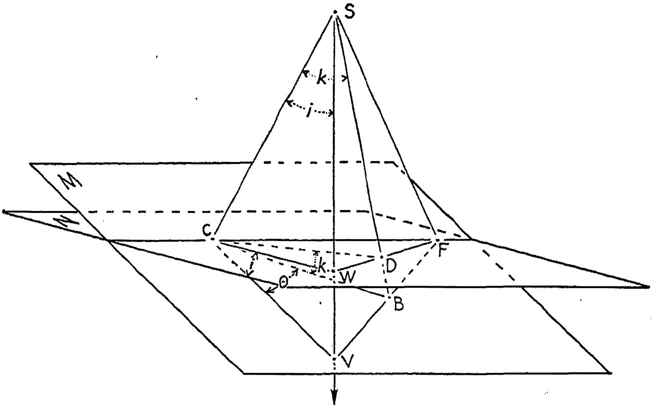


FIG. 52.—Components of tilt at the center of an aerial photograph.

170. Components of tilt.—The angle of tilt of an aerial photograph being the dihedral angle between the photograph and a horizontal plane, the purpose now is to determine the angular component of tilt along any line drawn from the center of the photograph. This angle (always less than the angle of tilt) will be measured in an *upright plane* which includes the *optical axis* of the photograph. Figure 52 represents a tilted photograph M whose principal point is at C, whose perspective center is at S, and whose vertical point is at V and a horizontal plane N whose intersection with the photograph is the line CF. The plane SCV is therefore the principal plane of the photograph, and the angle VCW is the angle of tilt i. Let SCB be any plane containing the optical axis which cuts the line CB from the photograph at an angle θ to the principal line CV. Let the angle in this plane between the photograph and the horizontal plane be designated by k. The value of k in terms of i and θ is to be determined.

Let SVF be a plane that contains the plumb line and is perpendicular to the plane SCB. Because it contains the plumb line it is also perpendicular to the horizontal plane N (CWF); it cuts from the photograph the line VF which is perpendicular to CB and from the horizontal plane the line WF which is perpendicular to CD. As a further consequence of the conditions of the figure, CD is perpendicular to SB, angle CSV equals i, and angle BSC equals k. Letting the focal distance SC be designated by f, the following equations may be set down:

$$CD = CB \cos k = f \sin k; \qquad CV = f \tan i$$

and

$$CB = CV \cos \theta$$

Substituting values from the first two of these equations in the last one, there is obtained

$$f \frac{\sin k}{\cos k} = f \tan i \cos \theta$$

or

$$\tan k = \tan i \cos \theta \tag{6}$$

Equation (6) is elliptical in form, but for small values of i it may be simplified, without introducing appreciable error, to

$$k = i \cos \theta \tag{7}$$

which represents a circle and is much easier to employ.

It is evident that, when θ is zero, k equals i; and that, as θ increases, k grows smaller until it reaches zero when θ is 90°. From the 90° line (which is the line of intersection of the photograph and the horizontal plane) k increases until θ reaches 180°, when it again equals i and so on round to zero and back to i again. This deduction is significant and

TABLE 4.—COMPONENTS OF TILT FROM EQUATION (7)

Angle θ, degrees	Values of k in minutes from $k = i \cos \theta$, when i is						
	½°	1°	2°	3°	4°	5°	10°
0	30	60	120	180	240	300	600
10	29.54	59.09	118.18	177.26	236.35	295.44	590.89
20	28.19	56.38	112.76	169.15	225.53	281.91	563.81
30	25.98	51.96	103.92	155.88	207.85	259.80	519.61
40	22.98	45.96	91.92	137.86	183.85	229.81	459.41
50	19.28	38.57	77.14	115.70	154.27	192.84	385.67
60	15	30	60	90	120	150	300
70	10.26	20.52	41.04	61.56	82.08	102.60	205.21
80	5.21	10.42	20.84	31.26	41.675	52.094	104.19
90	0	0	0	0	0	0	0

valuable in the processes of determining the tilt of an aerial photograph and in obtaining the elevations of objects pictured in aerial photographs.

Tables 4 and 5 show values of k for various values of i as obtained from equations (6) and (7).

Table 5.—Components of Tilt from Equation (6)

Angle θ, degrees	Values of k in minutes from $\tan \kappa = \tan i \cos \theta$ when i is						
	½°	1°	2°	3°	4°	5°	10°
0	30	60	120	180	240	300	600
10	29.55	59.09	118.18	177.27	236.36	296.46	591.07
20	28.19	56.39	112.77	169.19	225.52	282.00	564.49
30	25.98	51.96	103.93	155.92	207.93	259.96	520.93
40	22.98	45.96	91.94	137.92	183.97	230.05	461.56
50	19.29	38.57	77.15	115.77	154.42	193.13	387.98
60	15	30	60.02	90.06	120.14	150.29	302.30
70	10.27	20.53	41.06	61.61	82.20	102.84	207.07
80	5.22	10.43	20.85	31.30	41.74	52.22	105.23
90	0	0	0	0	0	0	0

The maximum disparity in corresponding values of k as obtained from equations (6) and (7) occurs when θ is 45°. To compare the corresponding values for $\theta = 50°$ and $i = 5°$, Table 4 gives 192.84 minutes; Table 5, 193.13 minutes, a difference of 0.29 minute, or about 18 seconds. For $\theta = 45°$ and $i = 5°$ (not shown in the tables), the difference is 0.36 minute, or about 22 seconds. It is therefore evident that equation (7) may be used instead of equation (6) for all tilts up to 5° without introducing an appreciable error and that, when the tilt is as great as 10°, the error is still not great.

171. Tilt circles.—Once the principal line and angle of tilt of an aerial photograph become known, tangent tilt circles of equal diameter may be drawn on the photograph or on a piece of tracing paper placed over the photograph, whereupon the component of tilt along any line of the photograph can be measured directly. It is necessary that the common tangent line drawn through the point of tangency of the two circles be placed at right angles to the principal line of the photograph and that the point of tangency be placed at the principal point. Figure 53 shows the arrangement. The diameter of each circle is the value of the angle of tilt i measured in a convenient linear scale, as 1 millimeter = 1 minute. The lengths of chords measured from the principal point in the same linear unit are then the components of tilt of the photograph in their various directions. Thereafter it is convenient to consider that the common tangent line divides the photograph into two parts, the lower side being the one that contains the vertical point, the other being the upper side. The diameter and chords of tilt circles, radiating from the principal point of the photograph, are highly accurate measurements of the arcs (or chords) of the angle of tilt and of the angular components of tilt for small angles.

172. Components of tilt along lines through the vertical point.—By use of a figure similar to Fig. 52, it can be shown that with reference to the vertical point of a photograph the corresponding equation for a tilt component is

$$\sin k = \sin i \cos \theta \tag{8}$$

In this equation i is the angle of tilt and k the component in the *vertical plane* that contains the plumb line and cuts the photograph at the angle θ with respect to the principal line. For practical purposes this equation may also be considered as equivalent to $k = i \cos \theta$.

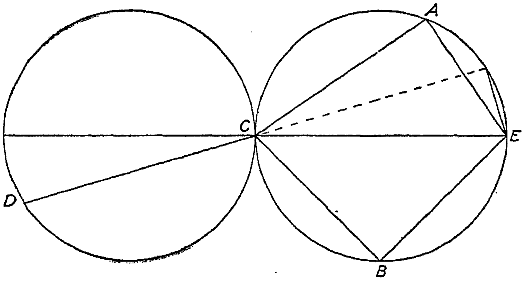


FIG. 53.—Tilt circles.

173. Components of tilt along lines through the center of distortion.—Figure 54 is employed to derive the tilt equation when the center of distortion is employed as the pivot point.

In it let S be the perspective center, C the principal point, M the center of distortion, A any point of the photograph, and W the intersection of the plumb line from S with the horizontal plane through M. The angle of tilt (WSC and CMF) is designated by i; the angle AMC, by θ; and the angle AMB, by w. Connect A with M, S with A, prolonging the latter to meet the horizontal plane at B. Also connect B with M and W with M, prolonging the line to F. From C draw a perpendicular to MA to intersect that line at N, and from W draw a perpendicular to meet BM prolonged at P. Connect both P and N with S.

Plane SCN is perpendicular to the photograph, and plane SWP is vertical; therefore SN is perpendicular to AM, and SP is perpendicular to BM. It follows, then, that angle PSN equals w, and angle PSM = angle $NSM = w/2$; whence $\sin w/2 = MN/SM$;
but

$$MN = MC \cos \theta = f \tan \frac{i}{2} \cos \theta,$$

and $SM = f/\cos i/2$. Therefore

$$\sin\frac{w}{2} = \frac{f\tan(i/2)\cos\theta}{f/\cos(i/2)} = \sin\frac{i}{2}\cos \tag{9}$$

This equation likewise for practical purposes may be considered equivalent to $w = i\cos\theta$, or w may be said to equal k of equation (7).

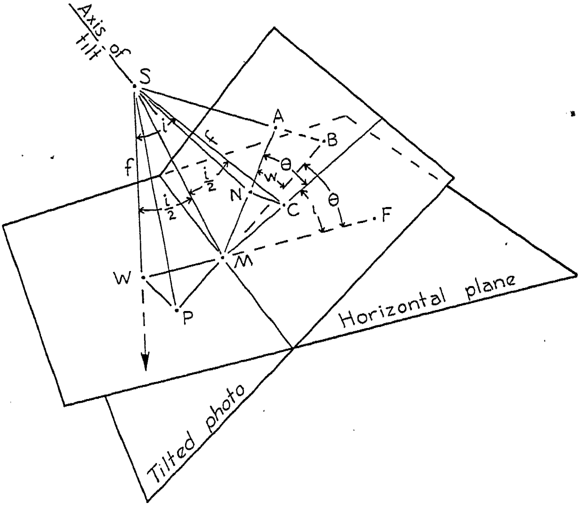


FIG. 54.—Components of tilt at the center of distortion.

174. Finding the angle of tilt and the principal line from tilt components.—Tilt circles provide a useful method of finding the angle of tilt and the principal line of the photograph from values of tilt components. Theoretically two components suitably disposed to give a resultant from a graphical plotting are sufficient to determine the tilt and the principal line of a photograph, but with only two components there is no check; experience shows that the accuracy obtainable in any single determination of tilt component is not high enough to make a satisfactory determination of tilt without three or more components. The process consists merely in laying off the respective component quantities along lines of the photograph from the principal point, using a linear scale, then erecting perpendiculars to the direction (radial) lines at the points thus found. If each component has been correctly determined, all the perpendiculars will meet at a point (E of Fig. 53) that is on the principal line of the photograph, and the distance EC is a measure in the same linear unit of the angle of tilt. It is essential to keep track of the sign of all components of tilt and to lay off the component values accordingly. For example, in Fig. 53 if CA and CB are on the lower side of the photograph and have been given the plus sign, and CD is found to be on the upper side, it must be considered to have the minus sign; then to use the component along CD with those along CA and CB its value must be laid off from C in the opposite direction, not toward D. The means employed to determine whether a component line lies on the upper or lower side of the photograph depends on the process used to determine its quantity. Explanations are given in connection with the processes. Tilt circles have particular application in the upright-planes method of determining tilt (Art. 219).

175. Effect produced by approximate component values.—In actual cases of tilt determinations through components it seldom occurs that three or more perpendiculars do meet at a point. Inaccuracies in determining components have the effect of producing a polygon (triangle for three lines) of error. If no accidental error has been made, the center, or midpoint, of the polygon is the most probable position of the principal line, and it may be accepted if the size of the polygon is reasonable in view of the character of the method employed for determining the tilt components. It should be understood that under the usual circumstances prevailing in such work any component value is liable to be in error by as much as 8 to 12 minutes. However, systematic errors in component values have a tendency to balance one another, and the error in the angle of tilt found from several components is of the order of the average error in components divided by the number of components.

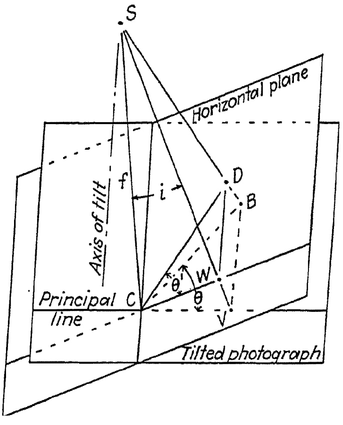


FIG. 55.—Angular distortion with respect to the center of a tilted photograph of flat terrain.

176. Angular distortion at the center of a tilted photograph when the terrain is flat.—The proposition is to find the value of an angle as measured at the center of the photograph so that it may be compared with the corresponding horizontal angle measured on flat terrain covered by the photograph. This will be accomplished by obtaining the value of the angle in the photograph in terms of the corresponding angle in a horizontal plane that intersects the photograph at its center.

In Fig. 55 let S be the perspective center of the photograph, i the angle of tilt, f the focal length, C the center, V the vertical point, CB any line of the photograph passing through C and forming the angle θ with the principal line CV, and SBV a vertical plane at right angles to the principal plane. Let the angle DCW in the horizontal plane that corresponds to θ be designated by θ'. The vertical plane intersects the horizontal plane in the line DW which is parallel to BV cut from the photograph. The two triangles SDW and SBV are therefore similar.

From the relationships established in the figure

$$\frac{DW}{CW} \tag{10}$$

The following equations may also be stated: $\tan \theta = BV/CV$; $CV = f \tan i$; $CW = f \sin i$; $SW = f \cos i$; $SV = f/\cos i$; and $BV{:}DW = SV{:}SW$. Substituting these values in equation (10),

$$\tan \theta' = \tan \theta \cos i \tag{11}$$

is obtained. By similar procedure it can be shown that at the *vertical point* of the photograph

$$\tan \theta' = \frac{\tan \theta}{\cos i} \tag{12}$$

A *right angle* laid off at either the center or the vertical point from the principal line of a tilted photograph is true. The error in angles increases gradually from the principal line until it reaches a maximum when the angle is 45°. Beyond it gradually diminishes until it becomes zero when the angle is 90°. This is true both for the center and for the vertical point. The maximum error occurs in 90° angles whose two limiting lines stand at 45° to the principal line. An acute angle at the center of the photograph turned from the line CV of Fig. 55 (the index direction being from C to V) will be too large; if turned from the opposite direction of that line, it will be too small.

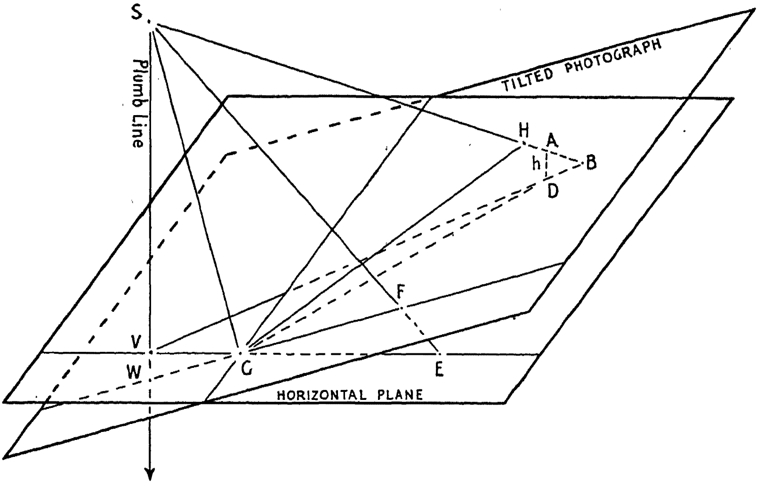
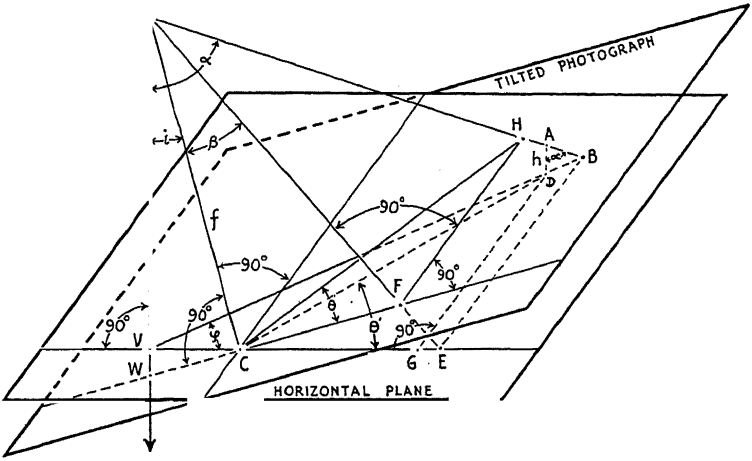


FIG. 56.—Angular distortion with respect to the center of a tilted photograph of rolling or hilly terrain.

177. Angular distortion at the center of a tilted photograph when the terrain is not flat.—Figure 56 represents a tilted photograph with center at C and a horizontal plane with a hilltop at A whose height above the plane is h. S is the perspective center of the photograph; D is the base of the hill; H is the image of A; and B is the point where the ray SA pierces the plane. B lies on the line drawn from V to D, V being the point where a plumb line swung from S pierces the plane. SVE, which contains both plumb line and the optical axis of the photograph, is the principal plane of the photograph. A comparison is to be made between the angle HCF in the photograph and the angle DCE in the horizontal plane.



$$\frac{CH \sin\theta \cos i \cos\beta \sin\varphi - h \sin^2\varphi \tan\alpha \cos(\beta \pm i)}{CH \sin\theta \cos i \cos\beta \cos\varphi - h \cos\varphi \tan\alpha \cos(\beta \pm i) \sin\varphi \mp f \sin i \cos(\beta \pm i) \sin\varphi}$$

FIG. 57.—Diagram used to obtain an equation for the angle measured at the center of a tilted photograph of rough terrain.

To avoid confusion, Fig. 57 is drawn to show the relationships and construction lines employed to furnish the desired comparison. In this figure the angle HCF is designated by θ; the angle DCE, by θ'; the angle DVE, by φ; the angle of tilt, by i; two other angles, by α and β, respectively, as indicated; and SC, by f. As FH, BE, and DG are all perpendicular to the principal plane, they are parallel to one another. It is evident therefore that

$$\tan\theta' = \frac{DG}{CG} \tag{13}$$

and

$$\tan\theta = \frac{HF}{CF}$$

The following relationships may also be stated from inspection:

$$CF = CH \cos \theta; \qquad FH = CH \sin \theta; \qquad \tan \beta = \frac{CF}{f}$$

$$VE = SV \tan (\beta + i) = f \cos i \tan (\beta + i); \qquad SF = \frac{f}{\cos \beta}$$

$$SE = \frac{f \cos i}{\cos (\beta + i)}; \qquad \tan \varphi = \frac{EB}{VE}; \qquad VB = \frac{EB}{\sin \varphi}; \qquad \tan \alpha = \frac{VB}{SV}$$

$$DB = h \tan \alpha; \qquad VD = VB - DB; \qquad DG = DV \sin \varphi$$

$$VG = VD \cos \varphi; \qquad CG = VG - VC$$

In the similar triangles SFH and SEB

$$SF:SE = FH:EB$$

therefore

Substitution of these various values in equation (13) gives

$$\tan \theta' = \frac{CH \sin \theta \cos i \cos \beta \sin \varphi - h \sin^2 \varphi \tan \alpha \cos (\beta +}{\sin \theta \cos i \cos \beta \cos \varphi - h \cos \varphi \tan \alpha \cos (\beta + i)}$$

$$\sin \varphi - f \sin i \cos (\beta + i) \sin \varphi$$

When θ is between 90° and 270°, $\beta + i$ becomes $\beta - i$, and the sign before the last term of the denominator changes to plus. The general equation therefore becomes

$$\tan \theta' = \frac{CH \sin \theta \cos i \cos \beta \sin \varphi - h \sin^2 \varphi \tan \alpha \cos (\beta \pm i)}{CH \sin \theta \cos i \cos \beta \cos \varphi - h \cos \varphi \tan \alpha \cos (\beta \pm i)}$$

$$\sin \varphi \mp f \sin i \cos (\beta \pm i) \sin \varphi \quad (14)$$

Another equation in which neither φ nor CH appears is

$$\tan \theta' = \frac{\tan \theta \sin \beta \tan (\beta \pm i)(f \cos i - h)}{\sin (\beta \pm i)[f \cos i \tan (\beta \pm i) - f \sin i \mp h \tan (\beta \pm i)]} \quad (15)$$

Equation (15) is somewhat simpler than equation (14). However, equation (14) is more direct in that it deals with a point H at a definite distance from the center of the photograph.

In making computations for a table of differences between θ and θ' a value may be assigned to either θ or φ for any setup of f, i, Ch, and h, and the value of the other computed. If f, i, CH, and h are kept the same for various assigned values of θ or φ, the following remain constant:

$$SV = f \cos i$$

and $VC = f \sin i$. After the angular differences have been computed, the lateral displacements corresponding to those differences may be determined. In Table 6 the results of a set of computations are given for a particular setup as stated at its head. Opposite each angular error is placed the corresponding lateral displacement (with respect to the ray from the center to the point) of the image point. The table is carried only halfway round the circle. From 180° on around to 360° the errors are repeated in reverse order, rising to a maximum at about 260°, at 270° equaling that at 90°, and gradually returning to zero at 360°.

TABLE 6.—ERRORS IN ANGLES TURNED ABOUT THE CENTER OF A VERTICAL AERIAL PHOTOGRAPH AND THE CORRESPONDING LATERAL DISPLACEMENTS

For $f = 8\frac{1}{4}$ inches, $i = 2°$, and $h = 0.3$ inch, when the distance of the point from the center is 4 inches

θ, degrees	Angular error	Lateral displacement of the point, inches
	′ ″	
0	0 0	None
10	1 14.3	0.0014
20	2 28.5	0.0029
30	3 42.0	0.0043
40	4 54.0	0.0057
45	5 29.1	0.0064
50	6 3.2	0.0070
60	7 7.2	0.0083
70	8 3.4	0.0094
80	8 48.9	0.0103
90	9 20.3	0.0109
100	9 34.6	0.0111
110	9 29.4	0.0110
120	9 2.9	0.0106
130	8 14.7	0.0096
140	7 5.6	0.0083
150	5 37.7	0.0065
160	3 54.3	0.0045
170	2 0.0	0.0023
180	0 0	None

Using that end of the principal line which stretches away from the vertical point (*CF* of Fig. 57) as the zero line of measurement, the angles at the center of the photograph are smaller than the corresponding horizontal angles, both to the right and to the left. When the lines forming the angle bracket the principal line, the error in the included angle is the sum of the errors in the two component angles (one on each side of the principal line). For example, the angle between a 45° and a 315° line would be about 11 minutes too small compared with the corresponding horizontal angle of about 90° or 11 minutes too large compared with the horizontal angle of about 270° measured through the other sector of the circle.

Figure 58 is a graph prepared from Table 6, the angular errors having been laid off as linear units along the radius vectors of the several values of θ, turned from the principal line, the end on the upper side of the photograph being zero. This shows the positions of the maximum errors to be approximately opposite the vertical point. When i, f, or h is changed, the effect is to shift slightly the directions of the maximum vectors as well as to change their values. The line through the center of the photograph at right angles to the principal line, being actually parallel to the true axis of tilt of the photograph, is labeled axis of tilt.

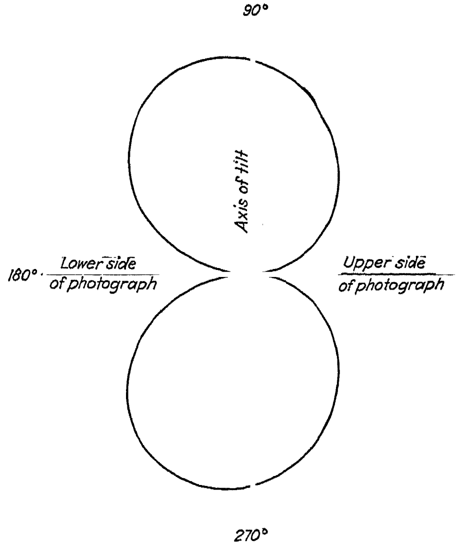


FIG. 58.—Diagram showing how angles measured about the optical center (principal point) of a vertical aerial photograph are deranged by tilt and relief combined.

Table 7 has been prepared from computations of errors in angles at the centers of tilted aerial photographs of three different focal lengths for $\theta = 110°$, which is close to a mean position of maximum error.

This table shows that the focal length of the photograph has but a small effect on the error and that the error increases rather rapidly as the angle of tilt increases. A significant fact obtained by comparing Tables 6 and 7 is that the angular error decreases as the distance from the center increases, but over the scope of single-lens and multilens verticals the corresponding lateral displacement remains nearly constant for given values of θ, f, and i.

Table 8 gives a comparison of near-maximum errors at the centers of aerial photographs of different focal lengths.

Summary.—At the scale 1:18,000, 0.3 inch corresponds to a height (ground relief) of 450 feet. The tables show that the maximum lateral displacement of any point, with respect to the center, which can occur from that amount of relief and a tilt of 2° is a little over 1/100 inch. Plotting from a photograph of that scale to a map scale of 1:24,000, the corresponding displacement of no point location would exceed 0.01 inch from this cause. It should be understood that this is the maximum displacement which would be liable to occur and that over much of the photograph lateral displacements would be too slight to have any appre-

TABLE 7.—ERRORS IN ANGLES AT THE CENTER OF A VERTICAL AERIAL PHOTOGRAPH AND THE CORRESPONDING LINEAR DISPLACEMENTS

For h = 0.3 inch and θ = 110°

Angle of tilt, degrees	Focal length, inches	Distance of point from center, inches	Angular error	Lateral displacement of the point, inches
			′ ″	
1	6	12	1 40.2	0.0058
2	6	12	3 42.6	0.0129
3	6	12	6 7.4	0.0214
4	6	12	8 54.2	0.0311
5	6	12	12 3.5	0.0421
1	8¼	12	1 33.6	0.0057
2	8¼	12	3 38.9	0.0127
3	8¼	12	6 0.8	0.0210
4	8¼	12	8 44.4	0.0305
5	8¼	12	11 49.7	0.0413
1	12	12	1 37.0	0.0057
2	12	12	3 35.8	0.0126
3	12	12	5 55.5	0.0207
4	12	12	8 36.5	0.0300
5	12	12	11 38.4	0.0406

TABLE 8.—NEAR-MAXIMUM ERRORS AT THE CENTER OF A VERTICAL AERIAL PHOTOGRAPH

For different focal lengths when i = 2°, h = 0.3 inch, and CH = 12 inches

f, inches	θ, degrees	Angular error
		′ ″
6	114	3 44.6
8¼	112	3 40.0
12	110	3 35.8

ciable effect. Therefore, when the relief is not greater than 500 feet, the tilt of photographs does not exceed 2°, and the map scale is not larger than 1:24,000, satisfactory plotting can be done by the radial-line method. If the map scale is 1:62,500, or 1 mile to the inch, relief as great as 1,000 feet can be satisfactorily dealt with.

178. Effect on plotting from one photograph to another.—In making a radial-line plot to establish the positions of secondary control points from the photographs themselves, lateral displacements of image points have a varied effect. If, as is the case with well-taken photographs, they are so tilted that the principal lines range in all directions, errors will tend to balance one another. Since the effect in any single photograph is that angles on the upper side of the photograph are too small, while those on the lower side are too large, the located position of a center point will be too far from the preceding center location if the upper side of the photograph is facing backward; too near, if facing forward. If the upper side of the photograph faces to one side of the flight line, the location will be displaced to the *other* side. It is evident that if a camera should be mounted in a fixed position so that all photographs are tilted in one direction and relief affects all alike, as would be the case when the flight was

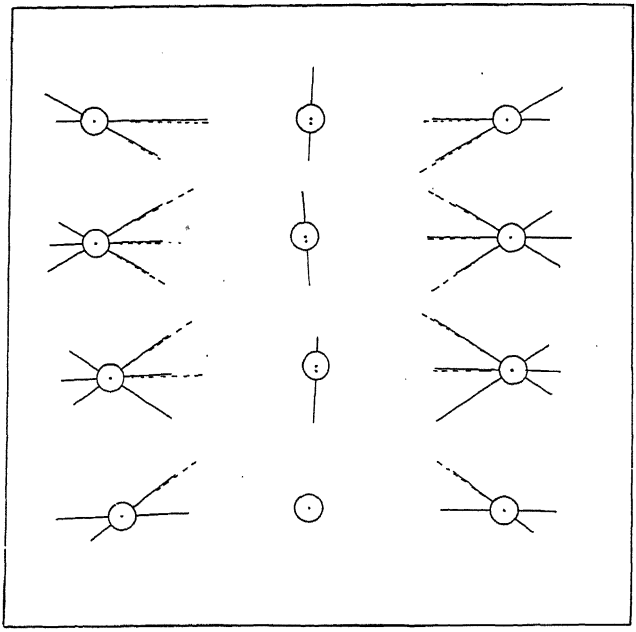


FIG. 59.—Radial-line plot to show the effect of tilt when the trailing edges of the photographs are on the upper sides. Only the center points are displaced. The displacements are greatly exaggerated.

over and along a valley and the photographs included higher terrain in their outer parts or the flight was along a ridge with a parallel valley on each side, the photographs stretching from one valley to the other, the plotting would be done with one of the following tendencies:

(*a*) All locations of center points would be beyond or short of their true positions, but locations of points along the lateral borders of the photographs would be practically true; *i.e.*, the scale in general would not be affected. Orientation from center to center would be correct; or

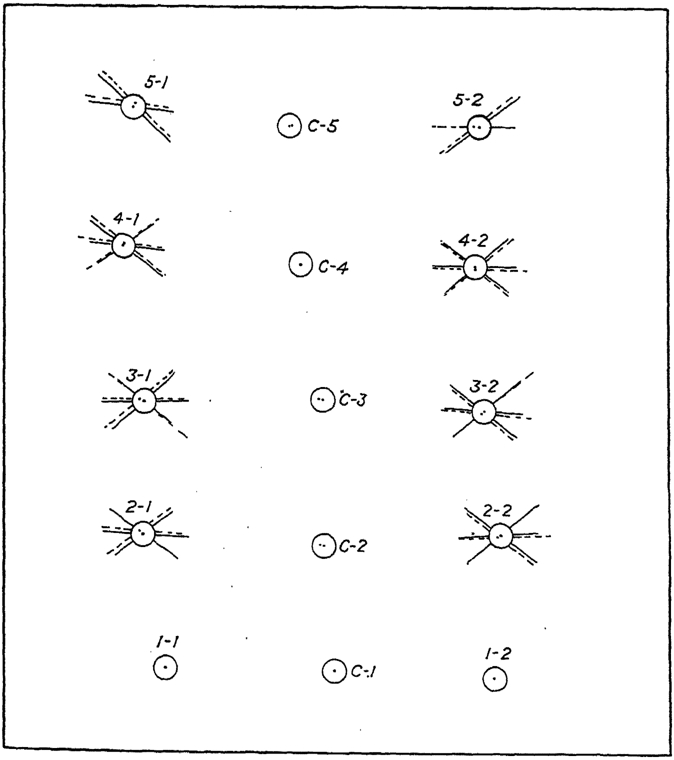


FIG. 60.—Radial-line plot to show the effect of tilt when all of the photographs are tilted upward to the left. Displacements are greatly exaggerated.

(*b*) The plot would have a bow to the left or right depending on whether the upper sides of the photographs were on the left or right side, respectively, and triangles of error would increase in size.

It should therefore be concluded that under present conditions, although it is not possible to obtain untilted photographs, the best procedure to follow in aerial photography is to adjust the camera by level bubbles or any other gravity indicator before each exposure. By so doing not only will the camera be kept as nearly level as practicable, but the axis of tilt will be varied over the entire range of azimuths, thereby avoiding tilts setting in one general direction.

Figure 59 indicates the tendency when the upper sides of the photographs face backward. Full radial lines indicate directions as obtained from the photographs; dotted lines, true directions. Points at the centers of circles are the positions that would be obtained; eccentric points are the true positions.

Figure 60 shows the tendency when all the photographs are tilted upward to the left of the flight line, the flight line is over a valley, and ridges appear in the left and right sides. Points 1-1, *C*-1, and 1-2 are correctly located in advance. Points *C*-2 and *C*-3 are shifted to the right; then the great bow sets in to shift to the left the center points later located. The skewing of the positions of the lateral points is evident.

It should be understood that angular errors are greatly exaggerated in Figs. 59 and 60.

Practical considerations.—In making a radial-line plot to be used as control, care should be taken to avoid selecting for location image points of objects that are much higher or much lower than the average level of the terrain. This is not always possible, but a preliminary stereoscopic examination of the photographs discloses points to be avoided. There is much more opportunity to select favorable points on photographs covering large areas than on those of more limited scope. Multilens photographs offer the greatest advantage.

CHAPTER VIII

PRINCIPLES OF PHOTOGRAPHIC TRANSFORMATIONS AND TRANSFORMING PRINTERS

179. Pinhole photography.—For the purpose of demonstrating the principles involved in making transformed photographs, it is helpful first to consider theoretical projections through a pinhole.

It is evident from Fig. 61 that pinhole photography, assuming the aperture to be very small and disregarding lighting difficulties, can be accomplished with the image plane either placed parallel to the plane of the object or swung obliquely to it. Moreover, other than placing it where its image will be cast, there is little restriction on the position of the object, and its plane may be placed obliquely to the image plane. For simplicity it is here assumed that the object has only width and length—a photographic negative, for example. Wide latitude is afforded both as to distance of the negative from the pinhole and in the orientation of its plane, the sole requirement being that the object must be placed so that its image will be cast through the aperture. Similarly there is wide latitude with respect to the image plane. The arrangement for obtaining an image unchanged in shape is, of course, to have the negative and image planes parallel to each other and perpendicular to the projection axis. In other arrangements (departure of either negative plane or image plane from perpendicularity to the projection axis) change in shape of the image will depend on the angle between the two planes. When that angle approaches 90°, the distortion becomes so great that there is little similarity between object and image. Within reasonable limits, however, practical use can be made of this type of projection, a lens being substituted for the pinhole.

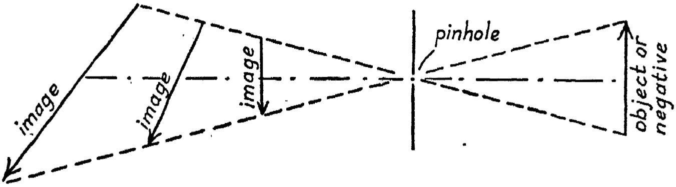


Fig. 61.—Setup for pinhole photography.

180. Use of transformation printing.—Transformation printing has two specific uses in aerophotography: (*a*) to project the image of one of two related photographs into the plane of the other photograph, (*b*) to project the image of a tilted photograph into a horizontal plane. In the first use, oblique negatives obtained with multiple-lens cameras are transformed into a common plane and may be joined together to form composite verticals. In the second, slightly tilted vertical photographs are transformed into true verticals.

181. Pinhole photography assists in analyzing the transformation problem.—Pinhole images are not restricted by the fundamental requirements of optics to obtain sharpness of focus with lenses, but images may be caught at any reasonable distance from the aperture provided there is enough illumination of the subject (negative). An understanding of pinhole photography clarifies the problem of transformation printing with a lens, making it easier to see why exact settings must be made. The particular task of printing from oblique negatives taken in a group about a central negative, so that all prints may be joined together to form a composite photograph of great field, will be dealt with first. In this, which is a special case, the photograph to be printed has the same focal length as the negative. This condition considerably simplifies the problem. A solution of the general problem of transformation printing, when the focal length of the photograph differs from the focal length of the negative, is given in Arts. 194 to 199.

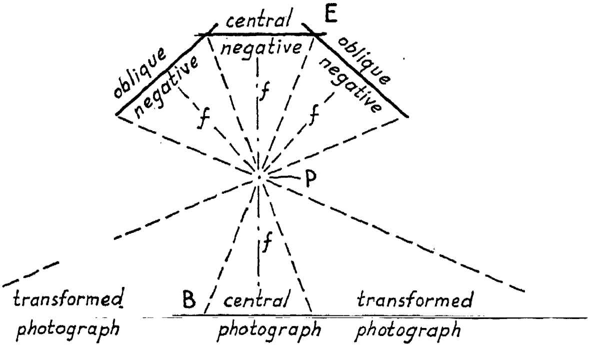


FIG. 62.—Vertical section through a multiple-lens camera showing relationships of negatives and photographs.

Figure 62 shows in its upper part a group of three negatives in vertical section arranged in position as in effect the central negative and two opposite oblique negatives of the five-lens camera are arranged; in the lower part it shows the corresponding prints made from those negatives in proper position with respect to one another. The focal length of all the negatives and prints, being the same, is designated by f. The print from the central negative may be considered as having been made by placing the sensitized paper parallel to the negative and at a distance from the pinhole P equal to the focal length of the negative. In order that the photographs made from the oblique negatives may join the central photograph properly to maintain a common perspective center and the full properties of a perspective, their images, too, must be cast into this same image plane. The figure shows how, by placing the negative at *DE* and the sensitive paper at *CB*, this might readily be done but for the limitations of pinhole photography. As pinhole photography is too slow for practical printing, it becomes necessary to make use of a lens to do the printing from the oblique negatives.

182. Transformation printing with a lens.—Use of a lens imposes very strict optical as well as geometrical requirements. Not only must the basic principle for sharp focus be adhered to in all parts of the image, but the angles between the plane of the objective and the negative and image planes must be accurately laid out. Placement of the negative at exactly the right place with respect to the lens is also essential to make the transformation accurate, and allowance must be made for the separation of the nodal points of the lens. Finally, it is necessary to correct for the refraction of the light passing through the thin glass plate on which the negative rests.

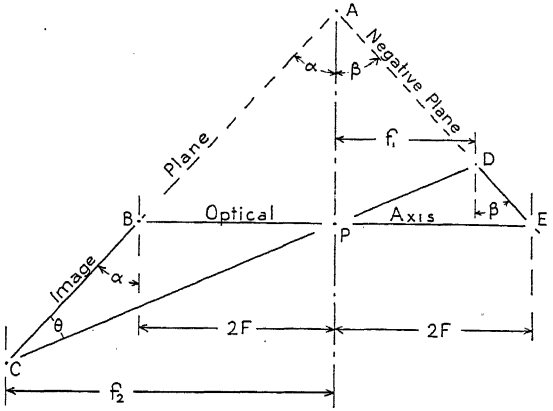


FIG. 63.—Setup for transformation printing when the focal length of the print is to equal the focal length of the negative.

183. Perspective problems solved by use of an important theorem.—The particular relationships that must be provided for are explained with reference to Fig. 63. This represents a plane that includes the optical axis of the lens and cuts through the negative, image, and lens planes at right angles to them. A theorem of geometry that is important in the study of this figure and indeed of many problems dealing with perspectives is: When two *mutually inclined* planes are intersected by another plane that is *parallel* to their *intersection* all three lines of intersection are *parallel.* For example: The line of the negative at *E* is necessarily parallel to the line of the image at *B*, because the plane *EB* (perpendicular to the plane of the figure) is parallel to the line at *A*, the intersection of plane *EA* and *BA*.

184. Analysis of the transformation problem.—(*a*) Keeping in mind that three dimensions are to be dealt with although only two appear in the figure, the line *E*, which is the same as *E* in Fig. 62, *viz.*, the intersection of the two negative planes, and whose length equals the width of the negative, stands perpendicular to the drawing and is to be photographed at its true length. Therefore it must be placed at the distance $2F$ (double the focal length of the projection lens) from the front nodal plane. Likewise, its image at *B* must stand at the distance $2F$ from the rear nodal plane. (In the diagram, the two nodal planes are assumed to be together and to coincide with the plane of the lens. The measure taken to allow for the distance between the two nodal planes is explained in Art. 187.) Hence that plane, which contains the two lines *E* and *B* and which is perpendicular to the plane of the drawing, necessarily embraces two similar and equal triangles whose common apex is at *P*. It cuts the base *E* of one of these triangles from the negative and the base *B* of the other triangle from the image plane. It is evident from Fig. 62 that because the focal length of the central negative equals the focal length of the oblique negative, *B* must equal *E* to provide a match in dimension between the contact print made from the central negative and the transformed print made from the oblique negative, since in each negative the line *E* is at the same distance from the perspective center *P*. The separation of the two lenses of the camera by but a few inches can be neglected without introducing an appreciable error, for the camera is ordinarily several thousand feet from the terrain photographed.

(*b*) The position of the image plane is established by any two lines of the negative whose positions in reproduction comply with equation (1), Art. 3. So employing the outermost edge *D* of the negative as the second line, the requirement is that it must fall at the distance f_1 from the front nodal plane, and the corresponding image line at *C* must fall on a line that is at the distance f_2 from the rear nodal plane, the distances f_1 and f_2 being in accordance with equation (1). Referring again to Fig. 62, it is evident that the line at *D*, which is perpendicular to the plane of the drawing, when reproduced at *C*, will be enlarged in the ratio $CP{:}PD$. Having proper preliminary data about the camera used to obtain the negatives, this ratio can be readily calculated.

(*c*) Returning now to Fig. 63 and still disregarding the separation of the nodal points, the two lines *BE* and *CD* in the plane of the drawing must cross each other at the center of the lens.

When the three conditions stated above are fulfilled, the entire transformed photograph will be true in perspective, because then all lines of the negative that are parallel to the lines at D and E will be reproduced so as to maintain the general equation (1).

(d) It is evident that the following equations are true:

$$\sin \beta = \frac{2F - f_1}{DE} \tag{16}$$

$$\sin \alpha = \frac{f_2 - 2F}{CB} \tag{17}$$

185. Transformation printing when the focal lengths of print and negative are equal.—Whenever the focal length of the transformed print equals the focal length of the negative, computations of α and β show that these two angles are equal. This being true, it necessarily follows that the only position in Fig. 63 that the line E can occupy to give an image at B equal to it in length is on the optical axis of the lens that is to produce the image, because the line BE must pass through the point P and both PE and PB equal $2F$. Therefore a fourth equation

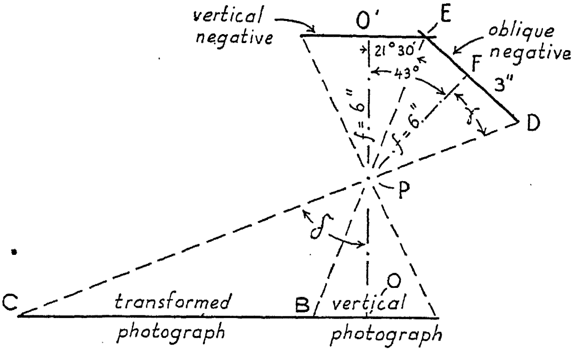


FIG. 64.—Geometrical relationships between the central and an oblique chamber of the T-3A camera.

$$AP = \frac{2F}{\tan \alpha} = \frac{2F}{\tan \beta} \tag{18}$$

completes the set of equations required to fix the positions of the negative and its image relative to the projection lens and its axis to make a correctly transformed print.

186. Example of computation for transformation setup when the focal lengths are equal.—The following concrete example is used to illustrate the procedure. Figure 64 is given to show the geometrical relationships existing in two chambers of an aerial camera, it being desired to determine the proper relationships for printing from the oblique negative a photograph that will be projected into the plane of the photograph made from the central negative by direct contact printing, the focal length of the oblique negative being equal to the focal length of the central negative.

Let the focal length of each negative be 6 inches; the angle between the two optical axes, 43°; the length of the oblique negative, 6 inches; and let each negative be centered on its optical axis. Using seven-place logarithms:

$$EF = O'E = OB = 6 \tan 21°30' = 2.3634632 \text{ inches}$$

which is the offset of the center from the optical axis of the projection lens.

$$DE = 3 + 2.3634632 = 5.3634632 \text{ inches}$$

$$\tan \gamma = \frac{FD}{f} = \frac{1}{2}$$

Therefore

$$\begin{aligned} \gamma &= 26°33'54.189'' \\ \delta &= \gamma + 43° \doteq 69°33'54.189'' \\ OC &= 6 \tan 69°33'54.189'' = 16.103452 \text{ inches} \\ CP &= \frac{6}{\cos 69°33'54.189''} = 17.184908 \text{ inches} \\ PD &= \frac{6}{\text{cosin } 26°33'54.189''} = 6.7082046 \text{ inches} \\ CB &= OC - OB = 13.7399888 \text{ inches} \\ \frac{CP}{PD} &= 2.5617742 \end{aligned}$$

The ratio CP/PD is used in connection with Fig. 63 ($f_2 = 2.5617742 f_1$) to obtain the values of f_1 and f_2.

Let the focal length of the projection lens be 6 inches. Then substitutions in the formula $1/F = 1/f_1 + 1/f_2$ give

$$\frac{1}{6} = \frac{1}{f_1} + \frac{1}{2.5617742 f_1}$$

whence

$$f_1 = 8.342128 \text{ inches}$$

and

$$f_2 = 8.342128 \times 2.5617742 = 21.37064 \text{ inches}$$

$$\sin \beta = \frac{2F - f_1}{DE} = \frac{12 - 8.342128}{5.3634632}; \qquad \beta = 43°0\,00''$$

$$\sin \alpha = \frac{f_2 - 2F}{CB} = \frac{21.37064 - 12}{13.7399888}; \qquad \alpha = 42°59'59.77''$$

$$AP = \frac{12}{\tan 43°} = 12.86842 \text{ inches}$$

In this case the angles α and β are actually equal to the dihedral angle between the central and oblique negatives, 43°. Lack of refinement in the calculations causes slight errors in the results.

187. Provision for separation of nodal points.—Having the data resulting from the computations and knowing the distance between the two nodal points of the lens as determined by optical test, a drawing can be made like Fig. 63 except that, instead of a single point P and a single objective plane, there will be two nodal points and two nodal planes, the right part of the figure being placed with P on the front nodal point and the left part with P on the rear nodal point. It has been shown that the line across the negative at E must be placed on the optical axis of the lens and that it must be oriented parallel to the intersection of the negative plane with the plane of the objective (front nodal plane). Usually collimation marks are placed on the negative holder for orientation and to indicate the position of the principal point, which is offset from the optical axis an amount equal to EF (Fig. 64).

188. Negative and image planes meet in the objective plane.—Instead of giving a general proof (which is tedious and lengthy) that the plane of the negative and the image plane meet in the plane of the objective, it will merely be stated that this is necessary to obtain a correctly transformed image and that numerous tests have verified the fact. A method of proving the statement in any concrete setup is given in Art. 197.

189. Lens suitable in a particular case.—Graphic construction of diagrams, giving F (of the transforming lens) various values, readily leads to proof of whether or not a particular lens is suitable for use in a particular printer. For example, if Fig. 63 is drawn for $F = 6$ inches, whereas the negative and image dimensions are the same in proportion as for Fig. 64, it is demonstrated that the lens must have a corrected angular field of 54°. Had F been taken at 5 inches, the lens would have to give a satisfactory image through more than 70°, a greater corrected field than is obtainable in most types of lens. On the other hand, if F were taken at 7 inches, the required field would be considerably less than 50°, but the dimensions of the printer would have to be somewhat larger than for a lens of 6-inch focal length, and the angle of incidence of the rays of the outer border C would be greater; *i.e.*, angle θ would be smaller.

190. Importance of matching lenses of multiple-lens cameras.—It is evident that for a single fixed transforming printer to suffice for making prints from negatives of the several oblique chambers of a multilens camera, the lenses of the oblique chambers must all be matched in focal length. Further simplification is obtained by having the central lens of the aerial camera matched in focal length with the lenses of the oblique chambers, because then α equals β. Although not important with respect to fixed transforming printers, this simplification is of value in rectifying cameras designed for correcting tilted photographs (the second use mentioned in Art. 180).

191. Correction for glass plate in printer.—In the construction of a printer, in addition to the relationship mentioned above, it is necessary to correct for refraction of light going from the negative to the lens through a glass plate that supports the negative. The final adjustment is made by trial printings; and when the parts have been brought together to give satisfactory prints, they are rigidly and permanently joined in a fashion to prevent change or derangement. It is customary to provide a printer for each multilens camera, and one printer is sufficient because all lenses of the oblique chambers of multilens cameras are carefully selected and matched as to focal length and quality.

192. Special features of the T-3A Printer.—Provision is made on this printer for using negatives in rolls, for setting them accurately into position, and for guarding them against the heat of the lamp. Convenient facilities are also provided for illuminating the negative while it is being brought into place and to open and close the shutter of the lens for exposures. A vacuum pump holds the sheet of photographic paper accurately in the image plane.

193. Limitations of transformation printing.—The practical limits in making transformed prints are reached when the angle θ (Fig. 63) becomes so small and the enlargement of the outer part of the image so great that details fail to register clearly. This limit has been found by experience to lie in the vicinity of $\theta = 20°$ for transformations made with the image of the nearest terrain approximately at the same scale that it has in the negative, which is the condition encountered in making the wing parts of five-lens photographs.

194. Setup for transformation printing—general case.—Figure 65 is drawn in a plane that stands perpendicular to two planes HA and CA, inclined to each other at the angle i. In the upper plane a photographic negative DH, having its perspective center at P and a focal length f_n, is to have its image projected into the lower plane. Lines drawn from D and H through P to intersect the lower plane give the true length of the photograph CG; and PK, the perpendicular to this plane, is its focal length f_p. Extending KP a distance equal to f_p upward to N, and drawing there a line parallel to the lower plane gives at its intersection E with the negative plane the position of that line of the negative which in the photograph will be exactly as long as the width of the negative; *i.e.*, the photograph will have at this line a width the same as the width of the negative. This is true because a line drawn from E through P to the lower plane gives at B a point that is at a distance from P equal to PE.

The line of the negative at E will therefore be the datum, or control line, of the setup. It will be designated the *control line.* The position of the image of the control line is therefore at B, for obviously it must be at a distance from P equal to PE. Letting F designate the focal length of the printer lens, both E and B must therefore be placed at the distance $2F$ from the plane of the objective, or, more exactly, from the front and rear nodal planes, respectively. Extending NE to meet at O the perpendicular drawn from P to the negative, the distance PO is obtained from the relationship $PO = f_p/\cos i$. The perpendicular PO passes through the principal point S of the negative. The evident relationships give

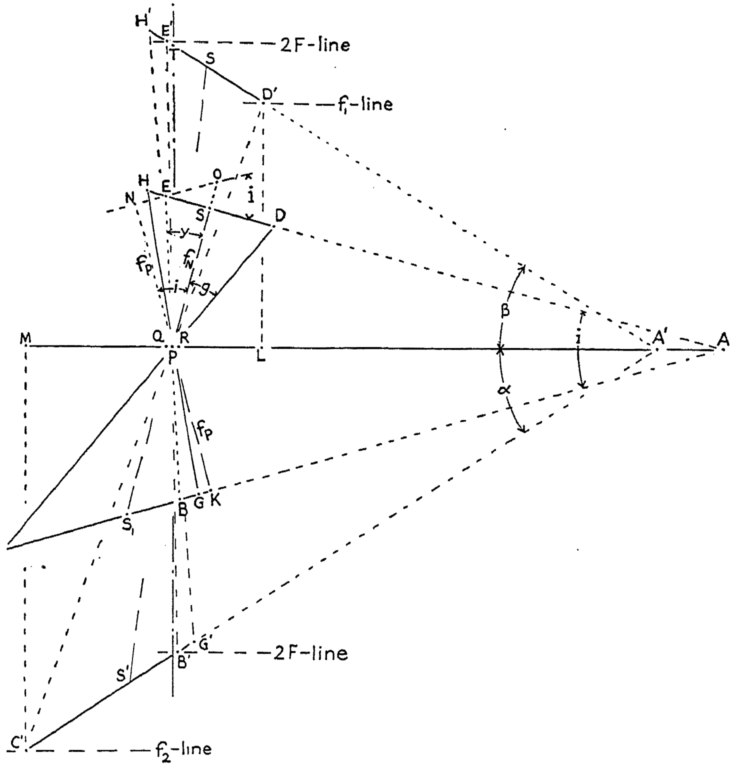


FIG. 65.—Relationships in the general setup for transformation printing.

$$SE = SO \tan (90^\circ - i) = (PO - f_n) \tan (90^\circ - i)$$

$$SD = f_n \tan g$$

$$\tan y = \frac{SE}{f_n} \qquad \text{and} \qquad PE = \frac{f_n}{\cos y}$$

The angle g being known from data of the negatives, PD is computed; then PC and CK from the right triangle PCK, the angle at P being equal to $i + g$. In the triangle PBK, the angle at P equals $i - y$, permitting BK to be computed. Then $BC = CK - BK$.

195. Position of the lens.—Now let P be the center of the lens to be used for printing by projection, and let PA be its plane. Its optical axis will then be perpendicular to PA. On both sides of the line PA let a line be drawn parallel to PA at a distance from P equal to $2F$. From the values of PD and PC as computed and the value of F known in advance, the values of f_1 and f_2 are computed by use of the relationship

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_1(PC/PD)}$$

in which $f_2 = f_1(PC/PD)$.

On the upper side of P a line is drawn parallel to PA at a distance equal to f_1; on the lower side, another parallel line distant f_2. In order to make a correct transformation, the negative must be so placed that E will be on the upper $2F$ line and D on the f_1 line. Thus placed, the following equation is true: $\sin \beta = (2F - f_1)/(ED)$, from which the angle β may be computed. To have a correct photograph the image of the line at E must fall on the lower $2F$ line and that of the line at D on the f_2 line, and the angle α between the image plane and PA must be such that $\sin \alpha = (f_2 - 2F)/(BC)$. As a fundamental condition of the optics involved, the lines E and B must either lie on the optical axis of the lens or else be on opposite sides and equidistant from the optical axis, since each must be equidistant from the plane of the lens.

196. Position of the negative.—Proceeding then to set the negative at $H'E'D'$ so that it stands at the angle β to PA, E' is on the upper $2F$ line and D' on the f_1 line, the image of the negative being cast in a plane $C'G'$ that extended meets the plane of the negative at A' and is inclined to PA at the angle α; so that C' falls on the f_2 line and B' on the lower $2F$ line, so that E' is as far to one side (left in this instance) of the optical axis as B' is to the other side of it, and so that the line $E'B'$ will pass through P, the following additional necessary data are obtained:

Let Q be the foot of a perpendicular dropped from E' to PA and R be the foot of a perpendicular dropped from B' to PA. Then $QP = PR$. Through similar triangles $A'B'{:}2F = A'C'{:}f_2$; so

$$A'B' = 2F \times \frac{B'C'}{(f_2 - 2F)} = \frac{2F \times BC}{(f_2 - 2F)}$$

and $A'R = A'B' \cos \alpha$. Also through similar triangles

$$A'D' = DE \times \frac{f_1}{(2F - f_1)};$$

$A'Q = (A'D' + ED) \cos \beta; PQ = PR = \frac{1}{2}(A'Q - A'R)$ and

The offset of the principal point S from the optical axis along the plane of the negative is $SE' - E'T = SE - PQ/\cos \beta$. The foregoing equations and established relationships are sufficient to make the required setup.

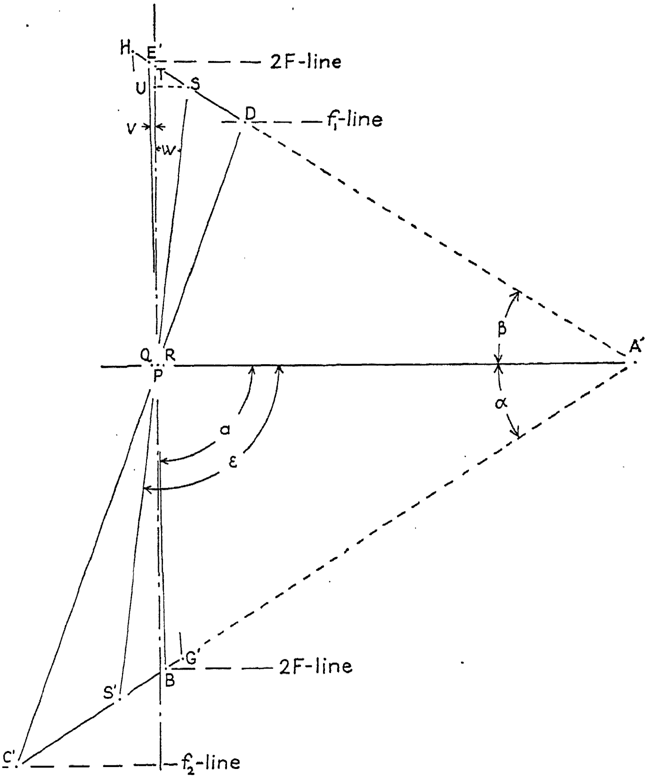


FIG. 66.—Diagram to illustrate the proof of the general setup for transformation printing.

197. Proof that negative and image planes meet in the plane of the objective.—In the preceding, two lateral lines of the negative and the corresponding lines of the image were made to fulfill the imposed conditions, assuming that the three planes concerned met in a common line at A'. Proof that this assumption was correct will be furnished if it can be shown that the image of a third lateral line of the negative will fall into its proper position under the setup. In order to avoid confusion, use is made of another diagram (Fig. 66), which is a duplicate in part of Fig. 65. Relationships are to be developed whereby in a concrete example it can be shown that the image of the lateral line through the principal point S of the negative will be reproduced at its proper position and length in the image plane.

Connect S with P and extend the line to S'. Then $\tan v = PQ/2F$; $E'T = PQ/\cos \beta$; $SE' = SE$ from above; $ST = SE' - E'T$;

$$US = ST \cos \beta;$$

$UT = ST \sin \beta$; $\tan w = US/(2F - UT - PQ \sin \beta)$; $\tan \alpha = 2F/PR$; $e = \alpha + v + w$. In the triangle $A'PS'$, two angles and the side $A'P$ being now known, $A'S'$ is computed from

$$A'S' = \frac{A'P \sin e}{\sin (180^\circ - \alpha - e)},$$

and $S'B' = A'S' - A'B'$. $S'B'$ should equal BS_1 (Fig. 65) which is found from $BS_1 = S_1K - BK = f_p\ [\tan i - \tan (i - y)]$. In making a computation according to these equations seven- or eight-place logarithms should be used to assure sufficient refinement.

198. Conclusions.—(*a*) Computations for transformation setups are greatly simplified if the focal length of the print is the same as the focal length of the negative. In this case the negative and image planes are inclined at equal angles to the plane of the objective (*i.e.*, $\alpha = \beta$). Moreover, the "control line" and its image fall on the optical axis of the printer lens.

(*b*) Rectifying cameras to be used for determining tilt as well as for making prints, or by which the setting is determined by an optical process of making the image fit control points, are materially simplified in operation by having the focal distances of the image and the negative equal to each other, *i.e.*, by having the focal length of the photograph equal to that of the negative.

(*c*) There is no particular advantage to be gained from using a printer lens matched in focal length with the aerial camera lens; but when the angle of inclination is large, it is well to have the two focal lengths approximately equal or not differing greatly. In printing to correct for small tilts (not exceeding a few degrees) there is no question of large angles of incidence, so it is best to use a lens having a longer focal length than that of the aerial camera.

199. Example of computation schedule for transformation setup—general case.—Constant data of the negative and projection lens are assumed to be:

$f_n = 150$ mm. $f_p = 155$ mm. $i = 30^\circ$
Negative length = 140 mm. Negative width = 140 mm.
Principal point is at center of negative, so $DS = 70$ mm.
$F = 160$ mm.

$\tan g = DS/f_n$	$\log (DS)70$	$= 1.84510$
$g = 25^\circ 1'$	$- \log (f_n)150$	$= 2.17609$
$i + g = 55^\circ 1'$	$\log \tan g$	$= 9.66901$
$PO = f_p/$	$\log (f_p)155$	$= 2.19033$
$PO = 178.98$ mm.	$- \log (\cos i)30^\circ$	$= 9.93753$
	$\log PO$	$= 2.25280$
$SO = PO - f_n = 178.98 - 150 = 28.98$ mm.		
$SE = SO \tan (90^\circ - i)$	$\log (SO)28.98$	$= 1.46108$
	$+ \log \tan (90^\circ - i)60^\circ$	$= 0.23856$
$SE = 50.077$ mm.	$\log SE$	$= 1.69963$
$\tan y = SE/f_n$	$\log (SE)50.077$	$= 1.69963$
	$- \log (f_n)150$	$= 2.17609$
$y = 18^\circ 27.67'$	$\log \tan y$	$= 9.52354$
$PE = f_n/\cos y$	$\log (f_n)150$	$= 2.17609$
	$- \log \cos (y)18^\circ 27.67'$	$= 9.97706$
$PE = 158.14$ mm.	$\log PE$	$= 2.19903$
$PD = f_n/\cos g$	$\log (f_n)150$	$= 2.17609$
	$- \log \cos (g)25^\circ 01'$	$= 9.95722$
$PD = 165.527$ mm.	$\log PD$	$= 2.21887$
$PC = f_p/[\cos (i + g)]$	$\log (f_p)155$	$= 2.19033$
	$- \log \cos (i + g)55^\circ 1'$	$= 9.75841$
$PC = 270.346$ mm.	$\log PC$	$= 2.43192$
$CK = f_p \tan (i + g)$	$\log (f_p)155$	$= 2.19033$
	$+ \log \tan (i + g)55^\circ 1'$	
$CK = 221.5$ mm.	$\log CK$	$= 2.34537$
$BK = f_p \tan (i - y)$	$\log (f_p)155$	$= 2.19033$
	$+ \log \tan (i - y)11^\circ 32.33'$	$= 9.30996$
$BK = 31.64$ mm.	$\log BK$	$= 1.50029$
$BC = CK - BK = 189.86$ mm.		
$\dfrac{PC}{PD} = \dfrac{270.346}{165.527}$	$\log (PC)$	$= 2.43192$
	$- \log (PD)$	$= 2.21887$
Ratio $= 1.6332$	log ratio	$= 0.21305$
$f_1 = 2.6332F/1.6332$	$\log (F)160$	2.20412
	$+ \log 2.6332$	$= 0.42048$
$f_1 = 257.965$ mm.	$\log f_2$	$= 2.62460$
$f_2 = 421.32$ mm.	$- \log 1.6332$	$= 0.21304$
	$\log f_1$	$= 2.41156$

$\sin\beta = (2F - f_1)/ED$	$\log\,(2F - f_1)62.035$	$= 1.79264$
$= 62.035/120.077$	$-\log\,(ED)120.077$	$= 2.07946$
$\beta = 31°6.4'$	$\log\sin\beta$	$= 9.71318$
$\sin\alpha = (f_2 - 2F)/BC$	$\log\,(f_2 - 2F)101.32$	$= 2.00570$
	$-\log\,(BC)189.84$	$= 2.27838$
$\alpha = 32°15.45'$	$\log\sin\alpha$	$= 9.72732$
$A'B' = \dfrac{2F \times BC}{f_2 - 2F}$	$\log\,(2F)320$	$= 2.50515$
	$+\log\,(BC)189.84$	$= 2.27838$
$A'B' = 599.56$ mm.		4.78353
	$-\log\,(f_2 - 2F)101.32$	$= 2.00570$
	$\log A'B'$	$= 2.77783$
$= A'B'\cos\alpha$	$\log A'B'$	$= 2.77783$
$= 507.02$ mm.	$+\log\cos\,(\alpha)32°15.45'$	$= 9.92719$
	$\log A'R$	$= 2.70502$
$A'D' = \dfrac{DE \times f_1}{2F - f_1}$	$\log\,(f_1)257.965$	$= 2.41156$
	$+\log\,(ED)120.077$	$= 2.07946$
		4.49102
499.32 mm.	$-\log\,(2F - f_1)62.035$	$= 1.79264$
	log	$= 2.69838$
$A'Q \quad (A'D' + ED)\cos\beta$	$\log\,(A'D' + ED)619.396$	$= 2.79197$
	$+\log\cos\,(\beta)31°6.4'$	$= 9.93258$
$A'Q \quad 530.34$ mm.	$\log A'Q$	$= 2.72455$

$PQ = PR = (A'Q - A'R)/2 = 11.66$ mm.
$A'P = A'R + PR = 518.68$ mm.

Offset of principal point $= SE - \dfrac{PQ}{\text{[illegible]}}$	$\log PQ$	$= 1.06670$
	$-\log\cos\beta$	$= 9.93258$
		1.13412

$= 50.077 - 13.618$ mm.
$= 36.459$ mm.

The influence of the ratio PC/PD on the results is marked. Its value should be determined with refinement. Although five-place logarithms will give results accurate enough for most practical purposes, it is recommended that seven-place tables be used for computations.

CHAPTER IX

PRINCIPLES OF STEREOSCOPY AND APPLICATION TO PHOTOGRAMMETRY

200. Basis of stereo measurement.—Although the image received at the retina of the eye does not afford an opportunity to make measurements useful in photogrammetry, employment of a transparent sheet in front of the eyes makes it possible to deduce significant fundamental facts about stereo observation, and these facts point the way to derivation of formulas and procedures of great practical value in making measurements with stereo pairs of photographs and in gaining quantitative information from such photographs.

In Fig. 67 suppose two eyes to be at L and R. The eyes are assumed to be looking horizontally toward objects (points) A and B lying on a line parallel to the eye base LR. A few inches in front of the eyes a vertical transparent sheet is placed directly across the lines of sight. Being at right angles to the vision for very distant objects, this sheet is necessarily parallel to the eye base. It is to serve as the plane of perspective projection for lines of sight extending from objects in the field of view to each of the eyes.

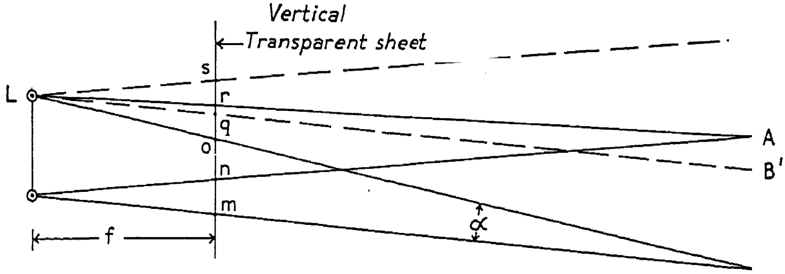


Fig. 67.—Fundamental condition of the stereo setup.

In this case parallax may be expressed as the angle between any pair of lines going from an object to the eyes, as α at the object B. It is generally not practicable to measure α at B; but if LB' is drawn parallel to RB, the angle BLB' will equal α, and the intercept oq in the transparent sheet provides satisfactorily for measuring it indirectly and hence of determining the position of B relative to LR. Therefore, by use of the interposed transparent sheet as the plane of perspective and measuring intercepted distances, such as oq, it is practicable to obtain equations that provide for fixing the positions of objects in the field of view. The position and orientation of the sheet may be completely controlled, and, since it is close at hand, measurements readily can be made on it. It is necessary to know the length of the eye base and the perpendicular distance from it to the sheet.

Let the eye base (distance between L and R) in Fig. 67 be designated by b; the perpendicular distance between the eye base and the transparent projection sheet, by f; and the perpendicular distance from the object to the eye base, by D. The lines of sight from B pierce the projection sheet at m and o, and LB' pierces it at q. Then, according to the meaning as herein employed, oq is the parallax of the object (point) B with respect to LR; and inasmuch as $BB' = LR = b$ and triangle Loq is similar to triangle LBB',

$$oq = \frac{b \cdot f}{D} \tag{19}$$

Consider also the parallax rs of the object A, which by a similar procedure gives $rs/f = b/D$, or $rs = b \cdot f/D$. Therefore $sr = oq$. A and B are any two points in the chosen horizontal plane and in the line parallel to the eye base; therefore all points in that line have equal parallax, or it may be stated that the parallax of all points in the line AB is a constant. Let this linear parallax be designated by p_1. It is evident that any other object in the horizontal plane will have a parallax p_2 which is less than p_1 if the object is farther from the eye base than the line AB or greater than p_1 if it is nearer. Therefore, for any given eye base b and distance f, D varies inversely with the parallax. Then letting p represent parallax, the general formula can be written

$$D = \frac{fb}{p} \tag{20}$$

It is practicable to have f quite small compared to D if its value can be accurately determined and if the parallax can be measured with care and refinement. If micrometric means of measurement are available, the ratio of f/D may be as small as 1/18,000 or 1/20,000 when the stereo base is as large as one-quarter or one-third of D.

THEOREM. *When measured in a plane that is parallel both to the stereo base and to the plane containing the objects, the parallax of such objects is a constant quantity.*

Figure 68 is a perspective view again showing the two eyes L and R looking horizontally through a vertical transparent sheet at a vertical plane that is parallel to the transparent sheet. The purpose now is to complete the proof of the theorem. In this general case parallax means the intercepted distance in the projection plane, measured along a line parallel to the eye base, the length of the intercepted distance being the shortest distance between two planes whose intersection is perpendicular to the eye base. Let *C* be any point in the vertical plane *BCEG*, and let *LRBG* be the horizontal plane which includes the eye base. Through *C* construct the vertical planes *RCB* and *LCB*, and through *L* construct a plane *LEG* parallel to the plane *RCB*.

The vertical plane *LCB* intersects the transparent sheet in the line *cb*, and the vertical plane *LEG* intersects it at *eg*. From the preceding demonstration with reference to Fig. 67, the distance *bg* is the parallax in the plane of the transparent sheet of the point *B*; and according to the explanation given in the preceding paragraph, *ce* is the parallax of the point *C*. But by construction *ce* equals *bg*. Therefore the parallax of the point *C* equals the parallax of the point *B*. In a like manner any other point in the vertical plane *BCEG* can be shown to have a parallax, in the plane of the transparent sheet, which equals the parallax of the point *B*.

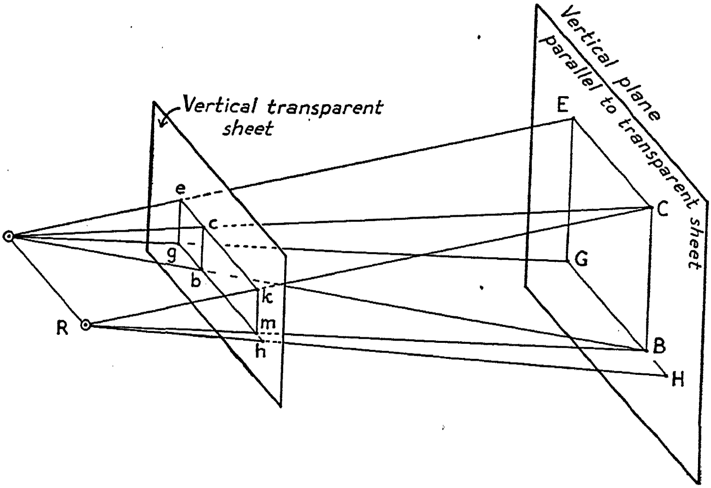

Vertical transparent sheet
Vertical plane parallel to transparent sheet
e
c
g
b
k
m
h
R
E
C
G
B
H

FIG. 68.—Diagram to illustrate parallax of all points in a plane which is parallel to the stereo base and the plane of measurement.

This theorem is important in stereo photogrammetry. It is used to reduce the labor of contouring with plotting instruments.

In Fig. 68, *h* properly may be considered to be the principal point of the perspective view seen by the right eye at *R* and the foot of a perpendicular drawn from *L*, which will strike the vertical transparent sheet at a point on the line *hg* near *g*, will be the principal point of the perspective view seen by the left eye. In a manner similar to that given above, it can be shown through similar triangles that *cb* equals *km*. Hence it can be stated:

COROLLARY. *On two perspectives (photographs) of equal focal length, taken in the same plane, the images of any point are equidistant from the line connecting their principal points.*

The statement is true regardless of the position of the plane of the perspectives. The fact is useful in making stereo measurements.

Figure 69 shows how a pair of photographs having the same focal length and taken with their optical axes parallel, both axes being perpendicular to the stereo base, affords a complete analogy to the stereo setup just considered. The camera stations (perspective centers) S_1 and S_2 of the left and right photographs, respectively, correspond to L and R of Figs. 67 and 68. The stereo base is lengthened as appropriate for the distances to objects in the scene, one of which is A. The two photographs, though standing in a common plane that corresponds to the transparent sheet of the previous figures, are detached from each other, so that it becomes easier to find the parallax of any point by measuring two components, one on each photograph. In order readily to accomplish the measurement of these two components it is necessary to employ reference marks on the photographs. If such reference marks give the principal lines of the photographs, measurements may be made from these lines, the sum of the distances measured being the parallax sought. For example, in Fig. 69, if o_2a_2 is added to o_1a_1, it gives the parallax a_1a_3 of the point A. The focal length of the photographs corresponds to f of Figs. 67 and 68.

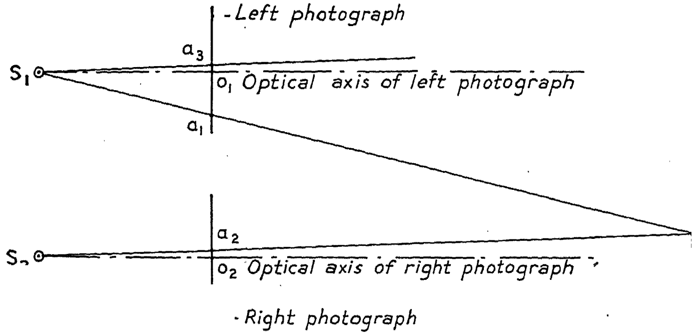

-Left photograph
a3
S1
o1 Optical axis of left photograph
a1
a2
S2
o2 Optical axis of right photograph
-Right photograph

FIG. 69.—The stereo setup with a pair of photographs.

In measuring the parallax of objects appearing in a pair of stereo photographs, it is customary to employ the principal lines and the horizon lines as the X and Y axes, respectively. Figure 70 shows the arrangement. It is also customary to apply plus and minus signs in the timehonored fashion as indicated in that figure. The general equation for parallax now becomes

$$p = \quad - x_2 \tag{21}$$

The perpendicular distance from the stereo base to any object having once been found, its elevation with respect to the level of either camera station can be obtained from the equation

$$\text{Elevation difference} = \frac{by}{p} \tag{22}$$

The quantity y will be the same for each photograph if the two camera stations are at the same level. When they are not at the same level, the elevation difference found is applicable to the station of the photograph measured. The latter condition is the one commonly encountered in surveying. The horizontal distance between the camera stations, not the slope distance, is then employed as the stereo base.

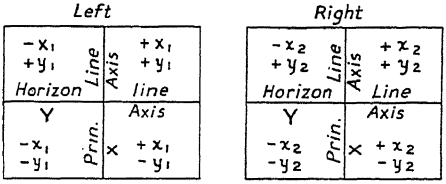


FIG. 70.—Coordinates in the plane of photographs taken on the ground.

Proof of equation (22) may be obtained from Fig. 68. From R draw a line perpendicular to LR to meet GB extended at H. This line pierces the transparent sheet at h. Then in the similar triangles RBC and Rmk

$$BC:mk = RB:Rm \tag{23}$$

and in the similar triangles RBH and Rmh

$$RB:Rm = RH:Rh$$

but $RH = D$; $Rh = f$; $mk = y$; and BC is the elevation difference desired Substitution of these values in equation (23) gives the required proof of equation (22).

The third axis required for a complete determination of the position of any object whose images appear in the two photographs is the stereo base. The origin in this base line is taken at one of the two camera stations L or R, and perpendicular distances as D in the figures are usually designated by Z, the three coordinates then being X, Y, and Z as hereafter employed.

201. Application to vertical aerial photographs.—So far this investigation has been confined to completely controlled setups under conditions that facilitate a mathematical analysis. The conditions are such as can be imposed readily in ground surveying where the stereo base may be measured at the time the photographs are taken and the camera leveled accurately and oriented properly to bring the pair of photographs into the same vertical plane. In aerial photography it is possible neither to measure the stereo base directly nor to keep the camera level. Under these conditions it is necessary to have enough control data based on ground surveys or other data to determine the length of the stereo base and the amount of tilt of each photograph.

Consideration of the difficulties that arise in this field will be resumed in another section. The present investigation will be continued on the basis of a theoretically controlled aerial setup. Figure 71 shows a pair of true vertical photographs that in part cover the same terrain; *i.e.*, they overlap about 60 per cent. It is evident that in this case the distance D (coordinate Z) applies to the height of the stereo base above the ground, that X applies to ground distances parallel to the stereo base, and that Y applies to ground distances at right angles to X distances. On the photographs, then, x values will be measured parallel to the line that connects the principal points of the photographs; y values will be measured perpendicular to this line; and values of Z will give various levels of the objects whose images are employed in measuring x and y quantities. Therefore all objects whose parallax is a given quantity will be at the same level. In the figure, A and B, being at the same level, will have a parallax corresponding to the distance Z_1; and C, D, E, and F, will have a parallax corresponding to the distance Z_2.

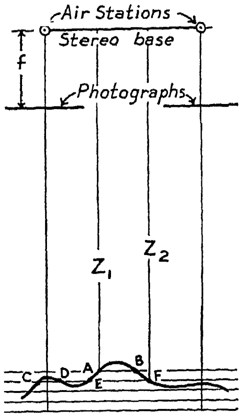


FIG. 71.—Contouring with true verticals.

202. Application to height difference.—Under this completely controlled setup, the differences in height of objects can be found from measurements of parallax. In Fig. 72 the simplest condition is assumed with two objects A and B (*e.g.*, top and foot of a bluff) directly under one of the air stations. Then the parallax of A is the distance p_1 as measured on the right photograph; that of B, p_2. Knowing the length of the stereo base b and the focal distance f, as well as the measured parallax values p_1 and p_2, the values of Z and AB can be calculated from equation (20), substituting Z for D and p_1 for p in the first instance and $Z - AB$ for D and p_2 for p in the second instance. This affords a method of determining the smallest difference in height that can be measured under given conditions. Suppose the parallax difference $(p_1 - p_2)$ to be 0.03 millimeter, which is about the smallest difference that normal eyes with moderate magnification can detect on photographs. Then if the photographs were taken at a height of 12,000 feet (Z) above the object B and 3,500 feet (b) apart, the height AB corresponding to the parallax difference of 0.03 millimeter is about 4 feet when $f = 12$ inches. (Determined from the relationships $AC:p_1 - p_2 = Z - AB:f$, and $AB:Z = AC:b$.)

This may be considered a fair measure of the refinement theoretically possible to attain in making stereoscopic measurements and in determining differences in elevation from aerial photographs by the stereo method under favorable conditions for aerial photographic mapping and with first-class instruments. Stereo perception is greatly sharpened by contrast in details of the photographs and by the shapes of objects as well as by the photographic quality of the prints employed. Two parts of a gentle slope appearing in monotone are more difficult to differentiate in height visually than are parts of objects of line shape such as embankments, walls, and houses. Because of these affecting factors the average power of stereo perception and hence the average refinement in making stereo measurements falls somewhat below the quality indicated by the example here given.

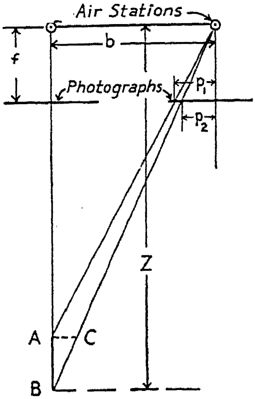


FIG. 72.—Simplest condition in which parallax is obtained from one photograph of a stereo pair.

203. Errors in parallax caused by tilt.—If actual vertical photographs with tilt are employed without compensation for the tilt, errors of considerable magnitude will arise in the values of Z. Figure 73 represents a setup similar to that of Fig. 71, except that the right-hand photograph is tilted about an axis at right angles to the vertical plane containing the stereo base. The tilt is purposely accentuated in the figure in order clearly to show the effects. For an object A whose image appears on the left photograph directly under its air station S_1, the parallax error caused by tilt in the right photograph is the difference between ca and $c'a'$, ca being the actual distance measured on the photograph, whereas the parallax ($c'a'$) should be measured on a true vertical whose center of perspective is at the air station S_2. A graphical representation is afforded by laying off a distance $c'a''$ equal to ca and connecting a'' with S_2, prolonging the line to strike S_1A at A'. AA' is then the error introduced. The parallax error is the same whether measurements are made from the centers or from the vertical points of the photographs.

For any particular setup this error depends on the angle of tilt i and the angle α between the optical axis of the photograph and the ray to the object. The parallax error is therefore

$$c'a' - ca = f[\tan(\alpha + i) - \tan\alpha - \tan i].$$

If, for example, $f = 200$ millimeters, $\alpha = 20°$, and $i = 1°$, the parallax error is 0.487 millimeters. Applied to photographs taken at a height of 12,000 feet aboveground and 4,000 feet between air stations, the corresponding error AA' in the determination of the level of the object A would be about 87 feet. This is the maximum error that could occur from a tilt of 1° under the conditions stated. The range of error in finding the levels of objects within the zone of overlap would in this case extend from 0 to about 87 feet. This error is too great for practical use of ordinary verticals without applying corrections for tilt.

The full effect of tilt in causing error comes into play only when measurements are made entirely across the field of overlap. The error in parallax differences as measured between two objects whose images are close together on slightly tilted photographs is microscopic in magnitude, and hence the error in determining such local height differences is negligible provided an accurate value of the stereo base is available.

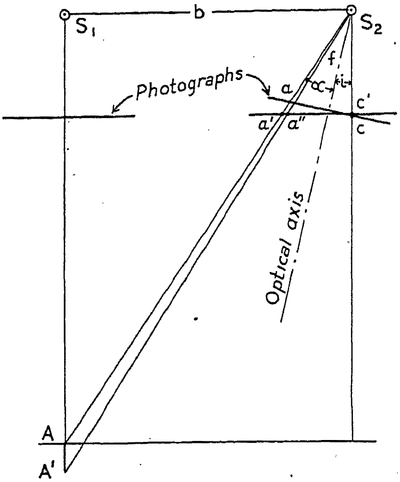


FIG. 73.—Errors caused by tilt of photograph of a stereo pair.

204. Wandering (floating or measuring) marks.—It has been indicated that when two stereo photographs have been brought into proper position for examination with a stereoscope the distance between a pair of images of any distant object will be found to be greater than the distance between the two images of a nearer object and that consequently the parallax of the nearer object will be greater than that of the more distant object. The difference between the two parallax distances is a measure of the relative distance apart of the two objects. Suppose that instead of measuring the distances between the image pairs directly, a small piece of thin and transparent material is placed over each photograph, each of these transparencies having a tiny black dot on its lower surface, and that then these are moved into position so that while viewing the photographs the images of the two dots are seen fused into a single image. This is possible because normal eyes have the capacity to see the dots stereoscopically at the same time that the photographs are so seen through the stereoscope. Then, if both dots are moved together over the photographs with the stereoscope until the left one comes into contact with the image of an object, and it is held there while the right dot is moved from left to right to take it toward the right image of the object, the fused image of the dots, seen as a single dot, will appear to float away from the observer until it "touches" the object. At this position the right dot will have come into contact with the right image of the object. If now, without changing the relative positions of the two dots, both are moved laterally with the stereoscope parallel to the stereo base and up or down at right angles to the stereo base until the left dot comes into contact with the left image of another object and it then becomes necessary to move the right dot to make the fused image appear to touch this second object, the amount that the right dot has to be moved to the right or to the left gives a quantity, which, under conditions of a proper setup and calibration, as has been explained, will tell the distance between the planes of the two objects. This is equivalent to measuring the parallax of each object and subtracting the lesser from the greater.

The wandering mark, therefore, may be employed as a convenient aid in stereo measurement, and its use gives many advantages over any other device, by lessening the labor of making stereo measurements and making it possible to devise instruments for rapid and automatic plotting of map details including contouring.

205. Condition necessary for use of wandering marks.—The movement of the wandering mark is apparent only when it is employed with a suitable stereo scene, as is furnished by any fairly good stereo pair of photographs. The floating movement cannot be detected against a flat background, because there is then no picture to give the required illusion of space in three dimensions. To obtain fusion of the dots, however, it is best to employ a background of white paper, for then the dots stand out more clearly than they do when viewed against a background of mixed tones.

206. Forms of wandering marks.—Considerable experimentation has been carried out to determine the most satisfactory form for the wandering marks. Tiny black dots are satisfactory so long as the images given by the photographs are not so dark in tone that the mark is lost to view from lack of contrast between it and the photograph. Arrowheads ↓, crosses either erect + or slanting ×, and angles, one turned upward ┘, the other downward ┌, are among the various forms found to be satisfactory. It is important that their size and boldness be commensurate with the magnification given by the instrument and the clarity of detail in the photographs as well as the refinement sought in measurements to be made with them.

207. Positions of wandering marks.—In the simple stereoscope the position of the marks necessarily must be in the plane of the photographs, for only there can a proper image of the marks be seen together with the stereo image given by the photographs. In the multiplex instruments a single bright dot is employed in the center of the little tracing stand which is moved about in the plastic model formed above a table. The single dot serves in this case because the model of the terrain is seen not through oculars of a stereoscopic instrument but by use of colored glasses independently of the projectors. In general it may be said that wandering marks (the case of the multiplex excepted) may be placed wherever images of the photographs are formed in the viewing systems of the instruments. For example, the focal planes of the oculars would be one set of positions available in all types of instruments having their own observing systems. In some of the more complicated photogrammetric instruments focal planes other than those of the oculars are employed for the marks.

208. Grids used as wandering marks.—A pair of grids of fine lines drawn on the lower surfaces of plates of glass constitute a series of wandering marks if used in contact with the photographs. If the squares forming the grids are all exactly equal, the use of this form of wandering mark economizes in movements necessary to scan the entire zone of overlap. Another advantage given by the grids is a greater facility for orienting and placing a pair of stereo photographs in proper position for making stereo measurements. Grids made of squares with diagonals also drawn are the types usually employed. Some measuring stereoscopes are equipped with such grid plates.

209. Character of parallax differences (differentials).—Figure 74 serves to demonstrate how parallax differs in a given setup according to the distance of the object from the stereo base and how parallax differences resulting from a given separation of two objects will vary according to their basic distance from the stereo base. Basic distance as here used means the greater of the two distances with which a parallax differential is concerned. It having been shown, in connection with Fig. 68, that the parallax of an object at a given distance from a stereo base is a constant quantity in any particular setup, no matter where in the perspective its image is caught, the setup for Fig. 74 is chosen to give the simplest possible relationships.

Two vertical photographs taken in the same plane and having equal focal lengths have a stereo base S_1S_2. Two objects A and B on the optical axis of the left photograph and separated by a vertical distance h will appear on the right photograph at the points where the rays S_2A and S_2B strike it. Since both objects appear at the center of the left photograph, the parallax of each is the distance of its image from the center of the right photograph. The parallax differential is indicated. Let C and D be two other objects also on the optical axis of the left photograph and separated vertically by the distance h, but lying nearer the stereo base than A and B. Now the parallax differential is greater.

Suppose now that a system of equidistant horizontal lines is drawn starting at the level of A. The distance between them may be any quantity such as 10, 20, 50, or 100 feet; hence the interval between any two consecutive lines may be considered a contour interval. By letting it be a desired contour interval, it is evident that the parallax differentials for that interval at all basic distances between any desired limits can be computed for any given setup and that the differentials will gradually increase in value as the basic distance becomes less. For a small interval and relatively large basic distances, the rate of differential change is slight, so that a straight line may be used to indicate the curve of change from one level to another if not extended very far.

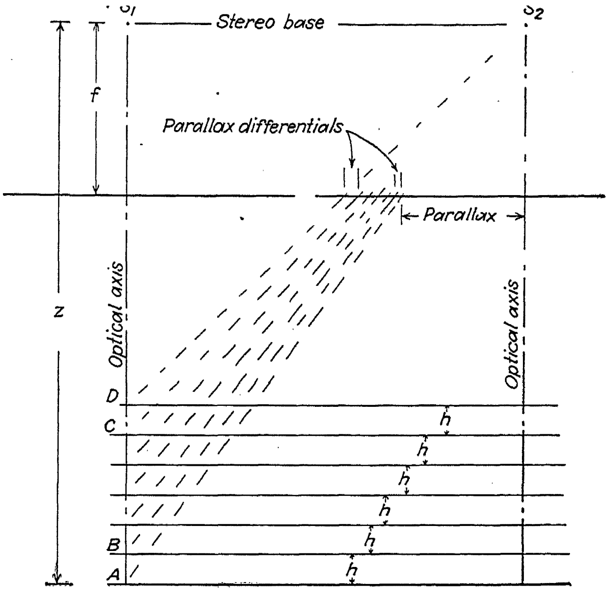


FIG. 74.—Relationships in a stereo setup of vertical photographs.

210. Deduction of equation for parallax differentials.—With reference to Fig. 74, let the parallax differential be designated by d, the focal length of the photographs by f, the stereo base by b, the height of the stereo base above any level by Z, and the height interval (as AB) by h. Then from equation (20), $d = p_1 - p_2$, or

$$\frac{fbh}{Z(Z - h)} \tag{24}$$

This equation is not simple enough for direct and rapid use, and ordinarily there will be too many unknown quantities. But it may be employed to prepare tables or curves of d for any particular pair of photographs as soon as the value of b has been determined; or, at the expense of great labor, tables could be prepared for a range of values of b as well as for a range of values of h. These could be used by interpolation. The values of d for any given value of b and for any value of h will depend on the magnitude of Z. If a chart were plotted for any given value of h (as a contour interval) letting abscissas be values of Z and ordinates be values of d, the plotted points would necessarily draw closer and closer to the ordinate axis as the value of Z increased. A line connecting the plotted points is found to be a curve slightly convex toward the ordinate axis.

As a concrete example, let f be 8¼ inches; b 4,000 feet; the interval h 100 feet; and Z 8,000, 9,000, 10,000, 11,000, 12,000, and 13,000 feet, respectively, in the six computations required. Then, using equation (24), values are obtained as given in Table 9.

TABLE 9.—PARALLAX DIFFERENTIALS

For $f = 8\frac{1}{4}$ inches, $h = 100$ feet, and stereo base = 4,000 feet

Values of Z, Feet	Values of d, Millimeters
8,000	1.3263
9,000	1.0464
10,000	0.84667
11,000	0.69908
12,000	0.58697
13,000	0.49982

Tables could be prepared in the same manner for parallax differentials with reference to any given value of Z (as the height of the stereo base above the datum plane) and for varying values of h and f, b being fixed in value.

A chart of curves giving parallax differentials drawn on this basis might well be for various assumed values of Z and b and a fixed value of f, in which case values of d could be plotted as ordinates and values of h as abscissas, or vice versa. It would be necessary to make two corrections —one for the actual stereo base and another for intermediate values of Z. The first correction would be merely a simple multiplication. The second would be rather complicated and could probably be most readily accomplished by use of a correction chart.

211. Values of parallax differentials from control stations.—Whenever control stations are available in the zone of overlap, values of parallax differences for pairs of stations in any particular pair of true vertical photographs can be obtained quickly by direct measurement if a suitable stereoscope equipped with wandering marks and measuring mechanism is available. If these stations differ in elevation approximately as much as the higher and lower parts of the terrain involved, they will furnish sufficient data to make a graph of parallax differences and heights.

This graph will serve to set the micrometer at any desired contour level and hence to contour the area common to the photographs. In this case the elevations of the several control stations are referred to a single datum plane, which, for example, may be the horizontal plane at the elevation of the lowest station. These values can be used to construct a graph of appropriate dimensions on which parallax differences are represented by abscissas and elevations by ordinates, or vice versa. The graph can be employed to find the parallax difference corresponding to any desired level, and levels may be taken to coincide with contours. When the correct parallax difference for any contour to be drawn has been obtained from the chart, the wandering marks of the instrument are set accordingly and the contour traced by merely moving the stereoscope about over the photographs, always keeping the mark apparently touching the surface of the ground.

The procedure therefore consists in finding on the graph the parallax difference for any given height above the datum plane and adding this to the basic parallax of the station at the datum plane, *i.e.*, adding it to the reading of the micrometer when the wandering mark stands at the base station. If the given level is below the datum plane, the parallax difference will have to be subtracted from the basic parallax. It would simplify use of a graph of this type to have the datum plane at sea level; but unless there is a control station at or near that level, it is not readily feasible to extend the graph to that plane. However, any contour level near the elevation of one of the control stations could be employed as the datum plane.

212. Graph of parallax differences and heights.—An example will serve best to explain the use of this type of graph. Figure 75 is drawn for the overlap zone between photographs 15 and 16 of a particular assumed set. The focal distance of the photographs was taken as $f = 0.68$ foot, the stereo base as 3,400 feet, and the height of the stereo base above the lowest control station as 11,000 feet. The control stations were assumed to be at the following elevations above sea level: No. 1 at 423 feet, No. 2 at 683 feet, No. 3 at 923 feet, No. 4 at 1,143 feet,

TABLE 10.—PARALLAX DIFFERENTIALS OBTAINED FROM CONTROL STATIONS

Control-station pairs	Height differences (h), feet	Parallax differences (d), millimeters
Station 2-Station 1	260	1.551
Station 3-Station 1	500	3.0506
Station 4-Station 1	720	4.487
Station 5-Station 1	970	6.1954

and No. 5 at 1,393 feet. The height differences between No. 1 as the base station and the other stations are as shown in Table 10 on page 169, which also gives the parallax difference corresponding to each height difference.

Equation (24) was employed to compute the values of *d*. The graph was constructed from this table. The curve connecting the several plotted points deviates only slightly from a straight line, so by careful drafting it may be extended short distances at each end to enlarge the scope of the graph. To trace the 500-foot contour the difference between 500 and 423 feet (77 feet) is the ordinate used to find the parallax difference, 0.42 millimeter, which must be added to the setting of the wandering marks on Station 1 to bring the apparent mark to the 500-foot level. To make the setting for any contour below 423 feet, the parallax difference found from the graph extended would be subtracted from the setting on Station 1.

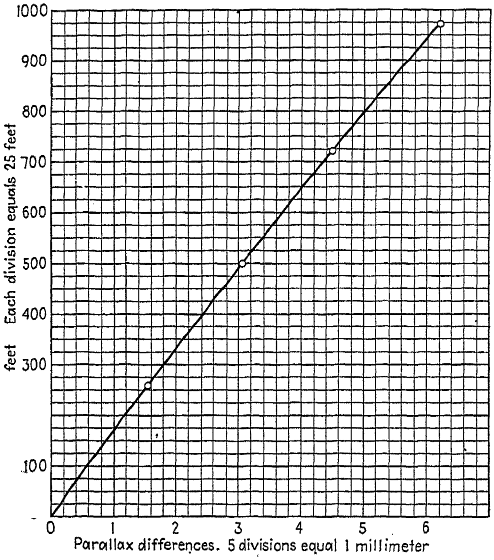


FIG. 75.—Graph of parallax differentials. The datum plane is at the level of the lowest station.

This graph could have been constructed with 400 feet above sea level as the datum plane. The parallax reading at that level would then have to be applied to a setting on one of the control stations (preferably Station 1) to obtain the basic reading to which subsequent values taken from the graph would be applied.

With a table containing values of *d* it is possible to compute the height of the stereo base above the datum plane and the length of the stereo base using two simultaneous equations formed by substituting values of

d and h in equation (24). In this case fb/Z is a constant and may be written as K. Taking values of h and d from Table 10, sets of simultaneous equations are formed as follows:

$$(a)\ 1.551 = \frac{260K}{Z - 260} \qquad (b)\ 3.0506 = \frac{500K}{Z - 500}$$

$$(c)\ 4.487 = \frac{720K}{Z - 720} \qquad (d)\ 6.1954 = \frac{970K}{Z - 970}$$

Any pair of these equations can be employed to compute the value of Z in feet; then, replacing K with fb/Z, the value of b can be found by substitution in any of the equations.

213. Accuracy attainable in determining heights.—In practical application when aerial photographs have been taken at a constant interval of time, as with an intervalometer, and the flight is held closely at a certain level, fair results can be obtained from vertical photographs of that flight (a single course) by employing as the stereo base the mean distance between the plotted positions of the centers of the photographs and the corrected mean height of the air stations as given by altimeter. In this case a special parallax table would be prepared for the photographs of the run. Suppose, for example, that 21 overlapping photographs have resulted from a flight and the total distance between the plotted positions of the centers of the first and last photographs as scaled on a map is 72,400 feet. The stereo base for each zone of overlap would then be taken as 72,400/20 = 3,620 feet. Let the mean altimeter height above sea level (or any other datum plane), after correction for temperature, be 11,870 feet. The maximum error in the stereo base liable to result from tilt (taken as 2°) in the first and last photographs is about 40 feet, and the probable error will not exceed 20 feet. In the height determined from the altimeter record the probable error would not exceed 50 feet. The errors from these two sources is about 1 per cent of the height of any object on the ground. Under these circumstances, if the relief of the area concerned does not exceed 500 feet, elevations determined should not be in error, from these causes, by more than 5 feet. It should be understood, however, that the probable tilt in any two photographs constituting a stereo pair would introduce errors far greater than those here indicated (see Art. 203 for errors in parallax caused by tilt); thus this method of determining the length of the stereo base to obtain heights with error no greater than 1 per cent would require the use of true verticals.

214. Accuracy attainable with control stations.—Whenever control by ground survey is adequate and means are available for determining the tilt of every photograph employed, a very high degree of accuracy is attainable by the stereo method of finding elevations. The limit is then set by the capacity of the observer to work accurately, the keenness of his perception as to when the wandering mark makes contact with a spot of any object image, and the refinements of the measuring mechanism. The scale of the photographs being employed and the ratio between stereo base and distances to the objects viewed play important parts. In general it may be said that a skillful observer working with properly taken and well-controlled photographs whose scale is about 1:20,000 can obtain elevations that will not be in error by more than 1/1,000 of the height of the air stations above the general level of the terrain.

215. Comparison of contouring with true verticals and with photographs taken at stations on the ground.—Contouring with truly vertical photographs can be accomplished with simple stereo instruments, because it is necessary only to set the wandering marks with a fixed parallax difference for any contour. When contouring with photographs taken on the ground, the plane of these photographs being vertical, it is necessary to have the position of the wandering marks relative to the photographs change vertically as the distances to the various parts of the terrain vary. Figure 76 illustrates in this latter case the required movements that must be imparted either to the wandering marks or to the photographs. It is seen that for any contour level higher than the stereo base the marks must be moved downward (or the photographs upward) as the distance to objects of the terrain increases. The direction of movement is reversed for contour levels below that of the stereo base. This movement is not proportionate to the change in distance to objects, but it varies in geometrical progression with arithmetical changes

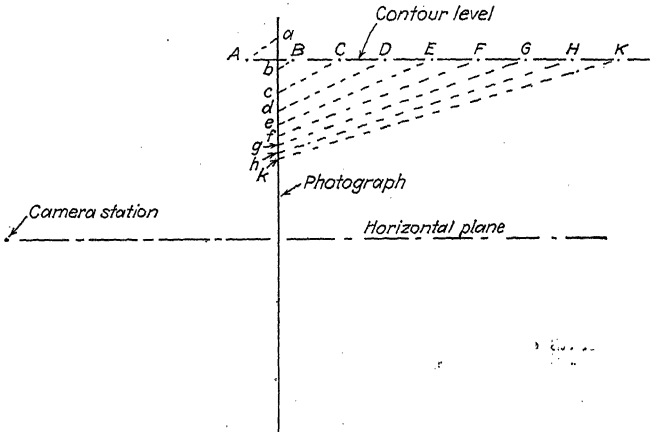


FIG. 76.—Condition when contouring with photographs taken on the ground.

in object distance. As a consequence of this requirement, instruments made for rapid tracing of contours with terrestrial photographs must have mechanisms that can be regulated to give to the photographs or to the wandering marks this geometrically progressive movement. With vertical aerial photographs the difficulty lies in obtaining photographs without tilt. With photographs taken at ground stations the difficulty arises in mechanical complexities of the instrument used to trace the contours.

CHAPTER X

METHODS OF DETERMINING THE TILT OF AN AERIAL PHOTOGRAPH

216. The methods.—Of the several methods of finding the tilt of an aerial photograph, the simplest is by means of the image of the visible horizon registered in the photograph. This is also one of the most accurate methods if the height of the feature that gave the horizon is known. It is possible to utilize photographs or diapositives in the more elaborate photogrammetric instruments without actually determining their tilts, but in the setups the pictures conform in tilting and orientation to the orientation and tilting of the camera at exposures. Angles of tilt may, however, be determined with them. Without access to such instruments it is necessary to employ a method that may be utilized with equipment commonly available.

Following is a list of some of the methods that may be employed in the drafting room or laboratory with comparatively simple equipment: (*a*) from an image of the horizon; (*b*) from a related photograph whose tilt is known; (*c*) by the upright-planes method; (*d*) with a mechanical tilt finder; (*e*) by the pyramid method; (*f*) by optical projection, and (*g*) from parallax differentials read with a companion photograph of a stereo pair, the tilt of the companion photograph being known.

217. Tilt obtained from an image of the horizon.—Necessary advance information consists of an accurate value of the focal length of the photograph and an approximate value of the height of the air station above the level of the land or body of water that has furnished the visible horizon. If the ocean is the visible horizon, the height for the purpose of finding tilt should be with reference to sea level; if a plateau produced the visible horizon, the air station height must be with reference to the elevation of that *plateau.*

Figure 77 represents a part of the earth with center at O and an air station S whose height above sea level is h. The base of a cone with apex at S, which is tangent to the sea, and the horizon which is visible from S are also shown. Because of refraction of light the apparent horizon, which will be registered in a camera placed at S, is actually beyond the base of the tangent cone. The distance from the air station to the base of the tangent cone (length of element) is given by

$$SP = \sqrt{(R + h)^2 - R^2} \qquad (25)$$

in which R is the mean radius of the earth and h is the height of the camera above sea level. If, for example, h is 5,000 feet and R is 3,956.5 miles, $SP = 86.593$ miles, or 457,209 feet.

The angle α at the center of the earth subtended by an element of the tangent cone is obtained from

$$\tan \alpha = \frac{SP}{OP} \tag{26}$$

Substituting in equation (26) the values stated above for SP and OP, α is found to be 1°15′25″. The length of the arc of a great circle of the earth subtended by the angle α is 457,142 feet. For practical purposes, therefore, SP may be considered equal to the length of the arc VP.

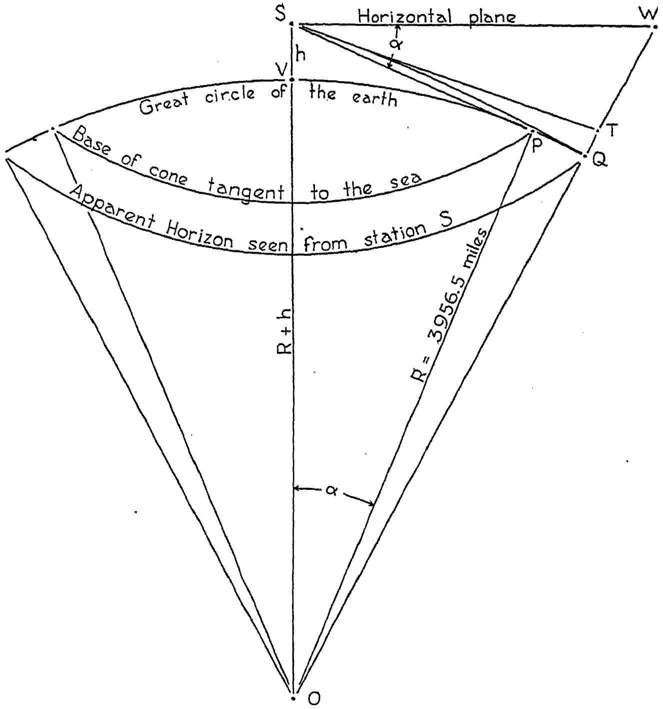

S
Horizontal plane
W
α
h
V
Great circle of the earth
T
P
Q
Base of cone tangent to the sea
Apparent Horizon seen from station S
R = 3956.5 miles
R+h
α
O

FIG. 77.—Diagram used to explain how the angle between the apparent and true horizons of a photograph may be obtained.

The distance from the air station to the horizon registered in a photograph is obtained from the empirical formula, long in established use:

$$D \text{ (in miles)} = \sqrt{\frac{h \text{ (in feet)}}{0.574}} \tag{27}$$

For $h = 5{,}000$ feet; $D = 93.33$ miles $= 492{,}791$ feet. Because of refraction of light, the apparent horizon is in this case nearly 7 miles farther from the air station than the base of the tangent cone. Under the conditions of the example, the angle *PSW* was found to be 1°15′25″ and the angle *PST* is 0°5′34″. Angle *TSW* is the difference between *PSW* and *PST*, or 1°9′51″.

Under equation (27) the angle between the horizon as registered in an aerial photograph and the horizontal plane through the air station (angle between apparent and true horizon) is

$$\text{Angle in minutes} = 0.9878\sqrt{h(\text{in feet})} \qquad (28)$$

The angle between the true and apparent horizon is added to the angular distance of the horizon image from the principal point to obtain the angle between the optical axis of the photograph and the true horizon. This sum is the complement of the angle of tilt. The principal line of the photograph is the line drawn through the principal point at right angles to the horizon image. The horizon image is actually slightly curved, so care must be taken to draw the principal line perpendicular to a tangent of the curve at the proper place.

Equations (27) and (28) are probably derived from observations made with telescopic instruments on the ground under conditions that are not necessarily the same as those encountered in aerial photography. Tables of refraction for use in astronomic computations do not appear to apply for angles of depression. Although there is not at hand proof that results obtained with these equations are thoroughly dependable, they undoubtedly are fairly reliable.

218. Tilt obtained from a related photograph whose tilt is known.—The problem may be stated as follows: With respect to a pair of photographs taken at the same aerial station, the optical axes lying in a common plane at a fixed angle to each other, knowing the principal line and angle of tilt of one of the photographs and the focal lengths of both photographs, find the principal line and angle of tilt of the other photograph. Two cases are considered, *viz.*, a special case and the general case.

(*a*) *Special case.*—The optical axes of the photographs are at *right angles* to each other.

In Fig. 78 let S be the common air station (perspective center), f_1 $(= SC_1)$ the focal length of the first photograph, f_2 $(= SC_2)$ the focal length of the second photograph, SC_1AC_2 the common axial plane, AB the line of intersection of the two photographs, i_1 the tilt of the first photograph, i_2 the tilt of the second photograph, and w the angle through which the first photograph is swung in its own plane. In the figure it is assumed that the tilt and principal line of the first photograph are obtained from a horizon line.

The line HB drawn through the center of the first photograph perpendicular to the horizon line is the principal line of the first photograph, and the angle between it and AC_1 is the angle w. The principal plane of the first photograph cuts the line DB from the second photograph; and since both DB and C_2A are perpendicular to the first photograph, they are parallel to each other. The principal plane of the first photograph also includes the common plumb line; therefore the vertical point V_2 of the second photograph must lie on the line DB. The angle of tilt i_1 is determined by the two lines SH and SC_1; but as the plumb line is perpendicular to SH, a line SD drawn in the principal plane SC_1BD perpendicular to SC_1 will form the angle i_1 with the plumb line. If, therefore, i_1 is laid off from this line, it will establish the position of the vertical point of the photograph on the line DB.

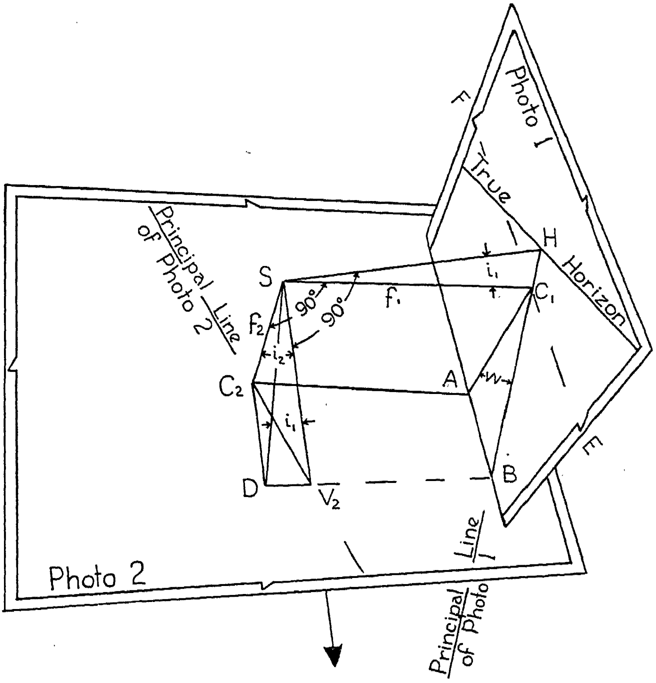


FIG. 78.—Two photographs at right angles to each other, the tilt of one to be determined when the tilt of the other photograph is known.

Practical procedure.—Assuming the true horizon to have been drawn on the first photograph and the marginal marks to give a line through C_1 that is parallel to the line of intersection of the two photographs, lay off C_1A equal to f_2 along a line drawn at right angles to FE, and draw AB parallel to FE. Then draw the line HB at right angles to the horizon line to pass through C_1, the two lines meeting at B.

On the second photograph draw the line AB at right angles to C_2A, the distance C_2A being equal to f_1. Lay off AB equal to AB as found on the first photograph, and draw DB parallel to C_2A. The line C_2D is drawn parallel to AB ($\perp$ to C_2A). From D, the intersection of these two lines, lay off toward B a distance DV_2 equal to $SD \tan i_1$, SD being computed from $SD = \sqrt{f_2^2 + AB^2}$. V_2 is the vertical point of the second photograph, and $\tan i_2 = C_2V_2/f_2$.

(*b*) *The general case.*—The condition here is that the optical axes are inclined to each other at a fixed angle, the setup consisting of a pair of photographs taken at the same aerial station, the principal line and angle of tilt of one and the focal lengths of both photographs being known, as well as the angle between the axes. The principal line and angle of tilt of the other photograph are to be determined.

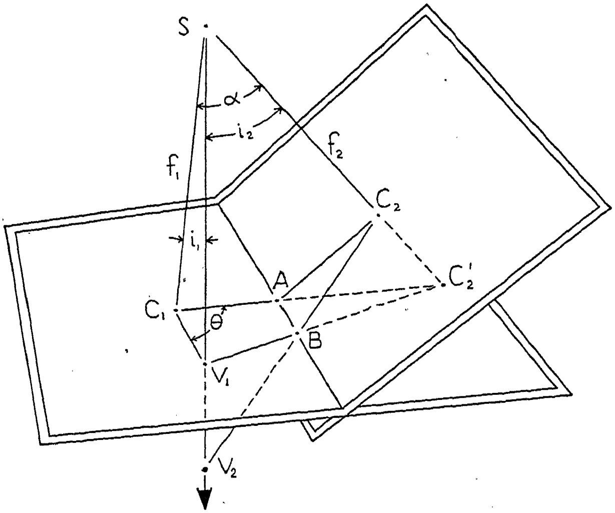


FIG. 79.—Diagram showing relationships between two photographs inclined to each other at any angle, the tilt of one photograph to be obtained when the tilt of the other photograph is known.

In Fig. 79 let f_1 be the focal length, i_1 the angle of tilt, C_1 the principal point, and V_1 the vertical point of the first photograph, all these being known; and let f_2, i_2, C_2, and V_2 be the corresponding items of the second photograph. Let α be the angle between the optical axes, S the aerial station from which the photographs were taken, and AB the line of intersection of the two photographs. The line of intersection ordinarily can be got from data of the camera or by inspection if the photographs overlap each other. Advance information leaves only the value of i_2 and the position of V_2 to be found.

Prolong SC_2 to meet the first photograph (extended) at C_2'; and join C_1V_1, C_1C_2', and V_1C_2'. The vertical point V_2 of the second photograph will be on the plumb line through S and V_1, in the plane of the second photograph. The common axial plane will cut the first photograph in the line C_1AC_2', and the second photograph in the line C_2A. The principal plane (SC_2V_2) of the second photograph cuts that photograph in the line C_2B (its principal line) and the first photograph in the line V_1B. The line of intersection of the two photographs AB, being at right angles to the common axial plane, C_1A and C_2A are both perpendicular to AB.

In the triangle SC_1C_2' the angle at C_1 is 90°, so

$$C_1C_2' = f_1 \tan \alpha$$

In the triangle SC_1V_1 the angle at C_1 also being 90°,

$$C_1V_1 = f_1 \tan i_1$$

The angle θ, between C_1V_1 and C_1C_2', is also known from initial information. Then knowing two sides and the included angle of the triangle $C_1C_2'V_1$, the length of V_1C_2' can be computed. Following this computation, the three sides of the triangle SV_1C_2' become known (for $SV_1 = f_1 \cos i_1$ and $SC_2' = f_1/\cos \alpha$), and the angle i_2 can be computed.

The distance BA can be obtained graphically by drawing the lines C_1C_2', V_1C_2', and BA on the first photograph extended as necessary, or it can be computed through the right triangles BC_2C_2' and $BC_2'A$, the angle at C_2 in the former and the angle at A in the latter each being 90° and $C_2C_2' = SC_2' - f_2$.

BA, having the same length on both photographs, and the position of the line AC_2 of the second photograph being known in advance, the principal line C_2B can be placed on the second photograph and the position of V_2 plotted along it at a distance from C_2 equal to $f_2 \tan i_2$.

The condition of this setup is that which exists between the center photograph and any oblique obtained at an exposure of multilens cameras, such as the five-lens camera, whose chambers are fixed in position so that the optical axis of each oblique lens lies in a plane containing the optical axis of the center lens, the angle between remaining constant.

The direction and length of the line V_1C_2' on the first photograph can be found graphically by laying off two lines inclined to each other at the angle θ and measuring from the apex along one line a distance equal to $f_1 \tan i_1$ ($= C_1V_1$) and on the other a distance equal to $f_1 \tan \alpha$ ($= C_1C_2$), then connecting the extremities. Placed on the first photograph properly, this line will give the point B at the intersection with the common line of the two photographs. The value of i_2 can be found approximately (within one or two minutes when α is from 40 to 50° and i_1 is small) by a similar graphical construction with α, i_1, and i_2 the sides of the triangle in linear units to represent minutes of arc, *i.e.*, by use of the familiar force diagram.

When a transformed copy of the second photograph is available, in addition to the first and second photograph, its principal line can be quickly found by prolonging the line V_1C_2' across the transformed photograph (oriented properly with respect to the first photograph), whereupon the corresponding line may be drawn on the untransformed print by relationship to points of images cut by the corresponding lines; for each will cut across identical image points.

219. The upright planes (displacement) method of determining the tilt of an aerial photograph. *Requirements.*—It is necessary that the photograph include three or more control points whose relative positions and elevations are known, and the control-point images must lie in the vicinity of the borders of, and be suitably distributed over, the photograph, preferably so that the center of the photograph will fall within the triangle formed by three of them. It is also necessary that the ground point corresponding to the center of the photograph be located correctly with respect to the control points. This can be done readily by the tracing-paper method of resection if the control points are correctly plotted on a map sheet.

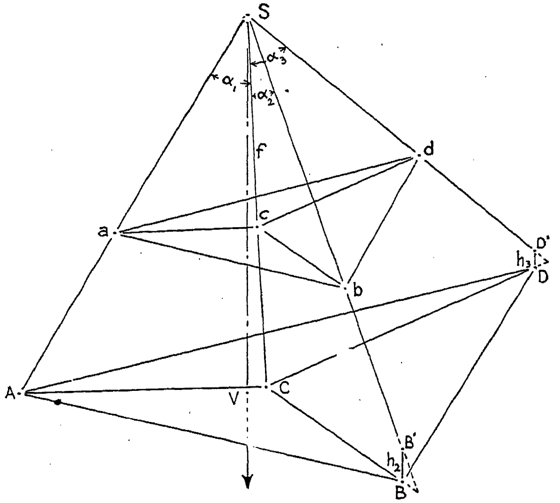


FIG. 80.—Diagram to explain the upright-planes method of determining tilt.

The upright planes.—In Fig. 80 the upright planes are *SAC*, *SB'BC* and *SD'DC*, each of which includes the optical axis of, and is perpendicular to, the photograph. These planes cut the lines *ca*, *cb*, and *cd* from the photograph and the lines *CA*, *CB*, and *CD* from the map sheet. The method consists in dealing with each of these planes separately for the purpose of (*a*) determining the distance *SC* and (*b*) determining the mutual inclination of the pairs of lines cut by these upright planes from the photograph and the map (horizontal plane).

Map position.—The distance SC of the map from the perspective center is taken to be the mean of the several values found by computation from assumed similar triangles in each upright plane, *i.e.*, by temporarily assuming the photograph to be without tilt and hence parallel to the map plane. On this assumption and letting Sc be designated by f,

$$SC = \underset{ac}{\qquad\quad} = \frac{f \times BC}{bc} = \frac{f \times DC}{dc} \tag{29}$$

Focal length of the photograph.—Knowing the calibrated focal length of the camera, the value of the focal length of the photograph f is determined by comparing the length and width of the photograph with the corresponding dimensions of the focal-plane frame or marks of the camera. A mean of the two values thus obtained is used in the preceding equation. By taking a mean value of f, so found, the effect of unequal shrinkage in the two directions is partially compensated.

Refraction correction.—If the camera has a glass plate in its focal plane, a correction must be applied to each distance (ac, bc, cd) measured on the photograph. A graph of refraction displacements supplies these corrections. In every instance the quantity is to be added to the distance measured on the photograph.

Datum plane.—It is necessary to select a datum plane for the construction. If the elevations of the control points do not differ greatly, it is best to use the horizontal plane at the level of the lowest of the control points; for the use of this plane reduces the number of map distances to be corrected, or the amount of work to be done, and places all the elevation perpendiculars on the upper sides of the map lines.

Allowance for relief.—It is evident that in determining the mean value of SC, measures must be taken to provide for differences in height of the control points. There are two ways of doing this: Either (*a*) those map distances (CB and CD in Fig. 80) affected may be changed to allow for the height difference displacement, or (*b*) a computation of SC may be made, temporarily neglecting the relief to get a value to which will be applied a correction for the height of the control station. The second procedure is preferred to the first; for it is less susceptible to errors, and it is in keeping with succeeding steps of this method of determining tilt. If the datum plane is taken at the level of the lowest control point, the computation of SC involving the other points will be corrected by adding the height difference in each case. In Fig. 80 the datum is taken at the level of point A, which is the lowest of the three control points; therefore the computation of SC from the map distance AC by use of equation (29) will require no correction, but the computation from a map distance

BC will be corrected by adding to it the difference between the elevation of B and A at the scale of the map. A similar addition will be made to the computation of SC from the map distance DC. All three of the determinations of SC will then be referred to the datum plane, and a mean of them will be a fair actual value of SC, the height of the air station (perspective center) above the chosen datum plane.

Graphical construction.—Having obtained a mean value of SC and using a suitable sheet of drafting paper, whose surface is to represent each of the upright planes in turn, two fine straight lines are drawn on it at right angles to each other, crossing at a distance from the top of the sheet equal to a little more than f, the upright line being set over to one side of the sheet as shown in Fig. 81. From the intersection, which may be labeled c, a distance is measured upward equal to f, and from this point S another distance downward equal to the mean value found for SC. From c, along the crossline, distances are laid off equal to ac, bc, and cd; then rays Sa, Sb, and Sd are drawn and prolonged as necessary. From C as pivot, using a beam compass or pair of dividers, the map distance CA is struck off to the ray Sa. A line drawn from C to the point A and prolonged will strike the line ca at an angle that is the component of tilt along the line ca of the photograph. To obtain the point B' on the ray Sb, a distance is struck off from C equal to the hypotenuse of a right triangle whose two legs are the map distance CB and the difference of height $B'B$ (h_2). Now with a radius equal to h_2 a short arc is drawn below B', and a line drawn tangent to it from C. This tangent line prolonged will intersect the line cb at an angle that is the component of tilt along the line cb of the photograph. In a similar manner the hypotenuse CD' is struck off; an arc whose radius equals the height h_3 is described; and a tangent drawn to it from C to get the component of the tilt along

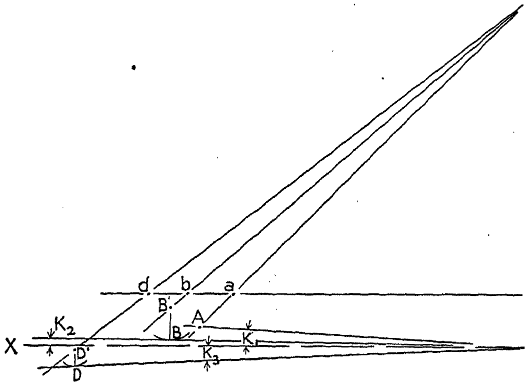


FIG. 81.—Geometry of the upright planes.

the line *cd* of the photograph. To facilitate the measurement of the several tilt-component angles, a reference line *CX* is drawn parallel to the photoline *ca*. Tilt-component lines (*CA*, *CB*) above *CX* indicate that along the directions *ca* and *cb* the photograph is tilted downward; tilt-component lines (such as *CD*) below *CX* show that along these directions the photograph is tilted upward.

The tilt circle.—Using a sheet of tracing paper over the photograph, the direction lines *ca*, *cb*, and *cd* are drawn, and from *c* along each of these lines the value of the proper tilt component in minutes is laid off by linear scale, the tilt circle then being completed as explained in Art. 171.

Scale of drawing.—When dealing with single-lens photographs such as those obtained with the K-3B camera, which has a focal length of about 210 millimeters, it will materially increase the accuracy of the work if *Sc* is laid off equal to twice *f*, and the distances *ac*, *bc*, and *cd* are multiplied by two. The map scale should be commensurately large, or the map distances and *SC* should be multiplied by a factor that will assure a like increase in the accuracy of the construction. For example: If the approximate scale of the photograph is 1:18,000 and the scale of the map 1:24,000, doubling *f* and the photograph distances should be accompanied by multiplication of the map distances and *SC* by three or four.

Errors in the method.—This method of finding the tilt of an aerial photograph cannot be classified as an accurate one. It is subject to many errors, which in some instances may tend to balance one another, in other instances to accumulate. Aside from personal care taken in measuring distances, in the order of their importance, errors evidently arise from (*a*) assuming the photograph to be horizontal for finding a mean value of *SC*; (*b*) unequal shrinkage of the photograph in the two principal directions of length and breadth; (*c*) large differences of elevation between control points or, what amounts to the same thing, choosing a datum plane far from the level of the ground at the point where the optical axis of the photograph strikes it; (*d*) the limitations inherent in graphical constructions; and (*e*) having to use slanting upright planes instead of vertical planes.

Probable magnitude of errors.—Under normal favorable conditions it is not likely that *SC* can be determined closer to its true length than the amount that corresponds to 20 feet in the actual height of the air station above the assumed datum plane. For a graphical construction at the scale 1 inch = 600 feet (which is about as large a scale as is practical to use) the pivot point *C* may be displaced $\frac{1}{30}$ inch, or nearly 1 millimeter. This may affect each value of *k* (any component of tilt) as much as plus or minus 10 minutes for low values of α and bring into the angle of tilt *i* a probable error equal to that amount divided by the number of control points used. It may also materially displace the principal line.

Unequal shrinkage of the photograph made on double-weight paper may cause an error in the angle of α up to a maximum of about 4 minutes and a consequent error in k, when α is 25°, of about 16 minutes; when α is 60°, of about 6 minutes. Errors in i will then be about 5 and 2 minutes respectively.

The error arising from the third cause increases directly as the tilt and height differences increase. If SC corresponds to 12,000 feet, i is 5°, α is 25°, and h is 1,000 feet, the error in k may be as large as 4 minutes; for a height difference of 500 feet, 2 minutes; if in the foregoing setup α is 60°, the maximum error in k is less than 2 minutes. When i is 2° or less, the error may be neglected for values of h up to 1,000 feet if α is 25° or more.

The error in drafting will depend on the scale of the construction. If this scale is taken large enough to permit laying off an angle of 2 minutes, it may be safely neglected, for excesses may be assumed to offset deficiencies.

The fifth error may be neglected as inconsequential except for large values of i and h (see Art. 298).

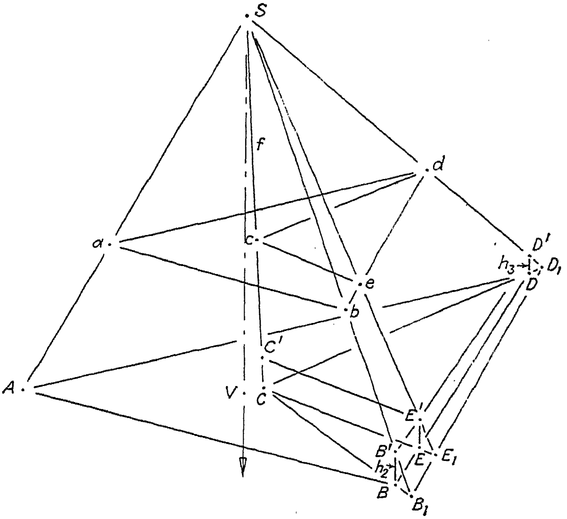
Accuracy of results.—Summing up the effects of the errors here analyzed, it may be said that the method should give values of i that are within 10 minutes of true and principal lines that are within 2 or 3° of their true directions when values of α are about 25°, the tilt does not exceed 2°, and the elevations of control points do not differ more than 1,000 feet. It follows that, if there is both considerable relief between control points and large tilt, the method is unreliable for ordinary vertical photographs, which give values of α that are comparatively small, unless several control points are available for use.

The method increases in reliability with increased field of the photograph. Wide-angle single-lens photographs will give better results than ordinary single-lens photographs, and composite photographs obtained with multiple-lens cameras give the best results. In general, the farther the control-point images are from the center of the photograph (*i.e.*, the larger α is) the more accurate the result will be. When α is 60°, the error brought to k from α is only about 1.3 times the error in α. Numerous determinations of the tilt of photographs obtained with the T-3A camera indicate that the method gives for them values of i that are probably not in error by more than 3 or 4 minutes. Moreover, the greater the field of the photograph the greater will be the amount of relief that can be satisfactorily dealt with.

Legs of triangles.—Inasmuch as equation (29) is based on the assumption that the plane of the photograph is horizontal, the lengths of pairs of corresponding triangle sides may be used instead of pairs of distances in the upright planes to find a mean value of SC, because

$$SC = \frac{f \times AB_1}{ab} = \frac{\quad}{ad} = \frac{f \times B_1D_1}{bd} \tag{30}$$

is just as nearly true as equation (29). When α_1, α_2, or α_3 is small, equation (30) may give a better value of SC than equation (29). The basis of equation (30) can be understood readily by reference to Fig. 82, which is like Fig. 80 but has additional lines. In the plane SE_1C, triangles Sec and SE_1C are similar (by assumption), so $f{:}Se = SC{:}SE_1$.



G. 82.—Determination of the height of an air station by use of distances between control points (legs of triangles).

In the plane SB_1D_1 triangles Sbd and SB are practically similar, so $bd{:}Se \doteq B_1D_1{:}SE_1$; therefore $f{:}bd = SC{:}B_1D_1$.

The allowance for relief is EE', whose value is

$$\frac{- h_3)ED}{BD} \quad \text{or} \quad \frac{{}_2 - h_3)ed}{bd}$$

The ratio ed/bd is found by drawing from c of the photograph a line perpendicular to bd to strike it at e. The ratio may also be found by a similar construction on the map. By drawing from c perpendiculars to the sides ab and ad, corresponding ratios are found for the height corrections in those instances. The correction for the side ab will be its ratio multiplied by h_2; that for the side ad, its ratio multiplied by h_3.

Height of camera.—The height of the air station (camera at exposure) above the datum plane used in the construction is given by $SC \cos i$, if the scale of the drawing is applied. Suppose SC to be 205 millimeters as computed from equation (29) when AC, BC, and CD were measured on a map sheet having a scale of 1:24,000 and i was found to be 1°40′.

The height in feet of the air station above the level of the datum plane is then

$$\frac{205 \times 24{,}000 \times \cos 1^\circ 40'}{} = 1$$

220. The mechanical tilt finder.—Tilt finders have been made in several designs. Figure 83 is a perspective drawing of one used for demonstration purposes at the Institute of Geographical Exploration of Harvard University. It consists of an adjustable platform to support a skeleton framework which simulates a camera to the extent of having a pinhole perspective center and a frame in which a glass plate may be inserted and held. This glass plate represents the negative so that when image points are spotted on it in their true relative position as given by a photograph, and it is placed in the frame, rays going from the points to the pinhole have the same relationships as the rays of light that caused their images to be formed by the aerial camera. It is necessary that the perpendicular distance from the pinhole to the plate be equal to the focal length of the photograph used to spot the points on the plate.

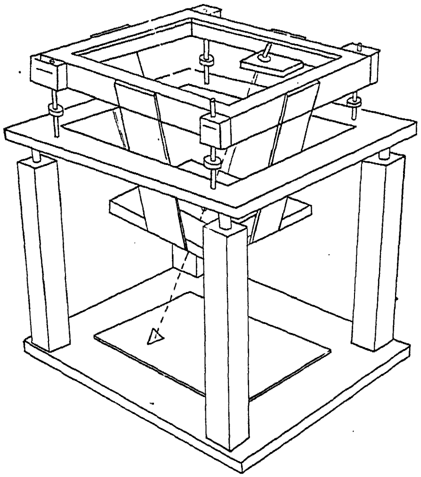


FIG. 83.—A mechanical tilt finder.

The framework is also fitted with four leveling screws which rest on the platform and provide for tilting in any direction. At the base of the platform is the horizontal reference plane on which a plate glass of suitable size rests. This glass gives a convenient and stable medium to represent a datum or map plane on which control points may be dotted, by merely placing it over the map and spotting the required points.

Having prepared the map plate with control points in their true relative positions and the photograph (upper) plate with the images of the control points all in their true perspective positions, the platform may be set at a convenient position for adjustment by means of the leveling screws to bring pairs of points (map and photograph) into alignment with the perspective center. When all corresponding points have been brought into line, the framework indicates with respect to the platform (which is always kept parallel to the base plane) the tilt of the photograph.

Index marks are placed on side plates, one near each leveling screw, so that the height of each mark above the platform may be measured. As all four of these index marks are placed in the photograph plane extended, differences between pairs of heights give two components of tilt by which the principal line and degree of tilt may be determined at once. The index marks are conveniently placed at the corners of an imaginary square of the photograph plane so placed that the center of the square coincides with the center of the photograph.

The datum plane of the map is ordinarily taken at the level of the lowest control station, and allowances are made for the heights of the other control stations above that plane by laying off the displacement on a line drawn from the plotted position of the center of the photograph of the station. The correction is only approximate, but it is exact enough for the purpose if elevation differences and tilt are not great. A requirement therefore is that the position of the center of the photograph be located on the map. Having its location aids materially in making a determination, for swinging of the map plate is always done about the optical axis of the photograph. Another procedure that facilitates operation is that of orienting the photograph plate to place one, and if possible two, control-point images in position to rest on or close to lines between leveling screws which determine the axes of the components of tilt.

To facilitate alignment, short tubes of small bore, mounted on ball-and-socket joints, are placed over the image points on the photograph plate and adjusted into line with the perspective center. With these properly in place it is necessary only to sight the control points on the map plate through the pinhole. An accessory used to spot points on the glass plates is made of a cutaway tube with centering marks and a magnifier. This prevents displacement of points in spotting.

Use of glass plates offers several arrangements that are suitable to overcome refraction distortions brought into negatives by a glass plate in the focal plane of the aerial camera or to maintain the freedom from such distortion in negatives taken with aerial cameras not equipped with focal-plane plates of glass. If the photograph plate is selected of glass having the same thickness and index of refraction as the glass plate of the aerial camera, and image points are placed on its top, the distortion will be compensated, because rays coming from the map plate to the eye will be refracted to the same degree that the image points were refracted at exposure.

If the photograph is not affected by refraction distortion, the image points are temporarily spotted on top of the photograph plate, then transferred to the bottom by respotting on that side, the spots on the upper side being wiped off afterward. With this arrangement rays from both photo-image points and map points are refracted equally on passing through the photograph plate, but this refraction has no effect on alignment. It is necessary to employ photographs printed on "nonshrink" paper or else correct for differential expansion or contraction in order to obtain reliable results.

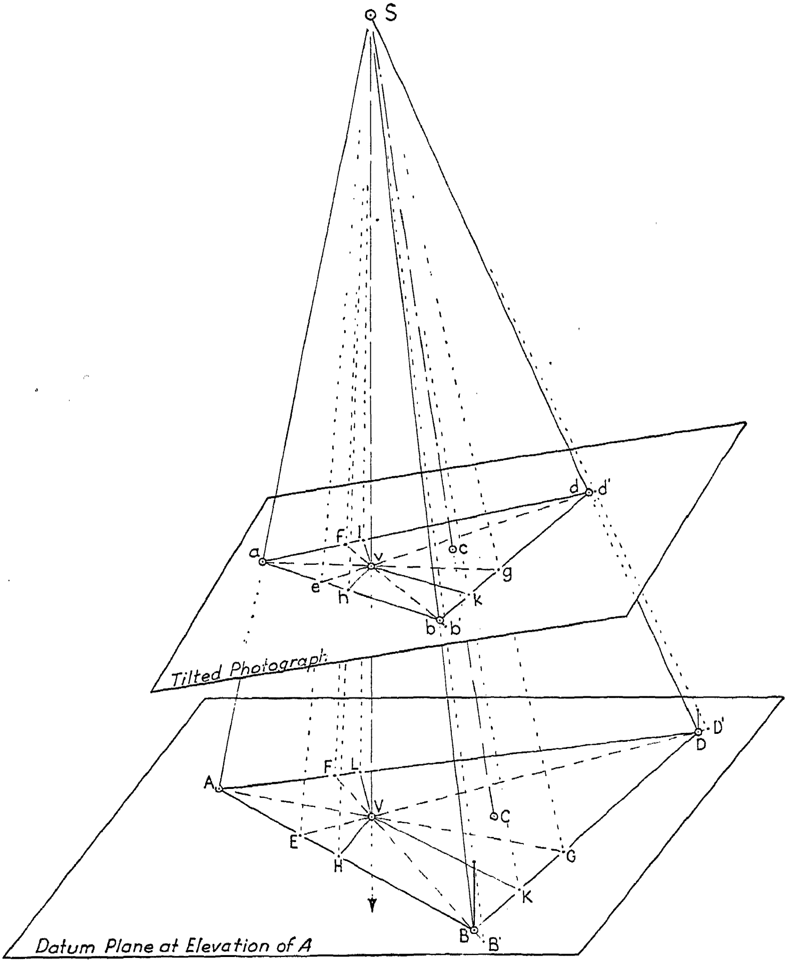


FIG. 84.—The pyramid method of determining tilt.

221. The pyramid method of determining tilt.—Figure 84 shows the setup wherein certain relationships that exist between a tilted photograph and three control stations are employed to find the degree and axis of tilt. It is necessary to know the positions and relative elevations of the three control stations, and these stations must be in positions that give a comparatively large triangle, preferably including within its perimeter the center of the photograph and the vertical point.

In the figure the three control stations are shown at A, B, and D. The datum plane employed is the horizontal plane through A, the lowest of the three stations. B and D are at the base of elevated objects whose heights are indicated by vertical lines drawn upward from their plotted positions. B' and D' are on radial lines drawn from V through B and D, BB' and DD' being equal to the displacement caused by the heights of these stations above the datum plane. Therefore, when the tilted photograph has been placed in correct position with reference to the control stations as plotted, the rays SA, SB', and SD' strike the photograph at the image points a, b', and d' of the stations. In order to obtain in the photograph the triangle that corresponds to the triangle ABD, it is necessary to correct for the two displacements BB' and DD'. Inasmuch as the position of V in the datum plane and v in the photograph are not yet known, this correction can be made only approximately; but as the tilt is ordinarily small, a partial and usually adequate correction can be made by shifting b' and d' toward c, the center of the photograph, by amounts equal to hd/f in each case, h being the height of the station above the datum plane, d the distance of the station from the center of the photograph, and f the focal length of the photograph. These computations give the distances $b'b$ and $d'd$ in the same unit used for h (usually feet), so it is necessary to know the approximate scale of the photograph to determine the actual value of these quantities for laying them off on the photograph. If the altimeter height of the flight to obtain the photograph is available, f/H gives the scale close enough for this purpose. If, however, the altimeter height is not available, find the approximate scale by comparing ab' on the photograph with AB on the datum plane (or any other pair of corresponding distances). A still better value of the average scale will result if all three of the sides of the triangles are compared, and the mean of the three determinations is used. The positions of b and d having been found, the three distances ab, bd, and ad are measured on the photograph. Note that because both B and D are higher than A, $b'b$ and $d'd$ are laid off *toward* the center of the photograph.

The procedure from this point on consists in making a graphical construction that will determine the positions of V and v, whereupon the axis of tilt and the distance vc become known and the degree of tilt i can be readily computed from

$$\tan i = \frac{vc}{f}$$

The figure shows all the planes and lines involved in making the construction. Two series of vertical planes are employed: (a) the planes SVH, SVK, and SVL, which are perpendicular to the respective sides of the triangle ABD; and (b) the planes $SAVG$, $SBVF$, and $SDVE$, which also include the plumb line *SV*. The first three vertical planes cut the straight lines *vh*, *vk*, and *vl* from the photograph, and the second three vertical planes cut the straight lines *avg*, *bvf*, and *dve* from the photograph.

The graphical construction consists of developing the faces of the pyramid *Sabd* in a single plane; placing on this development the developed faces of the pyramid *SABD*; drawing from S a line (*SH*, *SK*, and *SL*) perpendicular to the base of each face of the pyramid *SABD*; finding the points in the datum plane that correspond to *H*, *K*, and *L*; erecting a perpendicular at each of these points (*H*, *K*, and *L*) to the side of the triangle in which the point lies, thereby finding the position of *V*; then drawing the lines *AG*, *BF*, and *DE* which intersect at *V*, the intersection of the plumb line and the datum plane; finding the points *g*, *f*, and *e* corresponding to *G*, *F*, and *E*; and finally drawing the lines *ag*, *bf*, and *de* which correspond to *AG*, *BF*, and *DE*, thereby getting at their intersection the position of the vertical point *v* of the photograph.

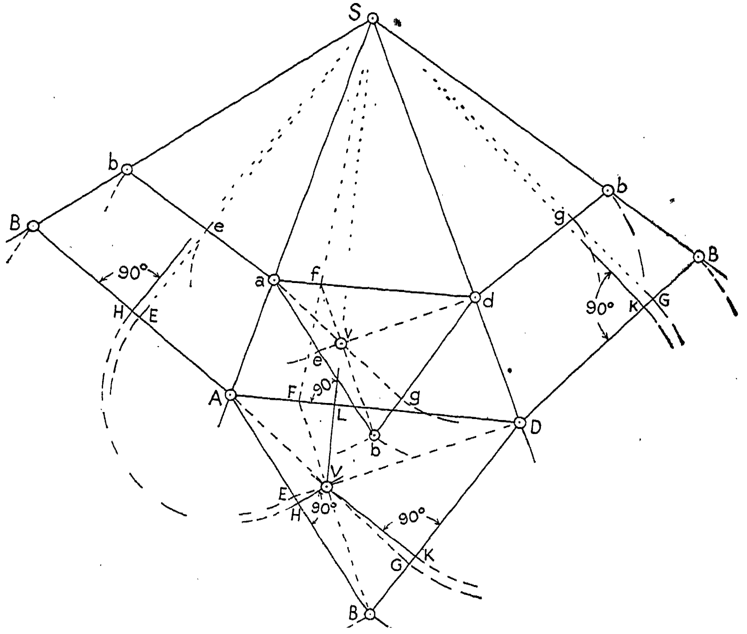


FIG. 85.—Faces of pyramids laid out in the plane of one face.

Figure 85 shows the developed pyramids laid out in the plane of the face *Sad*. The edges of the photopyramid are the hypotenuses of right-angle triangles whose bases are the distances *ca*, *cb*, and *cd* of Fig. 84 and whose common height is the focal length (*f*) of the photograph. Figure 85 is to a large degree self-explanatory, for the lettering corresponds to that employed in Fig. 84. The triangle *abd*, taken from the photograph (but modified for differences of elevation as explained above), is drawn to scale, and the pyramid spread out as indicated, the apex *S* (at the perspective center) being located by arcs from *a* and *d* as centers, each drawn with radius equal to its proper pyramid edge. The two lateral positions of *b* are also located by arcs from *S* and *a* on the left and *S* and *d* on the right, as centers, a proper side of the pyramid base and an edge being employed in each case. *Sa*, *Sd*, and *Sb* (on each side) of the developed pyramid are extended as necessary, dependent on the scale at which the triangle *ABD* is to be plotted. In the present case its scale being greater than the scale of the triangle *abd*, the extension is necessary. Usually the scale of a map from which control points are taken is smaller than the scale of the photograph. Then triangle *ABD* falls between triangle *abd* and *S*, and no extension is necessary.

The next step is to place the developed control pyramid in proper position relative to *Sbadb*. This may be accomplished by use of a sheet of tracing paper on which *AD* is laid off and arcs drawn from *A* and *D* as centers and *AB* as radius on the left, *DB* as radius on the right. Knowing that *SB* on the left must equal *SB* on the right, place this tracing over the drafting sheet and shift it until *A* falls on *Sa* prolonged, *D* falls on *Sd* prolonged, and each arc at *B* (left and right) falls on *Sb* prolonged. This requires special care and is a little tedious to do. It helps to mark a few equal distances along both of the lines *Sb* to bracket the positions into which *B* must fall in each case. Having placed the developed pyramid *SBADB*, close the triangle *ABD*, which falls in the lower part of the figure. Draw *SL* perpendicular to *AD*, *SH* perpendicular to *AB* on the left, and *SK* perpendicular to *DB* on the right. With *D* as center describe the arc *KK*, and with *A* as center describe the arc *HH*. At each of these new positions of *K* and *H* on the triangle sides erect a perpendicular to the side in which the point lies, and prolong *SL*. These three perpendiculars meet at *V*, the intersection of the plumb line and the datum plane. Draw *AV*, *DV*, and *BV* to intersect the sides of the triangle at *G*, *E*, and *F*, respectively. Again using *A* and *D* as centers, swing arcs *EE* and *GG*, and from these new positions of *E* and *G* draw lines toward *S* to intersect *ab* at *e* and *db* at *g*. Also draw a line from *F* toward *S* to intersect *ad* at *f*. From *a* and *d* as centers swing arcs *ee* and *gg*, and from each of these new positions of *e* and *g* on the triangle and from *f* draw a line to the opposite apex of the triangle *abd*. These lines intersect at the vertical point *v* of the photograph.

The method is most satisfactory when complete horizontal and vertical control is available; but when only the elevations of three points are known, their locations can be obtained by the radial-line method if suitably overlapping vertical photographs are available. The method is affected adversely by excessive tilt and excessive relief. Whenever either the tilt or a difference of elevation between control points is great, a second determination with better correction for the positions of *b* and *d* will refine the result.

222. Optical projection.—A special projecting camera is required to apply this method, and operation must be carried out in a darkroom or with subdued light in the vicinity of the image side. Strong illumination of the negative is essential. It is necessary that both negative and image (control) holders be provided with means for rotation in their own planes and for tilting. Shifting of the position of the negative is required, and provision must be made for scale adjustments. The principles that govern manipulations are explained in Chap. VIII. The procedure is so complicated as to be impracticable unless conditions are imposed to limit considerably the required range of operation. Ordinarily there is no need to provide for dealing with angles of tilt greater than 4 or 5°. Placing narrow limits on scale range tends to simplify the apparatus and its use. Further simplification of operation is gained by such aids as having the location of the center of the photograph along with the required number of control points and information as to the height of the air station above a datum plane.

Optical projection is employed in the Brock process (Art. 242), use being made of two projectors simultaneously to obtain the tilts of a pair of photographs and then to make corrected transformed prints on photographic plates.

223. Tilt may be obtained from parallax differential measurements when the tilt of one of the photographs of a stereo pair is known. This is done by a process closely related to that of obtaining elevations as explained in Art. 226. In this case, however, the line between the centers of distortion cannot be used, because the center of distortion of one of the photographs is not known in advance. However, if the tilt is not great, an approximate result can be obtained by employing the centers of the photograph instead of the centers of distortion and the line connecting them as the basic reference line. Then if a more precise result is needed, a second determination may be made using the centers of distortion, one, of course, being only approximately located.

The procedure consists in measuring the distances of a pair of corresponding image points from the basic reference line, correcting the distance obtained from the photograph whose tilt is known by applying $d \sin \varphi$, then comparing the corrected distance with that obtained from the photograph whose tilt is to be found. Equality between them indicates either that the photograph is not tilted (corollary, Art. 200) or that the point lies along the axis of tilt. Any difference between these two values is the component of tilt distortion along the X axis and must be divided by the sine of the proper angle (one of the series designated by φ)

to obtain the tilt displacement d of the image point along the radial line from the center of the photograph to it. By use of three or more pairs of image points chosen to give the displacements of tilt in directions spread through a large sector, a tilt circle (Art. 171) may be constructed to obtain the principal line and degree of tilt.

The curves of Figs. 46 and 47 are drawn for the center of distortion; but if the tilt is small and the field of the photograph moderate (α not greater than 40°), they may be used with reference to the center of photographs without bringing in much error.

CHAPTER XI

METHODS OF OBTAINING ELEVATIONS AND CONTOURS FROM AERIAL PHOTOGRAPHS

224. There are in common use but two general methods of obtaining the elevations of objects pictured in aerial photographs: the *stereoscopic* method and the *upright planes* (displacement) method. The stereoscopic method is applied in many ways, but all are based on the same fundamental principles. By this means all but a small percentage of the work of contouring maps is done. The chief present uses of the displacement method are (*a*) to obtain secondary vertical control for making topographic maps of terrain that is difficult to travel over and tedious to survey by ground methods and (*b*) for reconnaissance and exploratory mapping.

Two other possible methods are available, but they have not come into general use: (*a*) the *vertical plane projection* method and (*b*) the *vertical angle* method. The latter has been applied successfully to high obliques, the only type of aerial photograph with which it has been used. It is simple but requires the true horizon line of the photograph to be known and a means of measuring the required angles in vertical planes. It is analogous to trigonometric leveling with the plane table and telescopic alidade and is explained in Art. 247, which deals with the Wilson photoalidade, an instrument designed for applying the method. The vertical plane projection method is explained in Art. 229.

225. Applications of the stereoscopic method.—There are three variations on the application of this method to aerial photographs: (*a*) that in which tilted verticals are first rectified to true (or nearly true) verticals and brought to the same scale, measurements being made with a stereocomparator or similar simple stereo-measuring instrument; (*b*) that in which approximately parallel tilted verticals or low obliques are employed in elaborate stereo-measuring machines or outfits; and (*c*) that in which tilted verticals are employed to make measurements as though the photographs were true verticals and the effects of tilt eliminated or reduced by corrections applied either along with the making of observations or afterward. The Brock process is an example of the first variation; the aerocartograph is one of the instruments in which the second variation is employed; and the stereocomparagraph and contour finder are instruments used in the third variation. The multiplex method also belongs in the second category. The first two variations of the method give greater refinement in results but are comparatively expensive. The third is limited in its application to substandard mapping when the relief of the terrain is low and the contour interval small or to mapping of comparatively rough country, the contour interval being large. It is, however, much less expensive to apply than the first two variations.

The fundamentals of the first variation are given in Chap. IX; the details of the Brock process, given in Art. 242, are sufficient explanation of its application to obtain elevations and contours. The second variation is dealt with in Art. 241. The third, as applied when the tilts of the photographs are known, is explained in Art. 226. A method of its application without knowledge of tilt, which has been used in military mapping, is given in Art. 227.

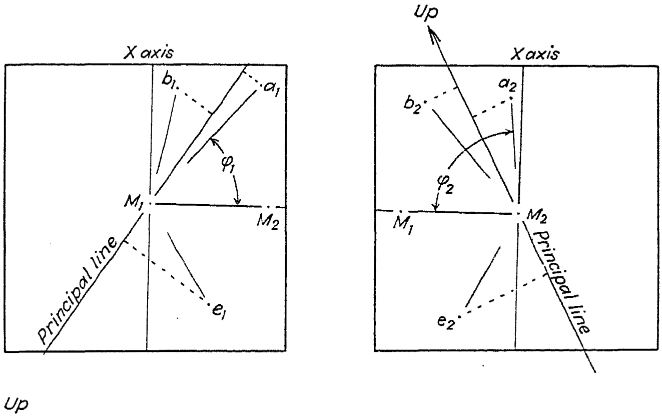


FIG. 86.—A pair of photographs as placed on a stereo-measuring instrument to make parallax readings and determine tilt distortions.

226. Elevations obtained from tilted photographs by correcting parallax differential readings.—If the tilt and the principal line of each photograph of a stereo pair are known, a stereocomparator or similar instrument (stereocomparagraph or contour finder) may be used to measure parallax differentials on the tilted photographs, and corrections made by employing graphs such as those of Figs. 46 and 47. The readings made are affected by tilt, and these effects must be eliminated to obtain the true differentials. If there are available in the zone of overlap three or more points of known elevation, which range through the relief of the area (*i.e.*, one point is at a level near the lowest part of the terrain, another near the highest part, and the third point near the middle contour), a parallax-height graph such as that of Fig. 75 may be made for the pair and used to obtain the elevations of points whose parallax differentials have been corrected for the effects of tilt.

If advance data are sufficient to give the length of the stereo base, the height of the air stations above a datum plane, and the focal lengths of the photographs, elevations may be obtained without control points. This procedure is less satisfactory, however, than that based directly on bench marks, because much more work is involved and the risk of error increased, no opportunity being given for checks.

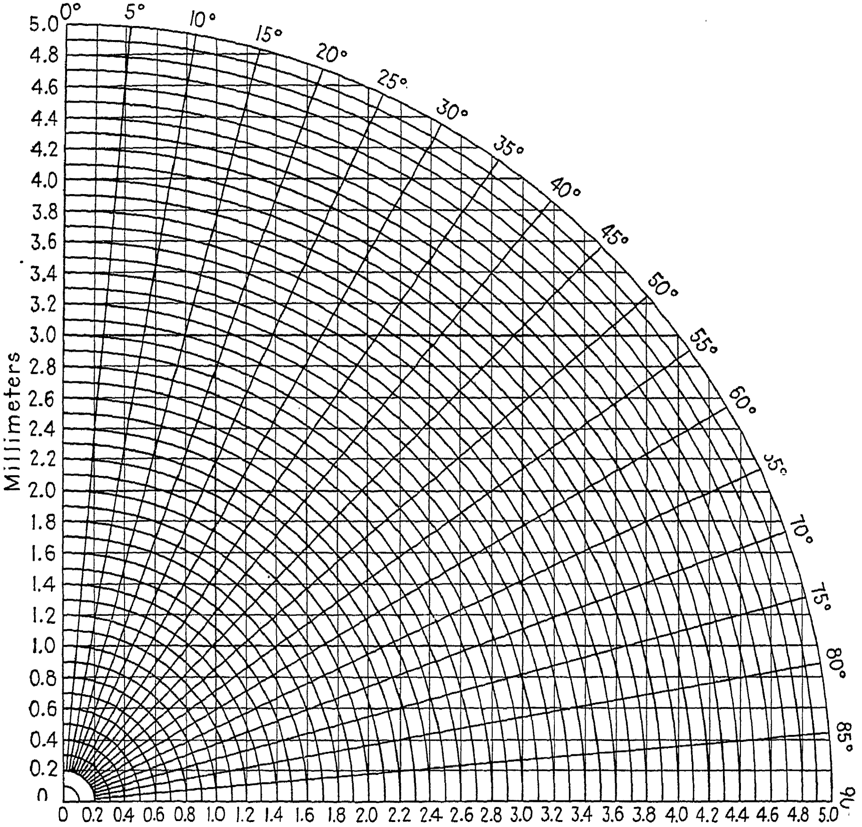


FIG. 87.—Special protractor on a square grid for use in determining components of distortion.

Figure 86 shows the setup of a pair of photographs. They are carefully oriented together on the stereo-measuring instrument by their centers of distortion M_1 and M_2 and parallax differentials read of the bench marks *a*, *b*, and *e*. The line M_1M_2 is to serve as the base line of reference. A perpendicular is dropped from the image of each of the bench-mark images to the principal line and the displacement of each point along the radial line from the center of distortion found as explained in Art. 166. This is done for each photograph. In order to obtain the effect of each displacement in the direction of the base line M_1M_2, the angles between the radial lines and M_1M_2 are measured with a protractor, and the component along M_1M_2 found by multiplying the displacement by the cosine of the angle. If, for example, d_1 is the displacement along the radial line M_1a_1, then its effect along the line M_1M_2 is $d_1 \cos \varphi_1$. The corresponding effect on the companion image a_2 is $d_2 \cos \varphi_2$. In a similar manner the effect on the parallax differentials of the other points is found.

To facilitate the measurement of angles φ_1 and φ_2 and the determination of the corresponding components of distortion, a protractor drawn on a grid of squares, as shown in Fig. 87, is used. It should be large enough in scale to permit distortions and components of distortion to be read to $\frac{1}{100}$ millimeter. Squares whose sides are $\frac{1}{4}$ inch, or 5 millimeters, taken to represent $\frac{1}{10}$ millimeter, give a convenient scale. A diagram 8 or 10 inches square provides a sufficient range for normal conditions. A parallax reading of the companion points a_1 and a_2 is then corrected by adding to it algebraically $\pm d_1 \cos \varphi_1$ and $\pm d_2 \cos \varphi_2$ as proper. If an image point is on the upper side of the photograph, the parallax component is positive; if on the lower side, negative. Parallax readings are considered positive.

A check on the work is obtained by applying $d_1 \sin \varphi_1$ (component at right angles to M_1M_2) to the distance of a_1 from the line M_1M_2 and $d_2 \sin \varphi_2$ to the distance of a_2 from this reference line. If the corrected distance of a_1 equals the corrected distance of a_2, it is an indication that the work has been done without error.

Rules for applying tilt corrections to readings on stereo-measuring instruments for parallax differentials when a leftward movement of the right collination mark or a rightward movement of the left mark lessens the reading are:

Position of image point	Left photograph	Right photograph
On upper side of photograph and to right of X axis.......	Minus	Plus
On upper side of photograph and to left of X axis........	Plus	Minus
On lower side of photograph and to right of X axis.......	Plus	Minus
On lower side of photograph and to left of X axis........	Minus	Plus

The rules may be modified as appropriate for readings made with instruments that do not conform to the stated effects of movements on readings.

227. Contouring from a stereo pair of tilted verticals without knowledge of their tilt.—A method[1] that is used in the Army and is being employed to some extent in making military maps is unique in that corrections are made to the setting of the wandering mark during the process of contouring with the stereocomparagraph. It is dependent on having in advance the elevations of several points (their positions are not required immediately) scattered through the area of overlap and is based on parallax tables prepared for the purpose. It consists of preparing from parallax readings made on the "bench marks" (spot elevations) a table for each stereo pair of photographs employed and then making from it a graph of curved lines which is drawn on the left photograph. This graph gives the correction to be applied to the setting of the wandering mark at crossing of a line when drawing the contours. The lines differ in value by 0.05 millimeter and are marked to indicate whether a correction is to be added or subtracted. Although the corrections are made by steps, not gradually, the height involved in each step is small so that large displacements of contour lines from their proper positions on the photograph do not occur if the terrain is moderately rough. In fact the corrections are at steps commensurate with the refinement given by the instrument. The process furnishes contours that are not free from the displacements caused by tilt and relief, so the plot must be adjusted to horizontal control on the map, as is the case in plotting by tracing (Art. 231).

The value of the method depends largely on the tolerances allowed for the map and on whether or not the required bench marks can be obtained without too great cost. The method has been used in conjunction with the multiplex instruments which supply the necessary secondary horizontal and vertical control points. Required advance data of heights of air stations and lengths of stereo bases also are obtained from systematic work with those instruments. Used in this connection, or in connection with another of the elaborate plotting outfits, the method has more value than without their aid.

The parallax tables furnish the parallax values for the bench marks available in the overlap of any stereo pair of photographs, and from these the parallax differential between each bench mark and the datum plane (sea level or any other convenient level) is obtained by subtraction. The photographs are set up in the stereocomparagraph and oriented by the line connecting their centers. A micrometer reading is made on each bench mark, and the differential mentioned above is added to the reading to obtain a figure for the datum plane at that place of the overlap. When the figures for the datum plane, which because of tilt differ in value and indicate the warping effect of tilt on that plane, have been obtained (at the several bench marks), a graph is made to counteract the deformation. It is assumed that the deformation is consistent and extends alike to all levels of the model. There is no proof that this is true, but it is reasonable to expect that the deformation is carried with little change to a considerable height in the model.

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[1] This method is explained in detail in "Instruction Manual for the Stereo-comparagraph," issued by the Engineer School, Fort Belvoir, Va., 1940. The manual also includes tables of parallax at 20-foot intervals for heights ranging from 10,000 to 25,000 feet and at 10-foot intervals from 5,000 to 10,000 feet.

The scheme employed to apply corrections to the deformed model is simple and logical. It consists in tracing the bench marks from the left photograph and placing near each the datum-plane micrometer figure obtained as explained above. The distances between pairs of bench marks are then marked with as many lines as there are 0.05-millimeter steps between the two figures. For example, between two bench-mark images whose datum-plane figures are 14.60 and 14.25 millimeters there will be seven spaces of 0.05 millimeter each, so the distance is divided into seven equal parts by ticks. The other distances between pairs of bench marks are similarly divided, each tick being given its number in the scale, as 14.30, 14.35, 14.40, and so on. When all the distances have been divided in this manner, ticks of equal value are joined by smooth lines to form a graph resembling contours, each line being labeled with its number in the scale. These lines are transferred from the tracing to the left photograph and drawn on it with removable ink or crayon.

Plotting of contours may now be carried out by setting the micrometer in turn for any desired contour level as obtained from the parallax tables and corrected for the effects of tilts. A start is usually made at the lowest contour, the micrometer being set correctly for that part of the model in view. As scanning proceeds to the next graph line, the setting is changed 0.05 millimeter plus or minus depending on whether the movement is toward lines whose numbers are larger or smaller.

Four bench marks distributed so as to have one near each corner of the model give an excellent distribution for making a graph. There are then six pairs of points between which graph lines may be located. The pattern of the graph will depend on the relative orientation of the principal lines of the photographs, and there is a great range of patterns. In many patterns there is considerable regularity so that it is possible to extend the lines some distance outside the polygon formed by the bench marks. The distance between lines of a graph depends on the degrees of tilt and the scale of the photographs.

228. The upright-planes, or displacement, method of determining the elevations of objects is the utilization of the principles employed to find the tilt of a photograph as explained in Art. 219. Once the setup of Fig. 81 has been established, elevations may be found readily. It is necessary to have the plotted position of each object whose elevation is to be determined as well as to know the tilt of the photograph. Accuracy depends directly on how well these two prerequisites have been determined and how accurately the position of the pivot point of the map line has been fixed.

The tilt circle (Art. 171) is employed to obtain the component of tilt along the radial line from the center of the photograph to the image point of the object whose elevation is to be determined. This angle is laid off on a construction like that of Fig. 81 from a reference line *CX* which is parallel to the photograph line *cd*. If the image point is on the lower side of the photograph, the angle must be laid off upward to give a line converging toward the photograph line; if on the upper side, downward. Along the line thus obtained, the map distance is laid off (*CB* for example). The corresponding distance *cb* obtained from the photograph is laid off along the photograph line, and a ray *Sb* drawn from the perspective center. The length of the perpendicular *BB′* drawn from *B* to meet *Sb* is the difference between the elevation of the object and the datum plane through *C* which was chosen for the construction. The elevation difference is measured at the scale of the construction. If the map distance is greater than 2 miles, a correction, which is always positive, should be made for curvature of the earth and refraction of light.

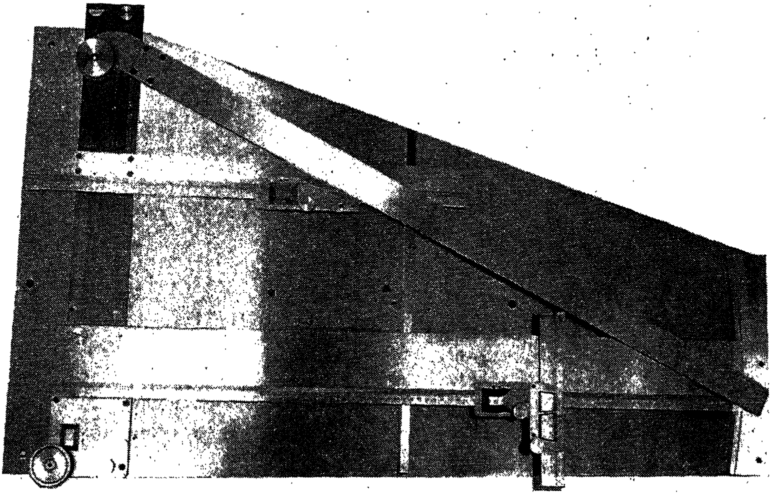

Fig. 88.—United States Army elevation calculator. (*Courtesy of D. W. Mann.*)

In order to expedite the work of finding elevations by this method and to obtain greater accuracy of results than are obtainable by graphical construction, the Army had constructed a few years ago several elevation calculators that are based on the principles here employed. Figure 88 is a photograph of one of these calculators. The instrument is made up of a base plate to which the map-scale pivot and a graduated circular arc are permanently fastened. A second scale slides in a guide at right angles to the reference line established by the center of this pivot and the zero mark of the circular arc. The second scale serves to lay off both the focal length of the photograph and the height of the air station above the pivot, which is always to be placed in the datum plane of a setup. Fastened rigidly at right angles to the second scale is the photograph scale, and at its upper end a slide carries an arm that may be swung obliquely across the photograph scale and the map scale. This slide carries a vernier that provides for setting the pivot of the oblique arm at any required distance from the photograph scale, and a vernier attached near the junction with the map scale provides for setting that pivot at the proper distance from the pivot of the map scale to accord with the height of the air station above the datum plane.

A slide with vernier may be moved along and set on the photograph scale. It carries a stop against which the oblique arm may be brought to lay off a ray such as *Sb* of Fig. 81. A slide with vernier also may be moved along and set on the map scale. This slide carries an elevation scale that may be slid along in a guide at right angles to the map scale, and a vernier is provided to make accurate readings on the elevation scale.

The instrument is made ready for work by setting it to accord with the focal length of the photograph and the height of the air station above the chosen datum plane. The photograph-scale slide is set at a distance equal to that of an image point from the center of the photograph, and the oblique arm brought to the stop. The slide of the map scale is then set at the map distance corresponding to that measured on the photograph and swung through an angle that equals the component of tilt for the direction concerned as found from the tilt circle. The circular arc is graduated in both directions from the zero mark, and a double vernier is provided on the end of the map scale, so that the angle may be laid off upward or downward as required. With the map scale so set, the elevation scale is slid into contact with the oblique arm, whereupon the difference in elevation between the object and the datum plane may be read.

229. The vertical-plane-projection method of obtaining elevations is a process in which the vertical plane, which includes the plumb lines of a pair of aerial photographs, is used as the projection plane, points in space being projected orthographically into that plane, and heights measured above a horizontal datum plane. The method is applicable to pairs of verticals and obliques that include a common area, and the larger the overlap the more satisfactory will be the application. Obliques whose axes are inclined more than about 60° to the plumb line cannot be satisfactorily employed. Aside from this limitation the method is general; for a vertical may be paired with another vertical or with an oblique, and it is immaterial whether the air stations are at the same or different heights. It is necessary to know the focal length and tilt of each photograph, the heights of the air stations, and the location of the foot of each plumb line (plotted positions of the vertical points).

Figure 89 shows a pair of oblique photographs set up in proper relation to each other, the vertical plane which includes their plumb lines, and a horizontal plane which is to be used as the datum for heights. It is a perspective view in which P is the vertical plane that includes the plumb lines S_1M_1 and S_2M_2, and the photographs stand behind it. The horizontal datum plane is represented by the line M_1M_2 and the two lines M_1N and M_2Q, M_1N being its intersection with the principal plane of the left photograph and M_2Q its intersection with the principal plane of the right photograph. The vertical point of the left photograph is at V_1, where the plumb line S_1M_1 prolonged meets its plane; the vertical point of the right photograph, at V_2. M_1 and M_2 are the projections of the air stations on the datum plane, *i.e.*, they represent the plotted positions of the air stations on a map sheet. E_1 and E_2 are the image points of an object E which is located in space at the intersection of the two rays S_1E_1 and S_2E_2. The planes $S_1E_1K_1$ and $S_2E_2K_2$ are each perpendicular to the plane P, and they intersect it at a point L which is at a height above the datum plane equal to the height of E above that plane, for each of the three lines E_1K_1, E_2K_2, and EL is perpendicular to the plane P and hence horizontal. The process therefore consists in finding the locations of K_1 and K_2 (projection of the image points E_1 and E_2 orthographically into the vertical plane), connecting each with its air station, prolonging

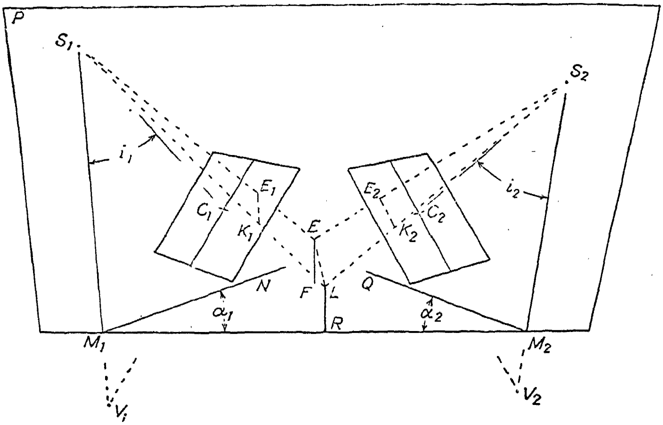

FIG. 89.—Perspective view of a setup of a pair of obliques.

the lines to intersect, then measuring the distance of the intersected point from the datum plane at the scale of the construction. The process is carried out graphically, and the vertical plane is the plane in which the setup is made and the necessary construction accomplished.

The principal line is drawn on each photograph, and a line through the principal point at right angles to the principal line. These two lines on each photograph are to be used as the coordinate axes to which image points are to be referred, the principal point being the origin. Either the principal point or the vertical point may be used as origin. It is better to employ the principal point for this purpose in obliques. In verticals the vertical point is close to the principal point, and for them there is a slight advantage in using the vertical point. For the purpose of this explanation the principal point is assumed to be the origin.

A large sheet of drafting paper or a thin sheet of aluminum or zinc painted white, which, being stable, is better, is used as the vertical projection plane. A transverse line is drawn across it near its lower edge; and having chosen an appropriate scale for the construction, such as 1 inch equals 1,000 or 800 feet, the scale being as large as the dimensions of the sheet will allow, a distance is laid off on it to represent the distance between the two plumb lines, and perpendiculars are carefully drawn at these points. The perpendicular on the left should be about the same distance from the left edge of the sheet as the perpendicular on the right is from the right edge. The height of the left air station is measured off on the left perpendicular from the transverse line, and similarly the height of the right air station, on the right perpendicular. The transverse line is to be used as the datum line, or datum plane, of the drawing, so the air stations will now be plotted correctly with respect to each other and the datum plane.

The position of each principal point (origin for coordinate axes) as projected into the vertical plane is now found and plotted. Let α_1 be the dihedral angle between the principal plane of the left photograph and the vertical plane of projection, and α_2 the corresponding angle for the right photograph. These are obtainable directly from the map sheet, on which the air stations and traces of the principal planes have been drawn. Let i_1 and i_2 be the angles of tilt of the left and right photographs, respectively, *i.e.*, the angles between the optical axes and the plumb lines. Let f_1 be the focal length of the left photograph and f_2 the focal length of the right photograph. Then with reference to the position of its air station the origin of the left photograph in the vertical projection plane will be at a position that is at a distance equal to $f_1 \sin i_1 \cos \alpha_1$ to the right and $f_1 \cos i_1$ below. This can be readily proved by descriptive geometry and will not be demonstrated. Similarly, the origin of the right photograph will be at a position that is at a distance equal to $f_2 \sin i_2 \cos \alpha_2$ from and to the left of the right air station and at a distance below it equal to $f_2 \cos i_2$. Having located the origins, axes are drawn through each parallel to the plumb lines and at right angles to them. These correspond to the axes drawn on the photographs. Elevations can now be obtained by measuring the distances of corresponding image points with reference to the coordinate axes of the photographs and converting them to values to be laid off from the coordinate axes established on the construction sheet.

It is necessary to establish equations to make the required conversions. Let a represent the distance of an image point from the principal line of a photograph, and b its distance from the transverse axis; *i.e.*, a will be a distance measured to the right or left, and b will be a coordinate distance measured up or down. On the construction sheet let x be the distance corresponding to a of the photograph and y be the distance corresponding to b. Then by descriptive geometry it can be shown that

$$x = \pm b \cos i \cos \alpha \pm a \sin \alpha \tag{31}$$

$$y = \pm b \sin i \tag{32}$$

in which i is the angle of tilt of the photograph and α the dihedral angle explained above. In applying the signs it is necessary to take into consideration the position of an image point with reference to the axes of the photograph and whether it is on the left or the right photograph. The rules are as follows:

FOR THE RIGHT PHOTOGRAPH

Image point above the horizontal axis $\left\{\begin{array}{l} b \cos i \cos \alpha \text{ is minus} \\ b \sin i \text{ is plus} \end{array}\right.$

Image point below the horizontal axis $\left\{\begin{array}{l} b \cos i \cos \alpha \text{ is plus} \\ b \sin i \text{ is minus} \end{array}\right.$

Image point to the right of the principal line, $a \sin \alpha$ is plus

Image point to the left of the principal line, $a \sin \alpha$ is minus

FOR THE LEFT PHOTOGRAPH

Image point above the horizontal axis $\left\{\begin{array}{l} b \cos i \cos \alpha \text{ is plus} \\ b \sin i \text{ is plus} \end{array}\right.$

Image point below the horizontal axis $\left\{\begin{array}{l} b \cos i \text{ c[illegible] is minus} \\ b \sin i \text{ is minus} \end{array}\right.$

Image point to the right of the principal line, $a \sin \alpha$ is plus

Image point to the left of the principal line, $a \sin \alpha$ is minus

Pairs of image points are measured, their coordinates converted by calculation or by slide rule, and the corresponding positions plotted on the construction sheet. A line is then drawn from the proper air station through each point, and each is prolonged to intersect the other. The distance from the point of intersection to the datum line at the scale of the construction is the height of the object above or below the datum plane. A correction should be made for curvature of the earth and refraction of light if the object is more than 2 miles from an air station.

CHAPTER XII

PLOTTING METHODS AND PLOTTING INSTRUMENTS

230. Methods of plotting data from aerial photographs range from the making of simple tracings, by means of which details are transferred to the map at the scale of the photographs, to the use of complex photogrammetric instruments, which not only give plottings on a true orthographic projection but permit the scale to be changed in the process and contours to be traced. Following is a list of methods that have been employed in North America. Some of them are suitable to plot secondary control points; and one, the slotted-template method, is used only for that purpose.

(*a*) The tracing-paper method
(*b*) By use of a pantograph
(*c*) The radial-line method
    (1) By use of a transparent sheet
    (2) By use of slotted templates (U.S. Department of Commerce and U.S. Geological Survey templates)
    (3) By use of a stereo-radial plotter
(*d*) By optical projection of individual photographs
(*e*) By optical projection of overlapping pairs of photographs, as in colors using the multiplex instruments
(*f*) By use of a simple stereo machine such as the Fairchild stereocomparagraph or the Abrams contour finder
(*g*) By observation of stereo pairs of photographs using elaborate photogrammetric machines such as the aerocartograph and the stereoplanigraph
(*h*) The Brock method, using the Brock instruments
(*i*) By use of instruments for employing obliques:
    (1) The Wilson photoalidade
    (2) The Miller plotting instrument
    (3) The Canadian oblique plotter
(*j*) The Canadian grid process

231. The tracing-paper method.—This is the simplest of all the methods but requires that (*a*) the map be drawn at the average scale of the photographs or the photographs be brought as near as practicable to the scale of the map, (*b*) every photograph have sufficient control to adjust its details to their proper places, and (*c*) verticals or transformed composite photographs be employed. Sheets of very thin and transparent tracing paper or sheets of plastic material (acetate and nitrate) are used. Where indistinct, photographs are studied under a simple stereoscope and inked before tracing. A tracing table having a glass plate illuminated from beneath aids in some cases. When completed, a tracing sheet is placed over the map, adjusted into position by control points, and held there by weights; a piece of carbon paper is slipped under it and the subject is retraced with a fairly sharp pencil. Ordinarily, details are taken from any one photograph only to about the middle of the area of overlap with the next photograph. Areas of considerable relief are traced as far as practicable by levels; *i.e.*, higher levels are traced by adjustment to control points lying near that level; lower levels, to lower control points.

A procedure that for a time was followed in applying this method consisted of inking with India ink all details to be transferred to the map, then bleaching out the photographic image before tracing. This increases the labor involved and is not usually practiced by experienced personnel.

The method is rapid but not so accurate as some of the others. It does give more reliable results than can be obtained from mosaics. Tied to a net of control points located by the radial-line method, it serves quite well for making general service maps at intermediate scales if the photographs have been well taken and the relief is not great.

232. Plotting with a pantograph.—This method has an advantage over the tracing-paper method in that it permits a change of scale from photograph to map. It is less convenient to work and requires the use of an accurately adjusted pantograph that responds without play or excessive friction. To be suitable for pantograph plotting, the photographs need not be taken at a common level, but they should be free of excessive tilt. The method is applicable to verticals of terrain with little or moderate relief. As in the tracing-paper method, a large amount of control is required, and attention must be given to adjustment to reduce errors caused by tilt of the photographs and differences of level. Plotting may be done directly to the map sheet or to tracing paper from which details are transferred to the map. More satisfactory results are obtained with the pantograph when the scale of the photographs is greater than the scale of the map, but plotting at slight enlargement is practicable.

233. The radial-line method.—The basis of the radial-line method is given in Chap. VII (Arts. 164 to 177). All applications of this method require that the vertical or composite photographs overlap one another more than 50 per cent, so that there will be a zone of overlap common to every triad group of photographs of a strip and that a moderate amount of overlap exist between photographs of parallel strips. The selection and identification of primary and secondary control points and the marking of these points is the same for all applications of the method. Particular attention should be given to orientation of one photograph with another when the plotting is to be carried out from photograph to photograph by means of a transparent sheet. This can be accomplished best by drawing a line entirely across the zone of overlap from one center point to the next. After these orientation lines have been drawn, an examination should be made to see that corresponding lines cut across the image points of the same objects in both photographs concerned.

Properly chosen, there will be three secondary control points in every overlap zone that is common to three photographs. One of these will be a center point (either the principal point or a definite image point close to the principal point), and one will lie near each end of the overlap zone. On selection of such groups of three control points, each photograph of a strip will have normally nine secondary control points to be located on the map to serve the purpose of controlling the placement of details, *i.e.*, every photograph except the first and last ones of the strip, which, since they have a photograph on one side only, will have but six selected points. Figure 90 is a diagram representing three overlapping vertical photographs to show the positions of the points in question. Photograph 1 will have the six points 1-1, *C*-1, 1-2, 2-1, *C*-2, and 2-2. Photograph 2 will have the nine points 1-1, *C*-1, 1-2, 2-1, *C*-2, 2-2, 3-1, *C*-3, and 3-2; and each succeeding photograph except the last will have nine points similarly placed. It will be noted that points 2-1, *C*-2, and 2-2 are common to all three of the photographs. It should be understood that only untilted photographs of flat terrain taken at a common level would match perfectly as shown in the diagram. Because verticals do not match perfectly, it becomes necessary to adopt such measures as these in plotting. The outer points 1-1, 1-2, 2-1, 2-2, etc., must also be spotted on photographs of adjacent strips. Any primary control points, such as *A*, will likewise be spotted, marked, and labeled on every photograph that includes them.

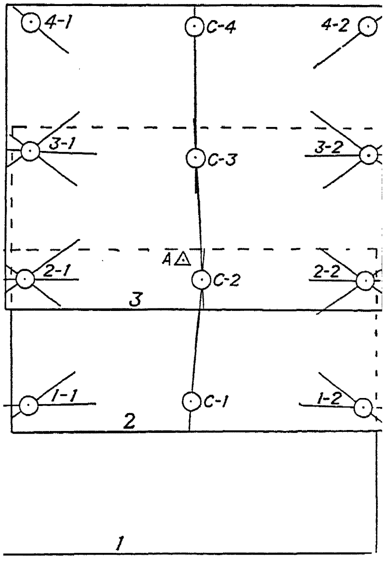

FIG. 90.—Diagram illustrating the preparation of vertical photographs for use in making a radial-line plot.

234. Marking the photographs.—The lines connecting *C*-1 with *C*-2 and *C*-2 with *C*-3 are the orientation lines mentioned. Drawn entirely across the zone of overlap, they serve to keep the orientation in going from one photograph to another. Short radial lines are drawn on each photograph from the center point to every other point embraced. Each point is pricked with a fine needle, and the hole filled with white chalk or black ink, depending on the tone of the surrounding detail. Each secondary point should be inclosed with a circle whose diameter is about ¼ inch, and each primary point with a triangle. Circles and triangles should be large enough to leave clear to be seen for inspection details immediately adjacent to a point. A labeling number is placed close to each point where it will not be crossed by a line. Radial lines are never drawn through a point but are stopped at the circumference of the circle or triangle. The length of radial lines to lateral points need be only a little longer than the difference between the scale of the photographs and the scale of the map; *i.e.*, if the distance from the center point to a lateral point on a photograph is 4 inches and the scale of the map is two-thirds the scale of the photograph, the radial line should be a little more than 1 inch long, and it should extend that distance from the lateral point toward the center point. These radial lines are required to carry out the plotting. They also assist materially in spotting lines on the transparent sheet.

235. Making a plot on a transparent sheet.—There are two variations of this method: (*a*) plotting at an unknown scale and (*b*) plotting at any predetermined scale, as at the scale of the map to be made. The first variation gives more freedom with respect to disposition of primary control points, but it requires a change in scale between the plot and the map. For plotting at an unknown scale two primary control points constitute a minimum required to determine the scale of the plot after it has been completed, and these points should be placed to lie one near each end of the strip within a zone of overlap and preferably about one-fourth to one-third the width of the strip from either lateral border. The two control points should ordinarily not be farther apart than six to ten times the width of the strip. If the photographs have been well taken and made on nonshrink paper, if the relief is not great, and if extreme care is taken, good results may be obtained even when the control points are as far apart as twelve to fifteen times the width of a strip. Although only two control points are theoretically required to establish the scale of the plot, it is best to have additional primary points, and all available should be used.

In order to align the transparent sheet with the strip covered by the photographs, the photographs are laid out on a table in matched position as well as practicable without taking much time. The sheet is laid over them and oriented to accord with the strip; the center point of the first photograph spotted; and a fine, straight, direction line drawn to the center of the next photograph. Withdrawing all except the first photograph, the sheet is placed over it with the mark for *C*-1 coinciding exactly with its position on the photograph and the orientation line falling exactly on *C*-2. If it is desired to make the plotting approximately at the scale of the photograph, *C*-2 is spotted at its distance from *C*-1, as taken from the photograph. If a smaller scale is desired, *C*-2 is spotted at a lesser distance. Whatever this distance may be, it will determine the scale of the plotting, which, however, will not be known until the two primary points have been located.

Direction lines are drawn from *C*-1 to each of the lateral points that have been marked and labeled. The sheet is placed over the second photograph with its *C*-2 exactly on *C*-2 of the photograph and the direction line to *C*-1 carefully brought over *C*-1 of the photograph. While held in this position, direction lines are drawn from *C*-2 to each of the remaining marked points. At this stage there will have been located on the transparent sheet the six points 1-1, *C*-1, 1-2, 2-1, *C*-2, and 2-2. Any primary points such as *A* lying in the zone of overlap, provided it is not too close to the line from *C*-1 to *C*-2, will also have been located. The transparent sheet is placed over the third photograph and shifted until points 2-1, *C*-2, and 2-2 on it fall on their respective radial lines drawn on the photograph from *C*-3 and the direction line on it from *C*-2 to *C*-3 falls on *C*-3 of the photograph. This amounts to making a resection (or tracing paper) location of *C*-3. Its position is spotted on the sheet, and direction lines drawn from it to the points to be located and to 2-1 and 2-2 as well, for the last mentioned lines indicate whether the plotting has been well or poorly done up to this stage. Thereafter each photograph is similarly treated first to locate its center point by resection, then to locate additional lateral points by intersection. The second control point is located by the intersection of direction lines drawn to its images as they are reached. As soon as it has been located, the distance between the two control points on the transparent sheet may be measured and compared with the known actual horizontal distance between them on the ground and the scale of the plot computed. If these two primary points have been plotted on a map sheet, the distance between them thereon may be scaled, and a computation quickly made to determine the scale of the plot. By reduction or enlargement as required, all points of the plot may be brought to the scale of the map sheet.

For plotting at a *definite scale* it is necessary that some of the control points be placed to give a proper start; *i.e.*, three or more points arranged in a satisfactory group for making resection locations of at least two consecutive center points must be available. Primary control points placed at or near 2-1, *A*, and 2-2 of Fig. 90 would serve well to locate both *C*-1 and *C*-2 and provide for starting the plot. These and any other available primary control points having been plotted on the map, they are traced off on the transparent sheet. By laying this sheet over photograph 1 and shifting it until each point falls on its corresponding radial line of the photograph, *C*-1 will be brought directly under its true map position and may be spotted on the sheet. Direction lines are drawn to each of the secondary points to be located including *C*-2, and the second photograph is placed under the sheet. A similar procedure gives the position of *C*-2, which should fall on the radial line drawn on the sheet from *C*-1. The center points of secondary photographs are thereafter located in turn by resection, and lateral points by intersection as for a plotting at an unknown scale. Additional control points are used to check the plotting when and as their image points are reached. In this case the control points of the group used to fix the positions of the first two center points control the scale of the plotting.

236. The slotted-template method.—In this method slotted templates take the place of the transparent sheet. Being an application of the radial-line principle, the method can be employed only with properly overlapping vertical or composite photographs. Two forms of templates are employed: One of these was devised and is employed in the U.S. Department of Agriculture; the other, by the U.S. Geological Survey. The template of the Department of Agriculture is cut from cardboard or thin sheets of aluminum, a hand punch being used to punch the center holes and a special hand- or foot-operated punch being used for the slots. On completion of the punching the cardboard has a round hole near its middle and elongated slots radiating from the center of the hole. The hole corresponds to the center of the photograph; the slots represent radial lines drawn toward the images of primary control points whose positions are known or toward images of secondary points to be located.

Each photograph to be used for making a template is put through a process of point selection and marking as described in Arts. 233 and 234. It is then laid on the cardboard and stuck in place with Scotch tape. A needle is pressed perpendicularly through each selected point into the cardboard, and the points thus made serve as guide points for the punching. A template is made for each photograph concerned in a given area and numbered to accord with the number of the photograph. The slot is long enough (about 2 inches) to provide for differences in scale ordinarily encountered in photographs of any area to be mapped. If there is a considerable difference between the scale of the map to be made and the average scale of the photographs, an allowance is made when punching the slots, radial lines serving for this purpose.

In making an assembly of templates the positions of all primary control points are plotted on a projection grid, and an accurately centered stud is fastened at each point. The stem of the stud has a diameter slightly less than the width of a slot, to permit the latter to slide with a minimum of side play and a minimum of friction. The stem of the stud is bored with an accurately centered hole entirely through to permit a pin to be inserted with a snug fit. A flange at the base of the stud supports it in a position perpendicular to the plotting sheet. In setting a stud over a control point the pin is set on the point in an upright position, and the stud dropped over it to be brought correctly into position.

Sheets of aluminum about 0.015 inch thick have been found satisfactory for use as the projection grid. Metal cement is used to fasten the studs in place. If more than one sheet is required for the assembly, extensions are made by soldering together abutting sheets.

A stud is inserted in the center hole of each template, and a floating stud for each secondary control point to be located is guided to its position by every slot pertaining to the point. There will be two or more slots over each floating stud, many being directed by several slots. Normally a center stud will have two slots astride it to keep it in alignment.

A stud having been anchored at each primary control point of the map sheet and placed in each center hole of the templates, the templates are laid down in their proper relationship, floating studs riding in the slots and templates being shifted automatically into their final positions by action of the studs sliding in the slots and templates swinging about the fixed studs. Templates are touched here and there to assist in overcoming friction. If there has been no error in the work, no excessive tilt in the photograph, no great relief in the terrain concerned, and no appreciable differential shrinkage in the photographs, and if there are enough primary control points properly distributed over the area, a satisfactory assembly will be obtained. With all templates in their proper positions, weights are distributed about to prevent dislodgment, and a pin is inserted in each stud to prick the position of all secondary control points.

The template of the U.S. Geological Survey is made up of stock articles that may be used again and again. The stock pieces consist of punched and slotted strips of thin hard steel, studs having a hexagonal base flange and a threaded stem, and clamping nuts which hold together the parts of the built-up template. A spanner wrench to grip the stud and a socket wrench to tighten the clamping nut are employed. The studs and nuts are made of brass, and brass washers are employed to prevent the slotted strips from being twisted out of position when the nut is being tightened. Built-up templates are employed in the same manner as the cardboard templates. Figure 91 shows an assembled template, ready for use, as made of U.S. Geological Survey stock parts.

Templates permit great numbers of photographs of many parallel strips to be employed for a single assembly. They eliminate the necessity of making tedious adjustments, which are usually required when a transparent sheet is employed for plotting. Assemblies of large areas have been made.[1]

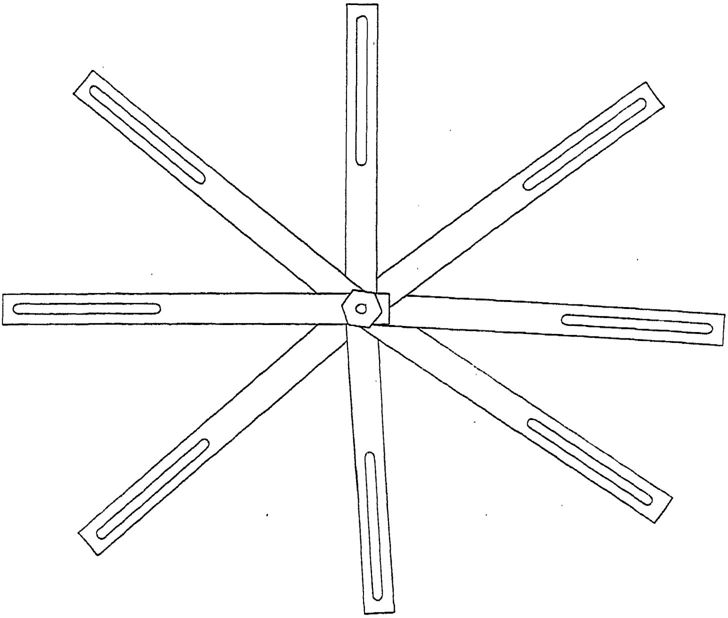

FIG. 91.—Assembled Geological Survey type slotted template, ready for use.

237. Plotting with a stereo-radial plotter.—Figure 92 shows an instrument of this type which is employed at the Institute of Geographical Exploration for plotting data from transformed wings of composite photographs obtained with the five-lens camera. It consists of an adjustable stereoscope mounted on a folding cantilever truss, two slotted radial arms, two pivots with weights about which the arms are swung, and a floating pencil held in the slots and by them made to move as the arms are swung. Accessories consist of a centering pin, a slot aligner, and a floating needle which may be substituted for the pencil.

A special feature of this method is that each photograph is swung exactly 180° about its center point from its true orientation on the map to place it in position for plotting. This takes the photograph entirely away from that part of the map to be plotted yet permits it to be viewed stereoscopically with its companion and to be employed with full advantage of the radial-line principle. Orientation is accomplished by aligning the image of a control point with the map position of that control point and the center of the photograph which is placed exactly on its previously plotted position. The stereo effect obtained by this arrangement is reversed, *i.e.*, hills appear as valleys, and lower parts appear to be above the hills, but this causes no inconvenience in plotting.

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[1] A complete description of the slotted template and its use is given in *Miscellaneous Publication* 404 of the U.S. Department of Agriculture, which is obtainable at small cost from the Superintendent of Documents, Washington, D.C.

Each arm is carefully slotted at the far end to receive the pencil carrier and to permit it to move along the slot practically without friction and without side play. The near end of each arm is cut out, and there is inserted a strip of transparent Plexiglass. A fine straight line is scribed on the lower surface of the Plexiglass strip, the line being on the center line of the slot prolonged through the center of the pivot hole. The left arm is made of one piece of bakelite, but the right one is constructed of two overlapping pieces to raise its slotted end enough to override the left arm.

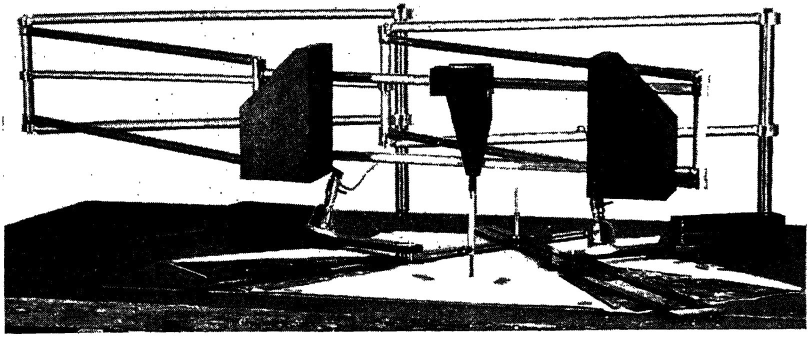

FIG. 92.—Stereo-radial plotter. *(Photograph by the Institute of Geographical Exploration, Harvard University.)*

Two pairs of arms are provided for the instrument, one pair being about 40 inches long, the other 48 inches long. They provide for plotting at scales ranging from about one-third the scale of the photograph to about 1½ times that scale.

The truss is supported by upright columns attached to the lower surface of a strong table at its back and provides for swinging the stereoscope over an area about 3 feet in diameter, the axis of the stereoscope being kept always parallel to the line of the supporting columns. The joints are fitted with ball bearings which permit easy movement. The eyepiece of the stereoscope, which is fitted with two 45° mirrors, is fixed to the front bars of the truss; but the lateral compartments, which house 45° mirrors and large reading-glass lenses, may be moved laterally along the bars for adjustment. The lenses in the compartments may be placed at any of three levels to aid in adjusting for focus. Electric lamps are attached to the compartments where they illuminate those parts of the photographs being viewed at any time. A hinged leg extends to the table from the ocular bracket to support the truss when not in use or at any time it is desired to study details.

For convenience in making adjustments between stereoscope and photographs, and to take care of differences of scale in a pair of photographs, the photographs, pivots, and map sheet are all mounted together on a thin board of pressed wood. This permits the assembly to be swung in orientation as required. The facilities for adjustment of the stereoscope are extensive enough to place photographs with corresponding image points anywhere between 9 and 40 inches apart. Very large photographs therefore may be viewed stereoscopically with the instrument.

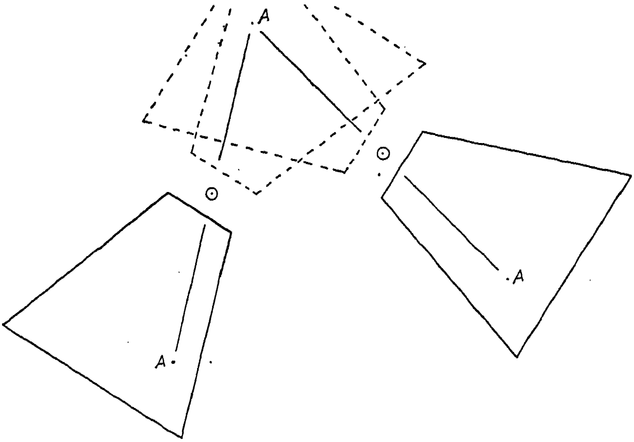

FIG. 93.—Placement and orientation of a pair of transformed photographs on a map sheet for plotting with the stereo-radial plotter.

When looking at photographs that have been properly placed under the instrument, a short stretch of each scribed line is seen as a cross, and the pencil occupies a position on the map corresponding to the intersection of the lines. As the arms are swung about the pivots, the cross appears to move over the fused photographic image. If the center of the cross is made to follow a line, such as a road, railroad, or stream, the pencil will plot that feature on the map. The scale of the plotting is necessarily the scale of the projection sheet on which the control points have been plotted. It is, of course, necessary to plot on the map beforehand the positions of the centers of all photographs and the other control points to be used for placement and orientation of photographs.

Figure 93 illustrates the placement of a pair of transformed photographs on the map for use with the instrument. Figure 94 shows how photographs may be taken with a five-lens camera to give well-arranged stereo pairs for plotting by this method. Parts of photographs made with two five-lens cameras mounted together or with a nine-lens camera may be used satisfactorily. Theoretically it is possible to employ single-lens verticals with the instrument by cutting them into two parts, but practically it is not profitable with photographs taken with cameras of moderate field. It is more satisfactory to use photographs obtained with wide-angle single-lens cameras.

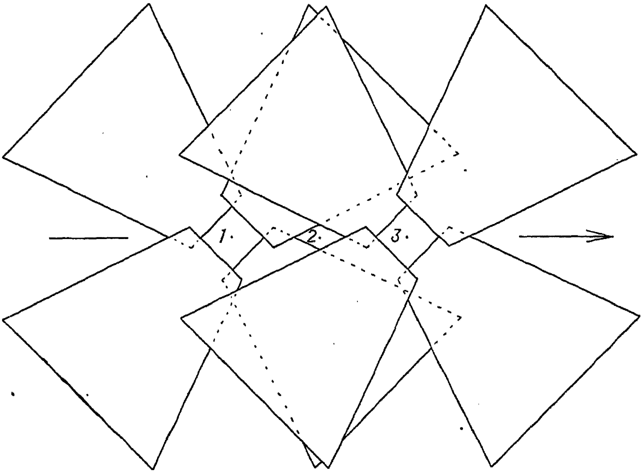

FIG. 94.—Arrangement of five-lens photographs by orienting camera to give satisfactory relationships for use with the stereo-radial plotter. Outlines of the second photograph are omitted.

A variation of this method, which permits two or three photographs to be used simultaneously, consists in setting up the photographs as shown in Fig. 93 and employing pivoted plotting arms of thin metal or plastic material, using a needle or sharp pencil to spot locations at the intersections of the arms. It is a rapid point-by-point plotting procedure and gives excellent results, particularly if three photographs are employed together.

238. Plotting from individual photographs by optical projection.—Unless the projector has provision for tilting both photograph and its image, this method is limited to verticals with but small tilt. Two types of projector for photographs with small tilt are employed. One of these is called a *vertical reflecting projector*, and the image is sent from a photograph placed in a vertical position above by means of a 45° mirror and a lens to a table below. A reflecting projector of this type made by J. G. Saltzman, Inc., New York City, is shown in Fig. 95. In this machine the photograph is placed in the panel at the front of the inclosed chamber, and the mirror behind reflects its image to the lens below, which transmits it to a table where it may be traced. Interior electric lamps illuminate the photograph. An electric blower, also placed inside the chamber, expels excessive heat. Facilities for moving the lens and the entire chamber up or down provide for focusing the image at the scale of the map. It is necessary to work in a moderately darkened room or to screen the table from normal room light. Projectors of this type in considerable numbers have been employed by government mapping agencies.

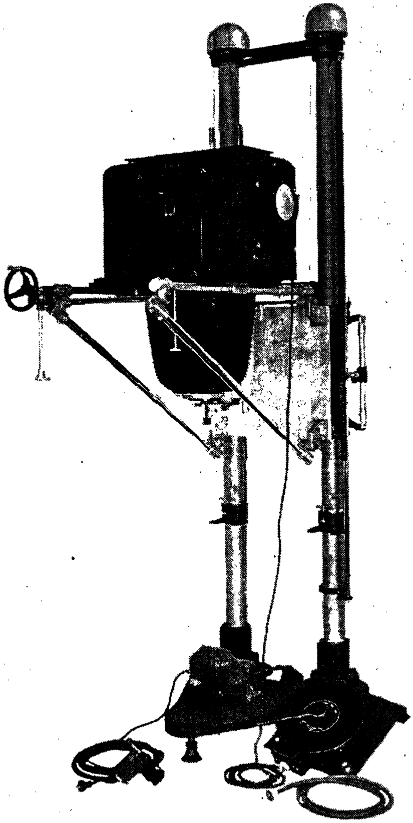

FIG. 95.—Reflecting projector. (*Courtesy of J. G. Saltzman, Inc., New York.*)

The second type of reflecting projector is equipped with two 45° mirrors. It is arranged so that the photograph is placed face down on a glass plate on top of a chamber, and one of the mirrors below sends its image in a horizontal direction to a lens that passes it on to the second mirror, which in turn casts the image upward to a glass-top table where it is caught on a ground surface. Interior lamps and a blower complete the equipment. The entire chamber carrying the lens with it, and the lens independently, may be moved horizontally for focusing and adjustment of scale. This type of projector has the good feature of obviating interference by pencil or fingers in tracing. It requires more floor space than the vertical projector, and the image must likewise be shielded from room light. A projector of this type has been used in an office of the Engineer Department of the Army.

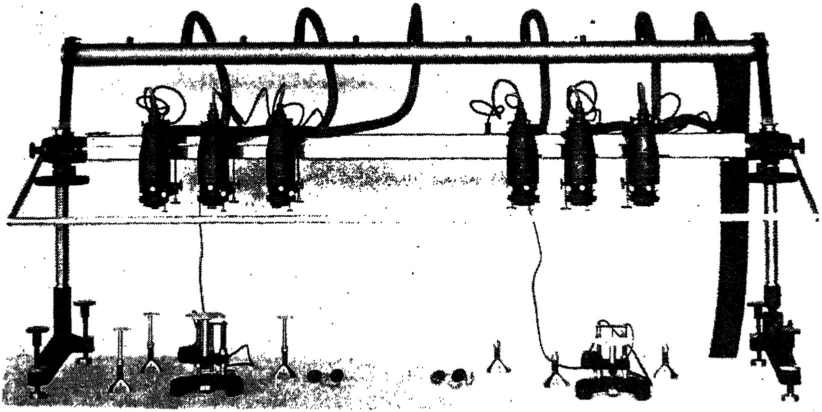

FIG. 96.—Multiplex outfit equipped with cooling unit. (*Courtesy of Bausch and Lomb Optical Company.*)

239. By optical projection of overlapping photographs.—A notable use of this method is the multiplex system in which the images are projected through colored light filters and observation is through colored spectacles to match the complementary colors of the filters. The plotting outfit consists of an adjustable framework with a beam that supports the projectors, a battery of projectors, several control stands, two tracing tables with 50- and 100-millimeter extensions, and spectacles. Figure 96 shows the outfit equipped with a cooling unit as made by the Bausch and Lomb Optical Company of Rochester, N.Y. It is necessary to employ the equipment on a table whose top surface is flat enough to serve as the reference plane for heights. Important accessories are special printers for making diapositives which must conform to the requirements of the projectors, a voltage-control regulator, and a special height gauge.

Other minor accessories are employed in the stages of preparation. Close-up views of a vertical projector, a tracing table with 50-millimeter extension, a 40-millimeter control stand, a height gauge, and a reducing printer are included in Figs. 97 to 101 inclusive.

Operation of the multiplex instruments must be carried out in a darkroom. With proper adjustment of the projectors, projection in colors gives a three-dimension model of a large subject, such as a landscape, which may be seen through the colored spectacles with as much plastic effect as is obtained by a natural view of a near-by object. When the model is adjusted to a datum plane by means of control stations plotted at a given scale, details of the map may be traced from the model by use of the tracing table. It is also possible to trace contours. This is one of the most valuable methods of preparing maps from aerial photographs, and it is being used to a large extent in the United States. The instruments are particularly useful in the study of mapping with aerial photographs.

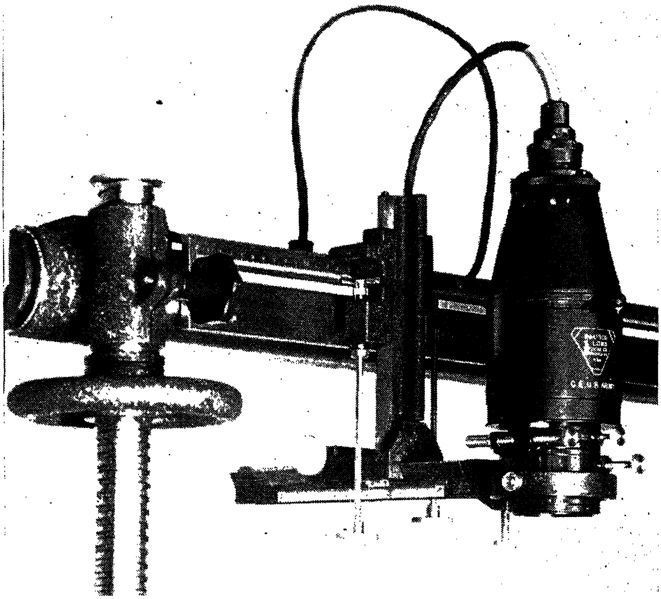

FIG. 97.—Multiplex vertical projector on frame. (*Courtesy of Bausch and Lomb Optical Company.*)

The principles involved in this type of projection are (*a*) pinhole photography, (*b*) placing the projectors in relation to control stations on a map to correspond to the relationship between the aerial camera and the actual control stations on the ground at exposure, (*c*) projection with light of complementary colors, (*d*) viewing with colored spectacles which enable the eyes to see the model. For projecting the images, polarized light may be substituted for colored light, and special spectacles used to see the model.

The projectors (Fig. 97) are equipped to apply six movements: (1) to tilt them parallel to the beam, (2) to tilt them at right angles to the beam, (3) to move them along the beam, (4) to move them laterally at right angles to the beam, (5) to move them up or down, and (6) to rotate them about the optical axis of the lens. Scales are provided for measuring some of the movements, and the height gauge is employed to determine distances above the base table. The movements are sufficient to place each projector in proper relationship to the others and to the datum plane for plotting at scales within certain range limits. Provision is also made for accurate placement of diapositives in the projectors. Illumination is by an electric lamp and a condenser. A slit is provided to receive the colored-light filter. The diapositive holder and the lens placed below are mounted together in a miniature camera and supported by a mechanism that permits movements (1), (2), and (6) to be made. Movements (3), (4), and (5) are made by sliding this unit in the directions stated. Diapositives made with the special printers must all be reduced to the

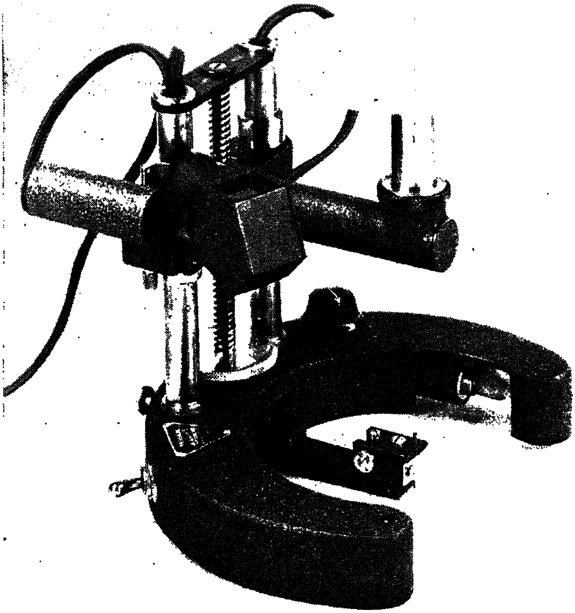

FIG. 98.—Multiplex tracing table with 50-millimeter extension. (*Courtesy of Bausch and Lomb Optical Company.*)

focal distance at which the lens is to be used. In some of the projectors this is about 1¾ inches. The average distance through which the image is projected is about eight times the distance of the diapositive from the lens. The depth of field within which a model may be formed ranges over several inches. The distance from the lens to the table top must therefore be approximately 24 inches to provide for considerable relief in the model and for the unutilizable space between the base and lowest position of the tops of tracing table and control stands. These dimensions do not require excessive size in the frame, and they permit all parts to be reached conveniently. They also permit plottings to be made at moderate scales provided the original negatives from which the diapositives have been made are not at too large a scale.

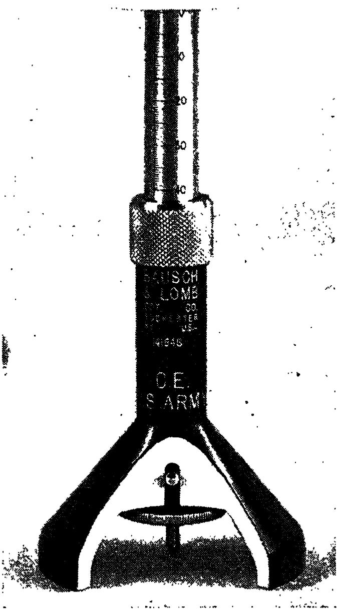

FIG. 99.—Multiplex control stand with 40-millimeter elevation extension. (*Courtesy of Bausch and Lomb Optical Co.*).

The tracing tables (Fig. 98) have a flat disk which may be moved up and down by a screw and a scale to measure the amount of movement. In this method of plotting, although parallax differentials are not read, the stereo effect is employed; and in order to make use of the convenience of wandering marks, a single bright dot at the center of the disk provides for keeping at given levels or for plotting details that vary from one level to another. Therefore to trace a contour the disk is set at the level of that contour; and by keeping the dot in contact with the apparent surface of the ground, while moving it through the model, the contour is followed. A pencil at the base of the table, which is directly beneath the dot, will trace the contour on a map sheet laid on the table. To trace roads and other features that run up- and downhill, it is necessary to keep the dot always in coincidence with the image of the feature in the model. Electric lamps are attached to the tracing table to provide for proper illumination of the wandering dot and to see the results of plotting. Each tracing table is equipped with extension disks to provide for tracing in models ranging from low to high relief.

The control stands (Fig. 99) are employed to set a target (center of the flat disk) at the positions of the control points above the datum plane. Each is provided with a scale, adjustment mechanism for setting the disk at the level of the station that it represents, and a centering point directly under the center of the disk which permits the stand to be placed accurately over a plotted control point. Control stands are made with superstructures (extensions) of two lengths to give the range required for dealing with considerable relief.

The height gauge (Fig. 100) is employed to measure the heights of the projectors above the datum plane. It is ordinarily clamped on the tracing-table disk for this purpose, and a boss on the projector mount is a reference mark with which the top makes contact. It may be used either to set a projector at a desired height or to read the height at which a projector has been set with reference to control data.

The special reducing printer (Fig. 101) is employed to make the diapositives on glass plates for use in the projector. It is equipped with an electric lamp, a reflector, a condenser, a negative holder, a lens, and a plate holder. Each type of printer is designed for the negatives to be used in it, in order that the diapositives to be made may have the proper reduction. Inasmuch as film negatives only are employed, brackets to hold spools of negatives are attached.

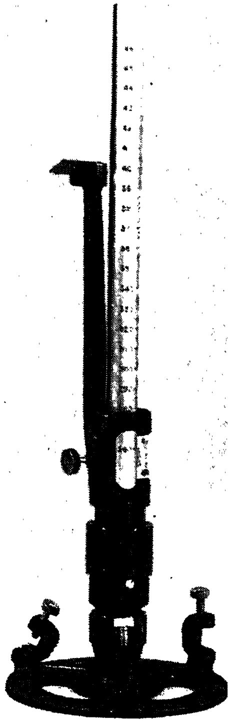

FIG. 100.—Multiplex height gauge. (*Courtesy of Bausch and Lomb Optical Company.*)

Sets of multiplex instruments have been made to employ verticals and others to employ oblique negatives obtained with the T-3A camera. The frames of most of the sets have been made to accommodate nine projectors, but some frames will permit a greater number of projectors to be mounted together. The usual procedure in using the instruments is to set up enough projectors to bridge between control stations and complete the adjustment of all before beginning to plot. To make a setting of a single pair of photographs, three control points (known in both position and elevation) within the area of overlap as a minimum are required, but better results are obtained if more are available. A thoroughly satisfactory arrangement for a single pair is to have six control points, one at or near each plate center and one in the vicinity of each corner of the area of overlap. This gives the maximum advantage for securing accuracy in setting the projectors. Once started with a pair well set, bridging can be carried over considerable spans, but it is advisable to have a few control points scattered through the strip concerned at intervals of three to four models and to close on a group of enough control points to give a thorough check. The amount of control required will depend on the scale of the map to be made, the amount of relief, and the contour interval. It is not practicable to lay down hard-and-fast rules with respect to control.

In organizations employing this equipment extensively it is customary to use some sets with a large number of projectors to carry forward control and to employ sets of three projectors for detail plotting, different personnel manning the sets and working simultaneously after a start has been made.

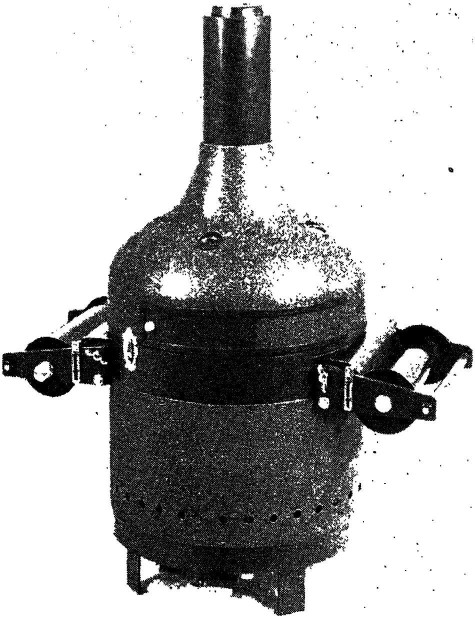

FIG. 101.—Multiplex Type II reducing printer, for making diapositives from negatives having a focal length of 8¼ inches. (*Courtesy of Bausch and Lomb Optical Company.*)

The U.S. Geological Survey, the Tennessee Valley Authority, and the Army engineers are using the multiplex equipment for a considerable part of their mapping.

240. Plotting with simple stereo machines.—With instruments of this type there are no means for correcting the displacements caused by relief or the distortions caused by tilt. They give plottings that are only approximately correct; but with considerable control under conditions of small tilt and slight relief, useful results can be obtained with them as with the tracing-paper method or with pantograph or by optical projection of individual photographs. Being a method in which stereo pairs of photographs are used, and as the instrument has means for making readings of parallax differentials, contours may be traced with it, provided the photographs have first been corrected for tilt or there are enough points known in elevation to make satisfactory adjustments for different levels. If the relief is great, considerable adjustments with respect to contour positions are required. For contouring, the instruments are more satisfactory for mapping at small scales and large contour intervals than for larger scales and medium or small intervals.

Figure 102 shows the Talley-Fairchild stereocomparagraph.[1] This consists of an adjustable mirror stereoscope with micrometer measuring mechanism, glass disks with wandering marks lying in contact with the photographs, a plotting arm, and a standard drafting arm which maintains a constant orientation of the stereoscope when it is moved about. The measuring mechanism is attached directly to the right disk so that any movement of this disk (with its mark) along the axial direction, independently of the instrument as a whole, may be measured with an accuracy of 0.01 millimeter. Provision is also made for shifting this disk at right angles to the axis of the stereoscope. This aids in making parallax settings on tilted photographs or photographs differing slightly in scale. The plotting arm merely records the movements of the entire instrument and is conveniently arranged to plot in a space behind the stereoscope where no interference occurs.

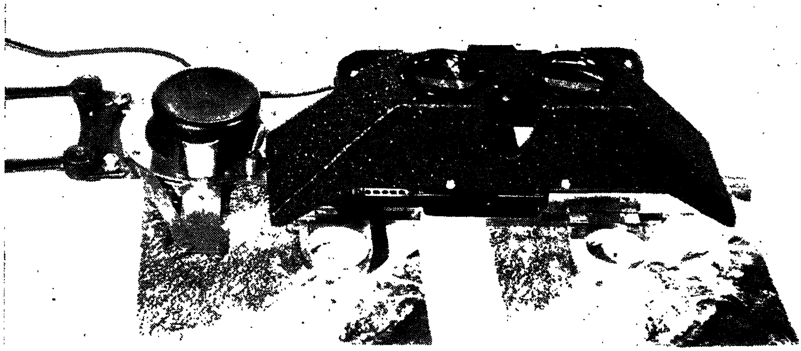

FIG. 102.—Fairchild stereocomparagraph. (*Courtesy of Fairchild Aviation Corporation.*)

Lenses of the oculars give a slight magnification of the image. The small mirrors directly under the lenses are equipped for tilting to make slight adjustments in the lines of sight. The left disk may be set at any one of several positions at 5-millimeter intervals along the stereo base, through a range of 45 millimeters, thereby making it possible to view photographs placed with image points of the same object at any distance apart from about 140 to 185 millimeters. As a consequence, photographs overlapping 60 per cent that range in width up to nearly 10 inches may be employed. Numerous small accessories and a handbook of instructions are furnished with each instrument in a wooden case.

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[1] Designed and patented by Maj. B. B. Talley, Corps of Engineers, United States Army.

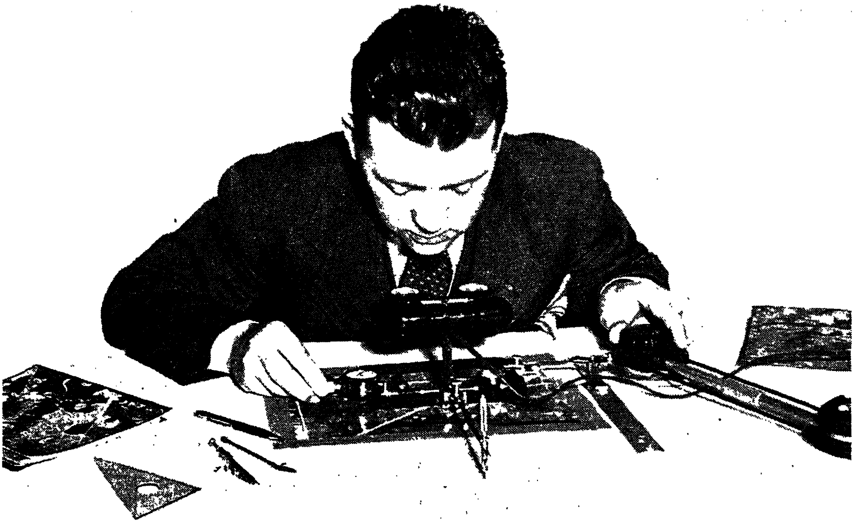

FIG. 103.—Abrams contour finder. (*Courtesy of Abrams Aerial Survey Corporation.*)

As with all stereo-measuring instruments it is necessary to align a pair of photographs by corresponding image points lying near the centers of the photographs. The image points (two on each photograph) are pricked and marked; then the photographs are placed side by side on a flat board (or glass plate) large enough to accommodate all contact points of the instrument and the plotting arm. The photographs are then brought into alignment by a straightedge or by using the wandering marks with the stereoscope held against a straightedge. Once in accurate alignment, the photographs are fastened securely in place with Scotch tape. The sheet on which the plotting is to be done is similarly fastened to the board. Plotting of surface data may now be begun. In order to trace contours or form lines (lines like contours but not necessarily in horizontal planes), either parallax tables or parallax graphs made from readings on control points of known elevation may be employed. If the principal line and the degree of tilt of each photograph are known, spot elevations may be determined (Art. 226), then contouring done about each control point by small areas and joined together by a sketching procedure while viewing the stereo model.

An instrument somewhat similar to the stereocomparograph, called a *contour finder*, made by the Abrams Instrument Company of Lansing, Mich., is shown in Fig. 103. This instrument is a simple stereoscope without mirrors coupled to a measuring mechanism, a plotting arm, and a drafting machine which keeps the orientation constant. Since it has a short stereobase (interpupilary distance), it is necessary to mount each photograph along its outer edges, leaving the overlapped parts free to override each other. When the plotting of part of the common area has been completed with one photograph uppermost, it is flipped up to permit the other photograph to come on top, whereupon the plotting may be completed. The magnification of image obtained is very satisfactory, but the scope of the instrument is somewhat less than that of the stereocomparagraph.

241. Plotting with the aerocartograph.—The aerocartograph is chosen as a representative instrument of a large group of complex and expensive stereoscopic machines used in preparing contoured maps from aerial photographs. It is selected because considerable mapping has been done with it in the United States and because the Army has made important improvements in its design and accessories for the purpose of rendering it more convenient in operation and more adaptable to standard mapping. It is not intended to give a comprehensive explanation of the aerocartograph but only to give a general description and to explain its principles. In order to make proper use of this or any other instrument of its class, such as the Zeiss stereoplanigraph, the Wild autograph, or the Barr and Stroud Thompson plotter, it is necessary to understand the basic principles on which it is constructed and either to receive instruction while using it or to follow elaborate written instructions which could not be included in this book.

Figure 104 shows the aerocartograph as remodeled by the Army for the U.S. Geological Survey. Neither the plotting table, which is separate from the instrument proper, nor the electric lamps, which illuminate the photographic plates, appear in this photograph. The mechanism of the plotting table may be coupled to the instrument at two joints near and back of the left handwheel. The cylinder, around which a supplementary plot may be made, is placed immediately below the oculars. It may be described as a double-direction instrument in which an object is located by sighting both its images on a stereo pair of photographic plates at the same time, this being equivalent to sighting an object from two separate stations simultaneously. Instead of being found at the intersection of two drawn lines, as with an alidade, or by solution of a triangle, as in ordinary triangulation, the position of an object is obtained merely by sighting, for it is plotted mechanically and automatically in accordance with the movements executed. In order to obtain horizontal directions, vertical planes are swung about a vertical axis (plumb line) through the perspective center of each plate (optical center of lens), and vertical directions are turned in these vertical planes from a horizontal reference plane.

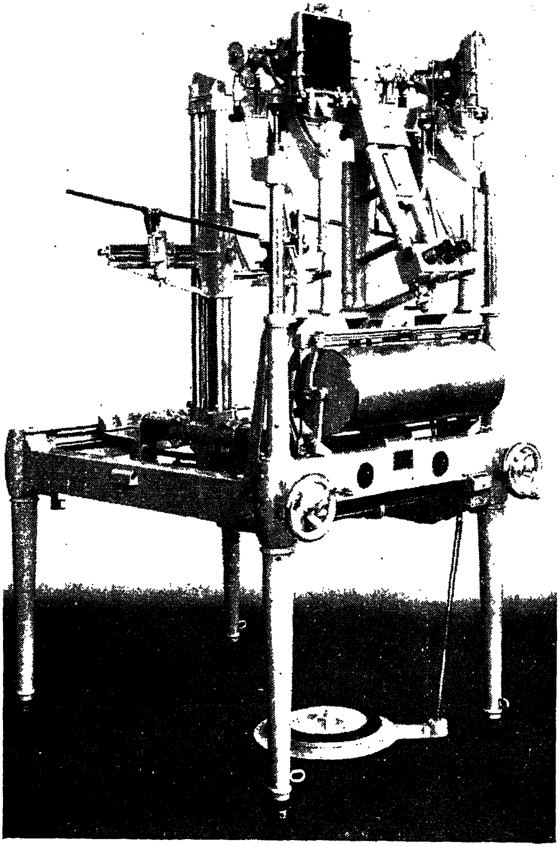

FIG. 104.—The aerocartograph of Hugershoff as remodeled by the Army for the U. S. Geological Survey.

To combine observation, computation, and plotting in one instrument three major mechanisms are joined together in the aerocartograph. These are (*a*) an elaborate stereoscope with telescopes made up of lenses, reflectors, oculars, and collimation mark; (*b*) a mechanism giving linear and rotary movements, which is equipped with and set by scales and verniers; and (*c*) a plotting device with pencil which draws lines to correspond to the apparent movements of the wandering mark. The instrument can be most satisfactorily examined by regarding these mechanisms separately.

In addition to the simple function of stero observation, the stereoscope provides for adjustment of the distance between the oculars to conform to the distance between the eyes of individuals and for focusing each one separately. An Amici prism in each telescope enables the field seen by each eye to be oriented so that there will be "parallelism" between the eye base and the effective stereo base. There are also focusing rings, which permit the images of the fields to be brought to the same scale, and still other rings, which vary the magnification of the oculars from 2 to 4½ times. Near and back of the oculars is placed an optical switch, made of 45° prisms on a carrier, which serves to transfer the image of the right field to the left eye and the left image to the right eye as desired. This has an important use in changing plates to make succeeding setups in a series of photographs, for it permits a third plate to be substituted for one of a pair and to be set with the undisturbed plate to form an extension of the model, without having to use additional control points. It also permits observations to be made on a stereo model seen as it should correctly appear or seen with hills and valleys in reversed (pseudo) appearance. A collimation mark, which takes the place of cross wires, is placed in each telescope not far from the cones. This position assures that errors in the elements of the telescopes will not be carried into the process of locating objects.

The *computing mechanism* properly may be called the *measuring* or *locating* mechanism. It is that part of the instrument by which directions or sights are taken and the relative positions and elevations of objects obtained. It is here called "computing mechanism" because something more than sighting is accomplished, for by its use point or line locations can be made without having to read angles or scales. This mechanism consists of two camera cones (bases) placed at the top of the instrument, each of which has a lens and a focal plane wherein a positive or negative photographic plate may be placed. The focal plane is, in fact, a plate holder and is so constructed that a plate may be inserted and then adjusted accurately into position to bring its principal point into line with the optical axis of the lens and at the proper focal distance, the cone having a focusing ring for the purpose.

For the purpose of setting a pair of plates into their proper relative positions, three independent movements are provided for each cone: (*a*) a rotary movement of the plate in its own plane about the optical axis of the lens, (*b*) a rotary movement of the entire cone about a vertical axis through the optical center of the lens, and (*c*) a rotary movement of the entire cone about a horizontal axis through the optical center of the lens. Screws for making these adjustments are provided at convenient positions. The movements ordinarily required are small. They take care of the varying angles of tilt and different orientations of aerial photographs and departures from strict level conditions in photographs taken on the ground.

The movements of operation will cause the two cones to rotate together about their vertical axes and the prisms in the telescopes to rotate together about their horizontal axis, so that, on completion of a setup of a pair of plates, the entire area of the stereo model, corresponding to the area of overlap of the plates, may be scanned, and any object appearing therein sighted and plotted. The two slender direction rods, which (Fig. 104) are approximately horizontal and extend from front to back, are the connecting links between the measuring and the plotting mechanisms. These rods are connected to the plotting mechanism by sleeves in which they may slide and to the measuring mechanism by hinges and other joints which impart the movements of the sleeve-supporting bracket (carrier) to the cones and the prisms of the telescopes. Through the jointing, all parts of the mechanisms are intimately tied together so that any movement is spread throughout the system.

The mechanism for operating the instrument consists of two handwheels and a foot wheel which appear at the front and on the floor. These are connected to carriers which are made to move in straight lines along tracks or guiding tubes. The left handwheel moves the entire central superstructure from left to right or right to left on tracks that are parallel to the stereo base (line between centers of the lenses of the cones). The right handwheel moves the vertical column of the superstructure along the horizontal tube that extends from front to back of the stand.

When the left handwheel is turned, the direction rods are rotated horizontally, and these cause the cones to swing about the vertical axes. When the right handwheel is turned so as to move the superstructure to the rear, the direction rods are changed so as to intersect at an increasing distance from the stereo base, and a more distant part of the model is sighted. When the foot wheel is turned, the carrier on the vertical column goes up or down, raising or lowering the far ends of the direction rods that revolve the prisms in the ends of the telescopes. The direction rods always remain parallel to the lines of collimation of the telescopes. Therefore, having a pair of plates properly set in the cones, they indicate the position of any object whose images are being sighted. The wheels must be turned simultaneously to keep control of the pointings.

The *plotting mechanism* is based on three axes which are in right-angle positions with respect to one another. These are known as the X, Y, and Z axes, and they correspond to the directions of movement given to the superstructure by the two handwheels and the foot wheel. The left handwheel therefore controls movements along the X axis; the right handwheel, movements along the Y axis; and the foot wheel, movements along the Z axis. Therefore it may be considered that the X axis lies in the horizontal plane that includes the stereo base, that the Y axis lies in that same horizontal plane and at right angles to the stereo base, and that the Z axis stands vertically. Plotting of maps is always done in a horizontal plane, so that image points on photographs taken on the ground are projected from vertical or nearly vertical perspectives orthographically into a plane that contains the X and Y axes. In this case heights are measured along the Z axis. On the other hand, maps made from vertical or low oblique photographs are plotted in a plane that contains the X and Z axes, and heights are measured along the Y axis. In this case, therefore, the X and Z axes are considered to constitute the horizontal plane for plotting and the Y axis to be vertical.

A gear shift is provided to change from plotting in one plane to the other. It relates only to the movements imparted by the right handwheel and the foot wheel; *i.e.*, its function is to switch the connections to the Y and Z axes, so that no matter in which pair of planes the plotting is done or whether photographs taken on the ground or aerial photographs are being used, the foot wheel controls elevations. In either case a contour is traced by setting the foot wheel so that the wandering mark will remain at the level of that contour in the model, and the handwheels are worked to make the mark appear to travel along the surface of the ground. Locking devices are provided to keep the wheels from being moved accidentally during the periods when movement is to be avoided. When plotting roads and other surface features whose points differ in elevation it is necessary to turn all three of the wheels in unison.

The collimation marks in the telescopes constitute the wandering (floating) mark of the instrument by which features and contours are traced in the model. The eyes can tell only whether or not a feature is being plotted properly; the hands and a foot must make the mark "travel" where the eyes want it to go. Therefore, it is only by perfect correlation of hands, eyes, and a foot that plotting can be done with completely correct placement of details. This requires practice.

It is evident from the character of the instrument that although the direction rods do indicate the directions to objects seen through the telescopes, the locations of the objects are not found at actual intersections of the rods. An explanation of the geometry involved in the mechanism that provides for plotting at various scales for various stereo-base lengths with a fixed distance between the centers of lenses (perspective centers) is therefore necessary at this time.

In Fig. 105 (upper part) *A* and *B* represent the centers of the two lenses of the cones placed at the ends of the stereo base *AB*, and they also represent the perspective centers of the photographic plates when properly set in the holders. *AP* represents the left direction rod, and *BP* the right direction rod. In any line, such as *P′P* parallel to the base *AB*, all points form on *AB*, as the common base, a series of triangles that are similar to a corresponding series of triangles on any intermediate line as base, such as *M′N*, if that line is also parallel to *AB*. *P* and *P′* being at the same distance from *AB*, it follows that $MN = M'N'$. If *P* is shifted along the line *PP′*, carrying with it the lines connecting it with *A* and *B*, a pencil placed anywhere in the line *MN* and perpendicular to the sheet will trace a line whose length is equal to the distance traveled by *P* multiplied by d/D.

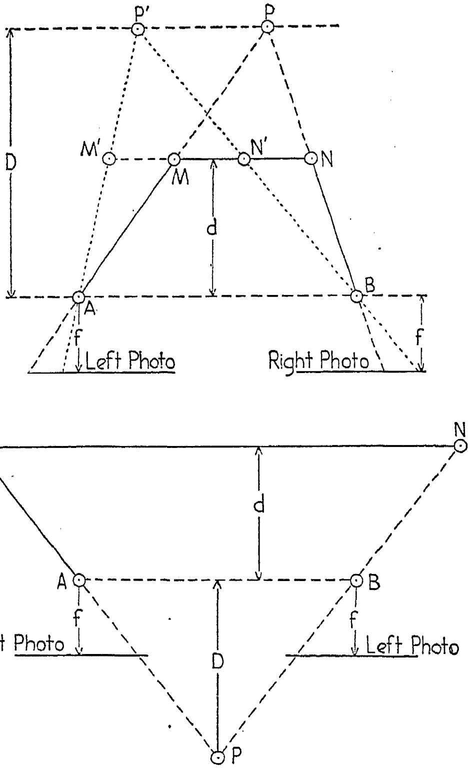

FIG. 105.—Plotting principle of the aerocartograph.

In the aerocartograph, although the perspective centers are fixed at a certain distance *AB* apart, the scale of the plotting can be varied, or kept constant for varying stereo bases, by changing the distance between *M* and *N*. If this distance is shortened, the line *MN* must be moved toward *P* for a particular pointing. In this case the scale of the plotting will be increased because *d* is increased. Movement in the opposite direction is, of course, necessary to decrease the scale of the plotting. Suppose that the length of *MN* has been fixed by setting, and let it be moved in or out or sidewise but always parallel to *AB*. When *MN* is moved away from *AB*, the lines *AP* and *BP* will point toward objects more distant from *AB* than *P*; when it is moved toward *AB*, the lines *AP* and *BP* converge more sharply and hence are directed toward points nearer *AB*.

In order to utilize this principle, the aerocartograph is provided with mechanism, on the carrier to which the direction rods are joined, for setting the distance between *M* and *N* (this is in the direction of the *X* axis) and for varying the distance *d* (this is in the direction of the *Y* axis). Scales, verniers, and clamps for making the settings in these two directions are provided on each side of the carrier. In order to provide for setting along the *Z* axis, *i.e.*, raising or lowering, a scale and a vernier are provided for the sleeve of the left direction rod.

The lower part of Fig. 105 indicates the relationships between *AB*, *MN*, *P*, *d*, and *D* when the right photograph is placed in the left plate holder and the left photograph in the right holder or the optical switch is thrown to give the same effect. This is equivalent to lengthening the distance between *M* and *N* greatly, and then the effect is as though the point *P* were being intersected on the near side of *AB*. The visual stereo effect then obtained, when the photographs are seen in stereo fusion, is the reverse of the normal, *i.e.*, distant objects will appear to be nearer the observer than nearer objects. When viewing vertical aerial photographs in this setup, hills appear as valleys, valleys as hills, and houses like cellars.

If, therefore, a sheet of paper is placed in a horizontal position below the movable superstructure of the aerocartograph (a position corresponding to the plane of Fig. 105) and a pencil, set erectly on the base of the superstructure, is brought into contact with the sheet, it will plot the movements imparted to it by the turning of the wheels. The plot will be an orthogonal projection in either the plane of the *X* and *Y* axes or the plane of the *X* and *Z* axes, depending on the position of the gear shift. If it is in the plane of the *X* and *Y* axes, elevations are obtained from the *Z* scale; if in the *X* and *Z* axes, elevations are given by the *Y* scale.

There are two other places at which plottings can be made with the instrument. These are, on the cylinder below the oculars and on the connecting drawing table. All three places may be used simultaneously or individually, and the plottings may be at the same or different scales.

Greater range of scale is possible on the drawing table than at the other two places.

In order to set up a pair of plates for correct and accurate plotting, it is necessary to obtain the orientation and adjustment relative to a given datum plane by use of control points. The first setup will therefore be of a pair of plates whose overlap includes the required number of control points known in both position and elevation. This requires considerable time and can be done expeditiously only after one has had considerable experience with the instrument. It is essential that first-class stereo fusion be obtained over the entire model. This is obtained by scanning and adjusting from one part to another until the model becomes sharp and distinct throughout all parts. When this is accomplished, the cones will occupy the same positions relative to each other that the aerial camera occupied at the instants of exposure. Having got a satisfactory model for the first setup and having completed the plotting with it, one plate is removed and another inserted in its place, leaving one plate of the original pair undisturbed. A model of an area adjacent to and overlapping the area of the first model is then obtained by adjustment of the new plate only. This requires no additional control.

Mapping with the aerocartograph, therefore, requires a certain number and distribution of control points which must be established by ground survey. The amount of such control depends on the scale of the map, the contour interval, and the character of the terrain. It is customary to carry out plotting from a series of aerial photographs taken consecutively of a strip of terrain and having an end lap of about 60 per cent. In all respects this is the most practical and economical procedure to follow; for it requires the least amount of ground work, and the arrangement of primary control points may follow a certain pattern.

In standard mapping, for each strip of 10 to 15 photographs, there should be in the area of overlap of the first pair four control stations that can be certainly identified on the photographs. Three control points arranged in triangular position at approximately equal distances apart constitute a minimum necessary amount, but a fourth point should be available to check the setup. In the last area of overlap there should be at least three control stations, which may be distributed approximately in a row across the line of flight. A point or two distributed between these two groups will serve well as check points but are not essential. All these control points should be known in position and elevation. Intermediate or secondary control points consisting of three approximately in a row through the middle of each photograph and extending across the strip are obtained from the plate pairs. This provides six secondary points for each stereo model or nine for each photograph as in radial-line plotting. The usual practice is to set up all the plate pairs of a strip and run through them for the purpose of establishing and adjusting the secondary control, then to reset the plates for the detailed plotting. Adjustments are made to eliminate or reduce cumulative errors, and corrections made for curvature of the earth and refraction of light. This is accomplished by a computation process that was adopted after it had been tested on specially controlled strips. The record of the setting of each plate pair is then corrected, and thereafter any pair may be quickly and accurately reset.

The procedure followed to obtain that setting which will give a plot at a definite and desired scale is to make a trial plot of the control points in the model, compare its scale with that of the map to be made, then make the necessary change in the setting of the verniers that control the scale. Elevations read from the instrument are likewise compared with the known elevations of the control points, and adjustments made to reach an agreement.

An important improvement made by the Army consists in having many plate holders so that, once properly set, the plate may remain in the holder to be placed in the instrument again as needed.

242. The Brock process.—The Brock aerial camera is described in Art. 42. To prepare contoured maps, four instruments are employed in utilizing photographs taken with the camera: (*a*) an enlarging projector (Fig. 106), (*b*) a large stereocomparator which is called a *stereometer* (Fig. 107), (*c*) a pair of duplicate correcting projectors (adjustable transforming printers) (Fig. 108), and (*d*) a tracing instrument which is a projector used for equalizing the scale of contours and for tracing them at the final map scale (Fig. 109). Photographic plates of glass are used through the entire process, and these are carefully selected for flatness. The process consists of several separate and distinct steps and is capable of producing high-quality maps of large scale and small contour interval, but it requires the relative or actual elevations of a large number of ground points.

The process is solely for the utilization of vertical aerial photographs. The aerial cameras are equipped with carefully selected, high-quality lenses, which are either about 7 or about 5¼ inches in focal length. The plates of each focal length are enlarged approximately two times to obtain working transparencies, all of which of any one set are brought exactly to the same focal length. The aerial camera is set with the longer dimension of plates in the direction of flight, and photographs are taken to have an overlap of about 55 per cent on parallel lines of flight, the flight lines being spaced to give the photographs a side lap of about 10 per cent. The correcting projectors are equipped with two sets of linkages, so that plates of either the shorter or the longer focal length may be employed in them. The method is unique in that it depends on differences of elevation between points on the ground and does not require the positions of those points to be known, provided there is enough horizontal control to carry out a satisfactory radial-line plot. Lines of spirit levels or vertical-angle (trigonometric leveling) surveys therefore furnish a high percentage of the data required for plate preparation and contouring. As stereo pairs of plates are corrected for the effect of tilt before contouring or elevation determination is begun, simple parallax conditions are attained and basic parallax laws are followed.

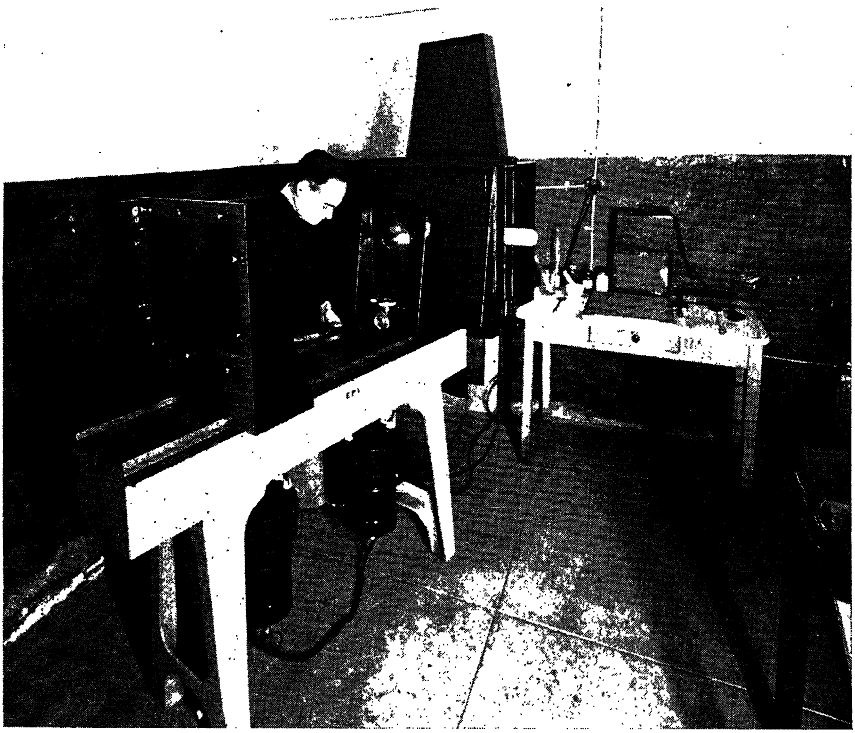

FIG. 106.—View of precise enlarging projector. (*Courtesy of Aero Service Corporation.*)

The first step in the process is to make the enlarged glass positives from the aerial negatives. The positives made from 7-inch negatives all have a focal length of about 13½ inches; those made from 5¼-inch negatives, about 10½ inches. The original negatives are 6½ by 8½ inches in size, so 14 by 17-inch plates are used for the enlargements.

The next step is to make an analysis of the tilts of the positive plates by pairs. This is done by computation based on parallax measurements made of a pair on the stereometer and a determination of the air-station heights. Here use is made of the elevations of the surveyed points. The tilt analysis of a pair is completed on the correcting projectors, and as a result of carrying out the process the projectors are set for making the next prints. Negatives are now made by removing the ground-glass screens and inserting loaded plate holders. After development these transformed negatives are placed in the enlarger for the purpose of making the final plate positives from which the map is actually prepared. In this last printing each plate is brought to the same scale, data for any change of scale having been obtained in the process of tilt analysis. Therefore, before starting to make the map, errors that might have arisen either from tilt or from difference in height of air stations are eliminated as far as practicable. From here on the process is one of great simplicity in which the basic principles of stereo observation and stereo measurement are applied (Chap. IX).

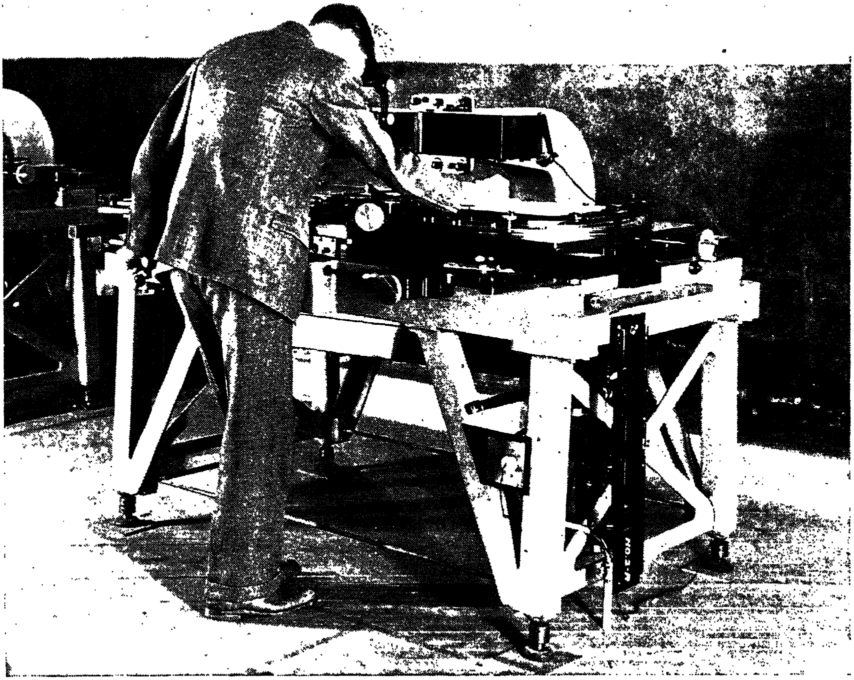

FIG. 107.—Aero Service Corporation stereometer. (*Courtesy of Aero Service Corporation.*)

The method also involves the making of a radial-line plot, and for this purpose sheets of vellum are carefully prepared by point pricking while the plates are in the stereometer. Contours and map details are also drawn on sheets of vellum, and the final map compiled from them.

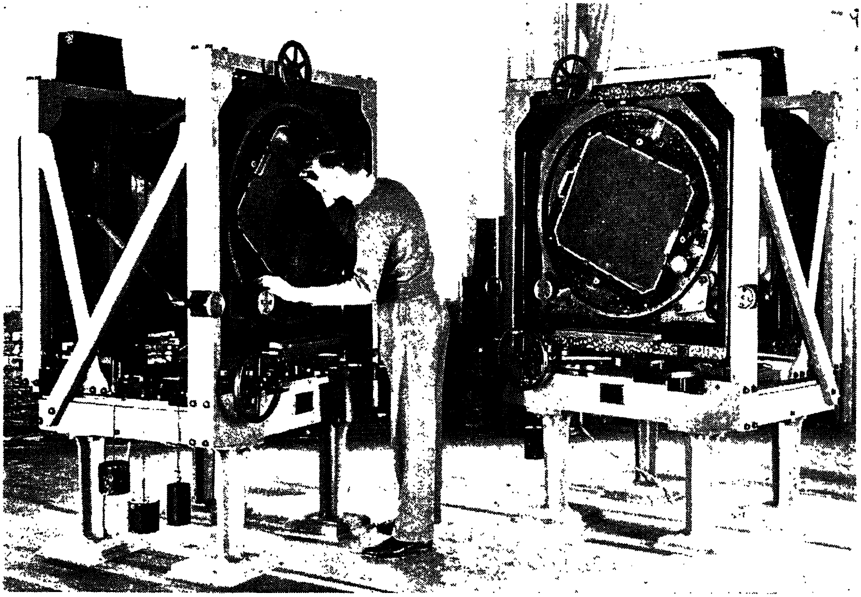

FIG. 108.—Aero Service Corporation duplicate projectors. (*Courtesy of Aero Service Corporation.*)

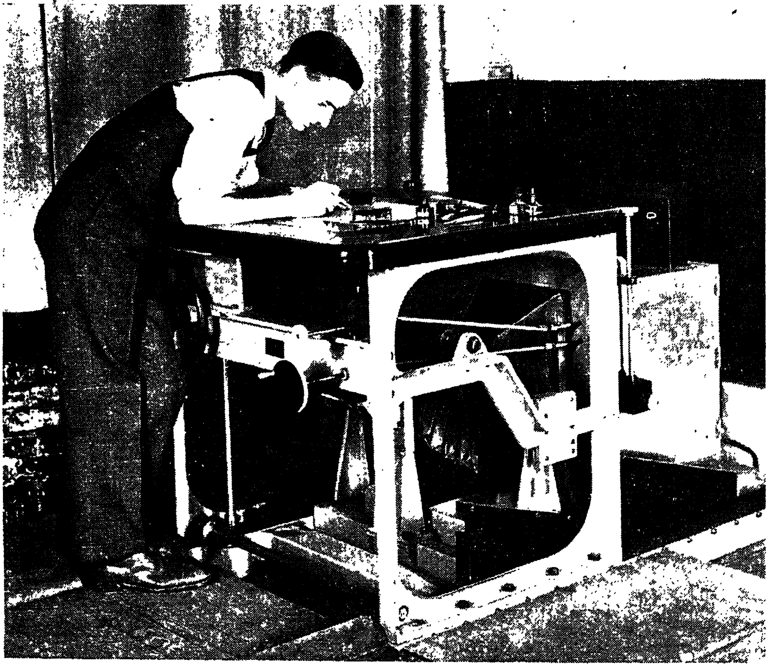

FIG. 109.—Tracing instrument used in the Brock process. (*Courtesy of Aero Service Corporation.*)

243. Details of the method.—The number of control points known in position that must be available is the same as for radial-line plotting and will depend on the scale of the map to be made and the amount of relief. At least six *vertical control* points (bench marks) are required for each pair of plates (stereo model), but it is preferable to have seven arranged as shown in Fig. 110. Three of each model are chosen where they will be in positions for satisfactory use in the next model. Under this arrangement four additional points are brought into use for each successive model of a strip. While the plates are in the stereometer, the principal point, each bench mark, and any horizontal control point within the model are carefully spotted by pricking on the vellum sheet which is to be used later as a template with other like templates to make a radial-line plot. Measurements are also made of the parallax differentials respecting each bench mark. These are recorded and are used to make the preliminary tilt analysis. The two plates are now placed in the correcting projectors and set in accordance with the preliminary tilt determination. Parallax measurements are then made of the image of each plate to check the setting, the screen of the projectors being gridded and special measuring mechanisms and micrometer dials being provided for the purpose. A record of the results is made at this time; if it is found that the correction is not satisfactory, adjustments are made of the settings and new measurements carried out.

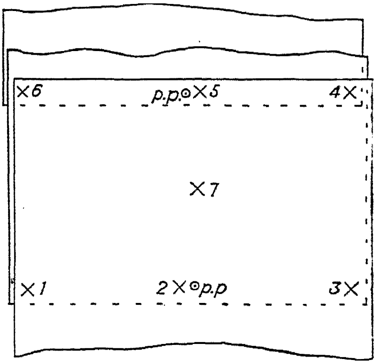

FIG. 110.—Ideal arrangement of bench marks for each stereo pair of plates employed in the Brock process.

This step of the process requires considerable computation and fitting, for it is one of trial and adjustment. The purpose is to correct the plates for tilt and difference of height of the two air stations. This consists in transforming the plate images by projection into those planes which give values of parallax differentials that are in agreement. This does not necessarily mean that the images are transformed into horizontal planes but only that, taking into consideration the elevations of the bench marks, parallax readings are obtained that fit the conditions. Owing to the lack of data as to the exact locations of the bench marks, it is possible that a solution will be obtained in which the image planes are slightly slanting with respect to the horizontal. It is therefore customary to treat each pair of plates independently and to determine a new setting regardless of whether or not a plate has been previously used as one of another pair. Tilt charts prepared especially on transparent sheets and parallax tables aid in making the tilt analysis and setting of the projectors. When the settings to accomplish this are found, the exposures are made. These give the negatives from which final positives are to be made in the enlarging projector.

The corrected positive plates are placed in the stereometer; using parallax calculations or a graph, settings of the wandering mark are made at the contours to be drawn, and one contour after another is determined. This is done point by point on a sheet of vellum placed over one of the plates, the points being connected by lines to form the contours. Surface details are also plotted point by point on the vellum. The plot thus obtained is virtually on the datum plane used for plotting, and hence all contours not in that plane are out of scale. This condition is remedied by use of the tracing instrument, the image of the plot being projected contour by contour to the proper scale in the datum plane, enlarging or reducing as necessary.

The radial-line plot is made of the entire area from the vellum sheets. These are treated as though they were the photographs themselves, but of course only the center points and the control points appear on them in the form of tiny pricked dots. Over an illuminated glass plate, fine radial lines are drawn from each center point in the customary manner. The control points have been placed on the vellum sheets by careful pricking while on the stereometer, so that chance of accidental error or misplacement is very slight. After the plot has been completed, the contoured vellum sheets are oriented over it and the positions of the vertical points are transferred from them to the plot. The distances between vertical points as thus plotted are used to compute the final air-station heights to be used in the process of equalizing the scale of contours. The procedure of tilt analysis, although rather empirical in character, is comparable to the method of parallax differential components as given in Art. 226.

The plate carriers of the stereometer may be moved together parallel to the stereo base (line perpendicular to the lines of collimation of the eyepieces) and at right angles thereto. The left carrier may be moved independently along the stereo base, and each carrier may be rotated in its own plane above a point on the stereo base. The stereoscope proper is fixed, but the plate movements are sufficient for all requirements. The rotary movements of the plate carriers provide for making exact observations and exact placement of the needle pricks to mark the positions of points on the vellum sheets, as well as for accurately orienting the plates together.

The correcting projectors are so arranged that when set at zero (normal position) the plane of the plate holder (back) and the image plane (front) are perpendicular to the optical axis of the lens, and the center mark of the screen grid of the image plane is on the optical axis. The first operation after inserting a plate is to bring its center to the optical axis. This is done by adjustment from the front while observing the image on the screen, using screws arranged conveniently for the purpose. The plate having thus been centered and having been rotated into a position that brings the axis of tilt (as found from the tilt calculation) into a vertical position, the plate and screen planes are turned each through an angle that depends on the degree of tilt. Both planes will be turned the same amount because the transformed image is to be virtually an equivalent vertical (1:1 copy). As they are turned in unison, a mechanism automatically shifts both plate and screen horizontally along the principal lines, so that, when the final proper setting is obtained, the center of distortion and its image point will be on the optical axis. Therefore, when the plate and its image occupy correct relative positions, their common principal plane will be perpendicular to each and include the optical axis of the printer as required for making transformed prints without disturbing their perspective quality.

244. Plotting instruments for obliques.—Three instruments of this class have been constructed and employed in North America: the Wilson photoalidade, the Miller plotting instrument, and the Canadian oblique plotter. All are designed especially for mapping with high obliques, *i.e.*, with photographs taken with the camera axis approximately horizontal or not so greatly depressed as to exclude the sky line of the terrain or the apparent horizon. All are based on the principle of revolving a telescope or a direction arm, or both, about the plumb line that passes through the perspective center (air station) of the photograph and of measuring vertical angles in planes which include that plumb line or of intersecting a horizontal plane with lines of sight. They are intended primarily for mapping at comparatively small scales, so they have greater usefulness in reconnaissance and exploratory mapping than in other classes of survey. The foot of the plumb line must be located by resection from sights taken on the photograph or plottings made from an assumed aerial position and adjusted to map control. As single photographs are quite limited in field, resection locations made from them are not wholly dependable. The instruments have a definite place in small-scale mapping of predominantly flat terrain or of extremely high mountains where relief tends to rob the ordinarily more precise methods of their advantage and great altitude requires specially equipped and expensive airplanes to photograph the areas satisfactorily from above.

245. The Canadian oblique plotter utilizes a parallelogram that couples an observing telescope to a plotting mechanism, whereby point plottings may be made at a desired scale in the map plane. This amounts to plotting an object at a position where the line of collimation of the telescope pierces the plane of the map. If all the objects sighted lie in one plane, it affords a rapid, economical, and satisfactory method of plotting from obliques. Any objects not in that plane either will be displaced or must be dealt with to eliminate such displacements, in which case the speed of plotting is materially reduced. The instrument was intended to replace or supplement the grid method (Art. 248).

It is essential to have the horizon line appear in a photograph in order that it may be properly set in the instrument or else to have enough independently obtained data or enough control points from which orientation data may be deduced. By plotting the same terrain from two different photographs it is possible largely to eliminate the effects of relief, but this is not done as a regular practice, because it greatly increases the cost of mapping by the method.

246. The Miller oblique-plotting instrument.[1]—In this instrument, instead of observing a photograph placed between the sighting element and the plotting sheet, the sighting element is placed between the photograph and the plotting sheet. In order to make sighting satisfactory under this arrangement, use is made of a cone, shaped like a camera, to which a mirror having a tiny center hole in lieu of a lens is attached at the perspective center, and a telescope mounted rigidly at a side to point directly at the hole of the mirror. The photograph (negative) is placed in a holder at the top of the cone, and adjusting devices allow for placing it exactly at the focal distance of the photograph from the aperture of the mirror and for shifting it to obtain proper centering and orientation.

The cone, the mirror, and the telescope are mounted together on a horizontal arm (axis) which is supported by a vertical column and may be shifted up and down for setting at any desired height above a plotting table. A screw is provided for swinging the cone assembly about the horizontal axis which passes through the center of the pinhole of the mirror, and angles of tilt may be read on or laid off by a graduated drum. Scales and manipulating screws complete the equipment required for this part of the instrument.

Stretching over the table is a plotting arm, arranged to swing about an axis that occupies a position in the vertical (plumb) line passing through the pinhole of the mirror. A conveniently placed hand crank is used to swing the arm. A carrier, which holds a pencil and a mechanism for sending out a slender beam of light, may be made to move along the bar by turning another crank. The light beam emerges from a pinhole placed directly over the pencil and is directed at the pinhole of the mirror, being passed through a condenser and regulated in intensity by a rheostat. Distances of the pencil from the center of rotation of the plotting arm may be read on a graduated drum. A device is attached for raising and lowering the pencil.

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[1] MILLER, O. M., "An Optical Device to Aid in Mapping from Photographs," *Journal of the Optical Society of America*, Vol. 25, pp. 185–189, June, 1935.

Figure 111 is a diagram that shows elements of the instrument in relation to a plotting table. *A* is the cone; *B*, the holder for the photographic negative; *C*, the mirror; *D*, the telescope; *E*, the vertical supporting column; *F*, the horizontal axis about which the cone may be rotated; *G*, the plotting arm; *H*, an aperture through which the beam of light is projected; *J*, the tracing pencil; and *K*, the center about which the plotting arm may be swung over the plotting table.

A setup is made by inserting a photograph in the holder and adjusting it into that position which centers it, places it at the proper distance from the mirror, and brings its horizon line parallel to the horizontal (tilting) axis of the instrument. The cone assembly is then adjusted at the height above the plotting table to give the desired scale to the plot, the relationship being as follows: Scale of plot equals the height of the horizontal axis above the source of the light beam divided by the height of the air station above the datum plane of the map to be made. The angle of tilt is now turned off on the drum that records the vertical angle through which the cone is rotated about the horizontal axis. If the plotting is to be done directly on a map sheet along with plotting from other air stations, the plotted position of the air station of the photograph placed in the instrument must be brought to the center of rotation of the plotting arm, and the map sheet correctly oriented on the table by sighting a control point. A centering device is provided for placing the station position at the center of rotation of the plotting arm.

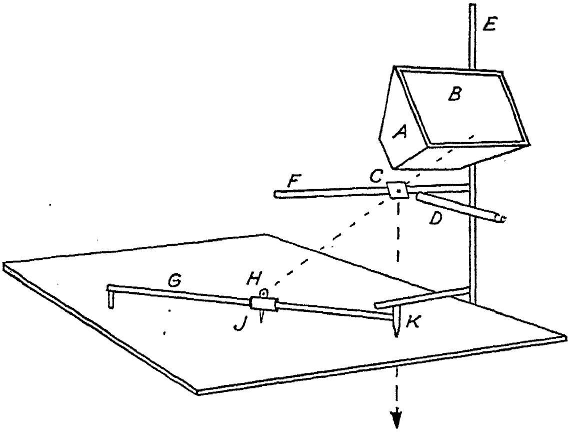

FIG. 111.—Drawing which illustrates the features of the Miller oblique plotting instrument.

Sighting is accomplished in the following manner: The beam of light passes through the hole of the mirror to the photograph where it is caught by the mirror and reflected through the telescope to the eye. Therefore, to plot a point it is necessary only to rotate the plotting arm and move the pencil-and-light carrier so as to make the spot of light coincide with the image of that point (object). If the terrain is flat or any feature (such as a shore line) lies in a horizontal plane, plotting may be carried out by making the spot of light traverse the feature or features to be plotted. The pencil will then automatically draw at the scale of the plot. If, therefore, the terrain is flat, the plotting is a simple and rapid process. If, however, as is the case generally, it is not flat but hilly or mountainous, plotting is carried out by using the instrument to obtain direction lines, locations being obtained by the intersection of two or more such lines from different photographs and different stations. In this case the object is sighted and a line drawn by moving the pencil a short distance toward the center of the plotting arm.

It is also possible to determine the height of an object above the datum plane of the plot. This is done by placing the pencil on the located position of the object, then raising the cone until the spot of light coincides with the image of the object. The amount that the cone must be raised is a measure of the height of the object, applying the scale of the plot.

The field of view obtained is not quite large enough to cover a 9-inch photograph taken with a lens of 10-inch focal length. To overcome this limitation a specially arranged optical linkage is added to the instrument. It is evident that a photograph as commonly printed cannot be employed in a simple construction of this type, because it is necessary to have the relationships among the photographic image, the perspective center, and the plotting sheet the same as those among the negative, the camera lens, and the scene at exposure. Printing through the back of the negatives is one way in which suitable photographs could be obtained, but the use of a print on opaque paper would require interior lighting, which would be troublesome. In fact film negatives are ordinarily employed, and they are placed between plates of glass to hold them flat. Lighting may then be got from a skylight or from a lamp placed above the negative holder.

247. The Wilson photoalidade.—Oblique photographs of lofty mountain ranges usually do not present a horizon line that can be utilized to determine the principal line directly. Although the sky line affords an approximate horizontal line that aids materially in finding the true horizon, it is necessary to have included in the photograph enough control points to make a tentative location of the aerial station and thereafter to find the true horizon by a trial-and-error process based on the elevations of the control points, the vertical angles, and simple computations. The Wilson photoalidade[1] has particular value in finding the true horizon under these conditions as well as for obtaining direction lines to locate objects and for determining their elevations.

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[1] WILSON, R. M., "Oblique Photographs and the Photoalidade," *Photogrammetric Engineering*, Vol. IV, No. 2, April-May-June, 1938.

The instrument is shown in Fig. 112. It consists of a triangular stand with three 9-inch legs on which a photograph frame and a telescope with folding straightedge are mounted. The telescope swings on a vertical axis, as does the telescope of a transit, and it is equipped with a vertical circle and vernier, clamp and slow-motion screws, and a level bubble like the equipment of a telescopic alidade. The limit of near focus of the telescope is such that photographs having a focal length as short as 4 inches may be sighted. The straightedge (ruler) is fastened rigidly to the telescope and is therefore rotated with it to give the direction to any point being sighted. Provision is made for centering the standard of the telescope over a point, the center line of the standard corresponding to the plumb line at the air station. The ruler is carefully adjusted so that its fiducial edge always passes through this center line.

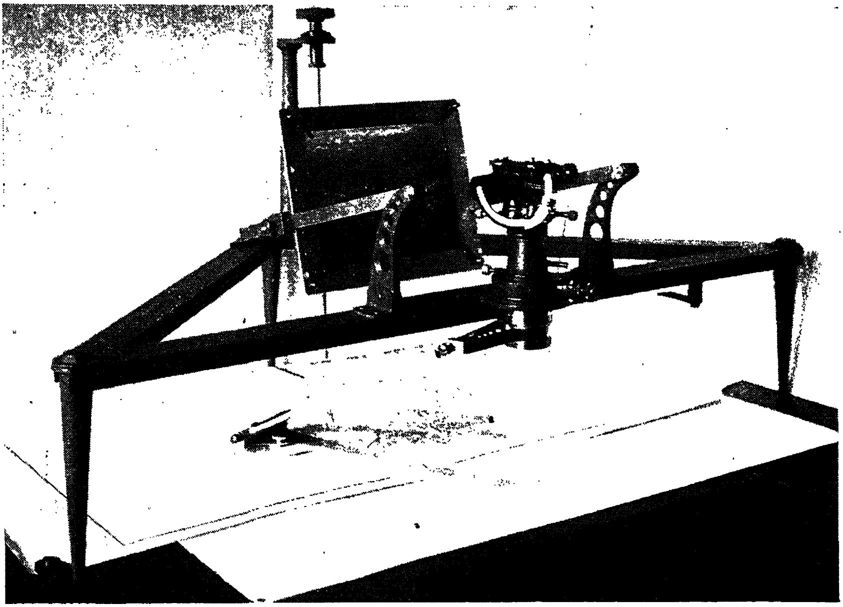

FIG. 112.—Wilson photoalidade. (*Courtesy of R. M. Wilson, U.S. Geological Survey.*)

The photograph frame may be placed at any distance from the vertical axis of the telescope required to employ photographs ranging in focal length from 4 to 14 inches, and it may be tilted to employ photographs whose optical axes are inclined at any angle between 10° above and 50° below the horizontal plane. It accommodates photographs up to 12 inches in width. The instrument is designed for convenient use on a table of average height, the observer sitting in a chair. The legs are long enough to permit ready access to a map laid on the table. The ruler may be folded up and held out of the way above or dropped onto the map as desired.

To make a setup the photograph is inserted in the frame with its horizon line parallel to an edge of the frame, and the frame is brought to the proper focal distance, then tilted until the horizon line is seen through the telescope set level. The ends and a mid-point of the horizon line are sighted to make sure that there is no cant. The map is now shifted under the instrument and centered at the station, then swung in azimuth until the ruler edge cuts through a control point at which the telescope is pointed. Sights may now be taken to any points in view, and short direction lines drawn along the edge of the ruler at their estimated positions. These lines are later crosscut by similar lines from other air stations. Vertical angles read to located points are employed with distances scaled from the map to compute their elevations. After sufficient data have been obtained, contours are sketched as when working with a plane table at stations on the ground. The instrument therefore serves as a telescopic alidade and gives the necessary horizontal direction lines to locate an object and the required vertical angles to determine its elevation. A mean of all determinations made is accepted as the actual elevation of an object.

As a rule the station position (foot of the plumb line through the perspective center of the photograph) will be obtained from the photograph itself. For this purpose sights are first taken at all control points, and lines drawn to them from a center point on a sheet of tracing paper. This is then placed on the map and shifted until each line goes through the plotted position of its control point, as is done in making a station location by the well-known tracing-paper method, and the station position pricked on the map. The map now may be set in proper position under the instrument.

If the horizon line is to be fixed on the photograph by reference to control points, the procedure is to place the photograph as near as possible to its correct position in the frame, make a tentative location of the station position by tracing paper, read approximate vertical angles, make tentative computations of the horizon line, then repeat the process with the better evidence thus deduced. All initial indications of the probable position of the true horizon are employed, and the better the quality of those indications the less laborious will be the work of finding it. Altimeter records assist materially in this process.

Although there is seldom, if ever, complete certainty about the location of an aerial station from a single-lens oblique photograph, errors from that cause are not carried into the map at their full magnitude but generally affect the accuracy of locations and elevations much less. If three or more direction lines pass through a point, it is substantial evidence that the object has been reasonably well located. If there is a triangle or polygon of error, the center will probably be an acceptable location. If the mean of three or more determinations of elevation is accepted as the true elevation of an object, it is probable that the error will not be so great as the error of a single determination, for the highs will be balanced to a certain extent by the lows. The instrument is employed with these considerations in view.

In addition to its orthodox use in mapping, the instrument also serves a special purpose in long-range alignment of prominent objects on a map, *i.e.*, it may be used to advantage to control or check the locations of details over a great extent of terrain. For this purpose it is profitable to take, in addition to those aimed across the area, a few long-range obliques with the camera aimed to photograph the terrain from end to end or in the direction of its long dimension.

248. The Canadian grid process.—This method[1] was developed by the Topographical Survey, Department of the Interior, Canada, and has been employed in making small-scale (1 inch = 4 miles) maps of many thousand square miles of sparsely inhabited and largely unexplored territory, where much of the time the atmospheric conditions are excellent for photography and the relief over the terrain is low. The oblique photographs are generally taken at heights ranging from about 5,000 to 8,000 feet above the general level of the ground, the airplane keeping as close as practicable to the chosen height during the photographic flights. Straight courses are flown, the photographs being taken in triad groups, one looking straight ahead, a second one aimed 45° to the left of the course, and the third aimed 45° to the right. All photographs of a group are taken in quick succession. These groups are timed for exposure at about 2½-mile intervals along the flight course, which is laid along a line provided with ground-survey control either throughout its length or at least near its ends. In another arrangement three cameras are mounted in the ship so that simultaneous exposures may be made. Flights are made along lines about 6 miles apart. At distances of about 30 or 40 miles along the main flight lines, cross flights are flown along lines that are also provided with ground control. All photographs are taken with the optical axis of the camera depressed below the apparent horizon line sufficiently to assure embracing terrain not too distant from the air station but not too low to catch an image of the apparent horizon. The camera is rigged with a sight for this purpose. A careful altimeter record is kept of each flight.

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[1] The grid process is explained in detail in *Topographical Survey Bulletin No.* 62 of the Department of the Interior, Canada, printed at Ottawa in 1932.

The method of plotting consists of employing specially prepared geometric (perspective) grids on glass or celluloid together with a corresponding grid of squares printed on plotting sheets. The perspective grid is placed over a photograph, and topographic features are drawn freehand on the plotting sheet by reference to corresponding lines of the two grids. A final map is prepared by compilation from a series of such completed plotting sheets, all of which are as nearly as practicable at the same scale.

Inasmuch as it is impossible to keep the airplane accurately at the chosen height above the ground and to take all the photographs at the same angle with respect to the horizon and impracticable to construct a grid for each photograph, a series of geometric grids is made for each camera employed. Each series of grids covers a sufficient range in angles of depression at intervals of a few minutes of arc and a range of elevations at intervals of 25 or 50 feet, to provide one to fit fairly closely any photograph obtained.

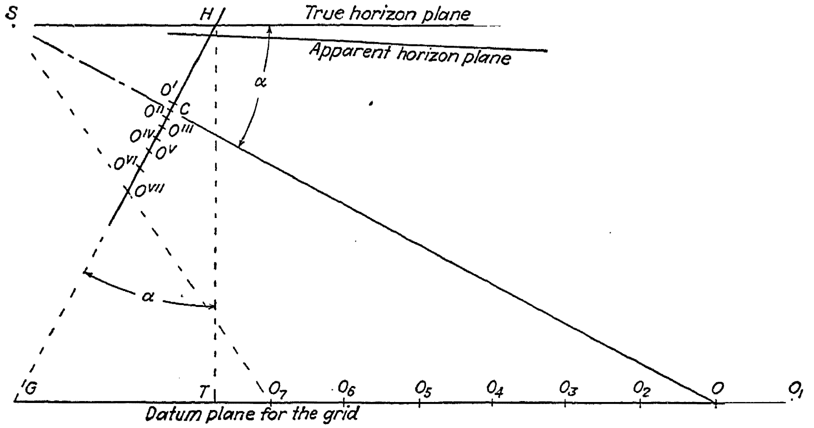

FIG. 113.—Diagram showing the setup of an oblique photograph to determine relationships required to make a perspective grid.

249. Principles of the grids.—The principles involved in making one of the perspective grids to accord with the grid of squares on the plotting sheets are explained with reference to Figs. 113 and 114. In Fig. 113 is shown a section of the principal (vertical) plane, which includes the optical axis *SC* of an oblique photograph, the air station (corresponding to the center of the lens at the instant of exposure and to the perspective center of the photograph), the horizontal plane *SH* passing through the air station (which contains the true horizon line of the photograph), the plane through the air station which is tangent to the apparent horizon, and a horizontal datum plane of reference *GO* at the general level of the ground which is identical with the plane of the plotting sheet gridded with squares. For convenience, *O*, the point where the optical axis pierces the plane of the plotting sheet, is taken as the origin of the grid of squares, *i.e.*, at an intersection of two lines of the grid, and the line *OG* is considered to be coincident with one of the lines of this grid; this is the line by which the grid of squares on the plotting sheet and the perspective grid over the photograph are oriented together. The point *O* of the plotting sheet corresponds to the point *C* at the optical center of the photograph. The height of the air station above the ground will then be represented by *HT*, the perpendicular dropped from *H* to the plane *OG*, and in the figure the length of this line determines the scale of the squares drawn on the plotting sheet.

It is evident that any line in the photograph that is parallel to the horizon line (perpendicular to the principal plane) will have a corresponding line on the plotting sheet that is also parallel to the horizon line. Hence such a line of any length on the plotting sheet will appear on the photograph with a length in proportion to the respective distances of the two lines from the air station *S*. For example, that line through *O*, which is parallel to the horizon line (note that it will also be parallel to the axis of tilt of the photograph) and has a length *L*, will appear on the photograph passing through *C*, and it will have a length equal to $L \times SC/SO$. Similarly, the length of any other line of the plotting sheet that is parallel to the horizon line can be found in the photograph.

If in Fig. 113 the plane of the photograph is extended far enough downward to intersect the plane of the plotting sheet at *G*, the line through *G*, which is parallel to the horizon line, will be at the same scale in the plane of the photograph as in the plane of the plotting sheet, for both are at the same distance from *S*. This is an important line in constructing the perspective grid.

250. Method of constructing a perspective grid.—Referring to Fig. 114, which is drawn in the plane of the photograph, if *HG* is laid off equal to *HG* of Fig. 113 and a line *MGN* is drawn at right angles to *HG*, this line may be divided, starting at *G* and going both ways, by laying off distances *GA*, *GB*, etc., each equal to the side of the squares of the plotting-sheet grid. *GH* is the principal line of the photograph, and *H*, at its intersection with the true horizon line, is the vanishing point where, in accordance with a basic principle of perspectives, all lines of the photograph representing vertical planes parallel to the principal plane come together. If, therefore, the points *M*, *A*, *B*, *N*, etc., are connected with *H*, the lines thus drawn will correspond to one set of lines of the square grid of the plotting sheet.

Referring again to Fig. 113, if from *O* distances OO_1, OO_2, O_2O_3, etc., are laid off in both directions from *O* along the line *OG*, each equal to a side of a square of the plotting-sheet grid at the scale of the figure and lines are drawn to connect the air station with the points O_1, O_2, O_3, etc., these lines will intersect the line *HG* at the proper distances from *C* to correspond to lines of the plotting grid which are parallel to the horizon line. From *C*, in Fig. 114, distances CO', CO'', etc., are laid off along *HG* equal to the corresponding distances in Fig. 113, and at the points thus found lines are drawn parallel to the horizon line.

The extent to which the perspective grid may be carried toward completion without auxiliary construction is limited by the expanse of the construction sheet, so to keep this sheet within a moderate size a line parallel to *MN* may be drawn through any point as O^{VI}, the distance HO^{VI} being equal to one-third or one-fourth of *HG* or any other convenient

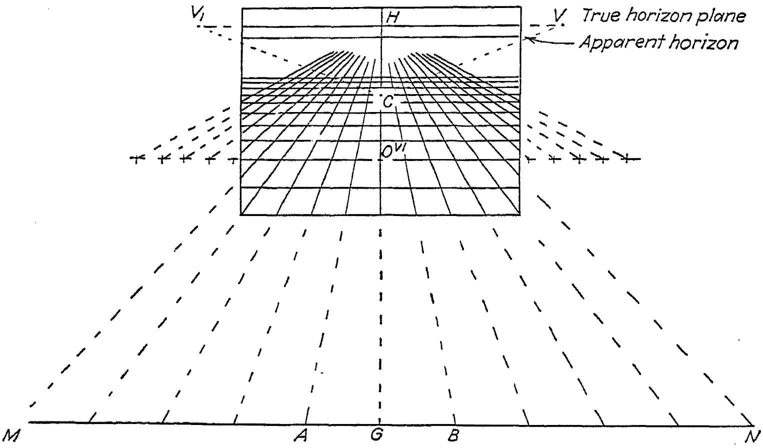

Fig. 114.—Perspective grid as drawn from distances furnished by Fig. 113.

fractional part. Points dividing it into parts one-third or one-fourth the size of those along the line *MN* will furnish the means of completing the grid. Nothing is to be gained by carrying the grid into the upper part of the photograph beyond the point at which images are distinct enough for plotting. It is customary in practice to carry the lines out to about halfway between the center of the photograph and the apparent horizon line.

251. Practical example.—As a practical example in grid construction, suppose the angle of depression α to be 20°, the focal distance of the photograph 8 inches, and the height of the air station to be 5,000 feet. Then *SO* (Fig. 113) will be drawn making an angle of 20° with *SH*. *SC* will be laid off equal to 8 inches, and *HC* drawn at right angles to *SO*. From *H* draw a perpendicular to *SH*, and lay off *HT* equal to 10 inches. At *T* draw *TO* parallel to *SH*. Prolong *HC* to intersect *OT* at *G*. Inasmuch as *HT* was made 10 inches long, representing a height of the air station of 5,000 feet, the scale of the plotting sheet in the horizontal plane of reference will be 1 inch = 500 feet. If it is desired to have the squares of the plotting-sheet grid 500 feet on a side, then the distances OO_1, OO_2, O_2O_3, etc, should be 1 inch long, and likewise the spacings from G along the line MN in Fig. 114 should be 1 inch long.

The length HG may be obtained either by careful graphical construction or by calculation using the equation

$$HG = \frac{HT}{\cos \alpha}$$

Instead of laying off from C (on Fig. 114) the distances CO', CO'', etc., diagonals connecting corners of the squares, including the corner at C, may be drawn, and the points at which these cut the lines MH, AH, BH, NH, etc., furnish the points for drawing those lines of the perspective grid which are parallel to the horizon line. It is necessary only to lay off HV and HV_1 equal to the focal length of the photograph divided by the cosine of the angle of depression. (In the case cited above, this would be $SC/\cos 20°$.) V and V_1 are the vanishing points for all horizontal lines that cut across the photograph at 45° to the horizon line, so any corner of the grid already located may be joined to V or V_1 to furnish at the intersections with grid lines already drawn the positions of additional corners.

The true horizon is found from the apparent horizon by use of equation (28) (Art. 217), the height of the air station being taken from the altimeter record.

252. Mapping with the grids.—If there is available a series of perspective grids to cover the focal lengths of the photographs, all ranges of altitude, and all ranges of angles of depression encountered in a lot of oblique photographs covering an area to be mapped, the plotting can be taken up and completed expeditiously. It should be understood that although the plotting sheets all have the same size grid of squares (printed on with a single printing plate), there will be considerable variation in orientation and scale among the map details taken from the several photographs. Rough orientation of the plotting sheets could be accomplished by matching details common to adjacent sheets, but to obtain the most satisfactory orientation it is best to pick out certain distinct objects lying in the area of overlap and to plot these very carefully for orientation purposes.

Because of the unavoidable slight differences in scale in the map details drawn on the plotting sheets it is customary first to run through a skeleton plot of control stations established by ground survey together with the objects picked for orientation purposes, make any adjustments found necessary to bring the plotting sheets all to a common scale, then to go through a second time for the actual plotting of details. Substitution of other perspective grids to take the place of those first chosen will often give better matching of scale. The plotting having been completed at the chosen scale, the final map may be drawn by tracing the details of the plotting sheets and transferring them to the map sheet or by trimming the plotting sheets and joining them together on a board.

253. Apparent horizon line is necessary.—It is essential to have the apparent horizon line registered in each photograph. Unless the atmosphere is particularly clean and free of moisture, the apparent horizon line will be neither well defined nor continuous across a photograph. When the haze layer terminates abruptly at the upper level, a false apparent horizon line is liable to be registered instead of the earth horizon. However, under favorable atmospheric conditions a horizon line of the ground or of haze will usually be well enough defined to afford the direction line of tilt with sufficient accuracy for the purpose of mapping by the method. Any necessary correction to obtain the true horizon from whatever horizon is given in the photographs can usually be found by trial on ground-control stations. Film sensitive to infrared rays give better images of the horizon than can be obtained with panchromatic film.

254. Curvature of earth and refraction.—The surface of the terrain is here being dealt with as though it were a plane and that therefore the horizon line merges with that plane at an infinite distance away. This, of course, is not a true assumption, and the errors caused by curvature of the earth and refraction of light in plotting objects shown in the photograph will increase in proportion to their distances from the air station. At a distance of 10 miles the combined effect of curvature and refraction will be the same as though the object lay about 57 feet below that plane; at 5 miles the effect will be equivalent to a drop of about 14 feet. Plottings should therefore be kept within the range of a few miles of the air station. Directions are not so greatly affected by curvature and refraction and may be satisfactorily obtained for objects many miles away, particularly if the images of these objects fall near the principal line of the photograph.

255. Control required.—Control established by carefully run traverses and leveling is satisfactory for mapping by the method. It is best to arrange the flights for photography along such traverse lines, so that the photographs taken directly ahead will include images of the traverse stations near their middle (principal) line. Whenever it is not practicable to have a flight course follow a traverse line, it is necessary to have two or more control stations registered in some of the photographs, especially those falling near the beginning and the end of a flight. The control stations are commonly laid out on the ground to be between one and two miles apart, and the stations should appear well in the foreground of a photograph. If the control stations are not near the same level, a correction for the difference in elevation is made.

The images of any two prominent objects lying in suitable positions within the zone of overlap between two photographs may be employed for orientation of one photograph with another and for maintenance of a common scale for the plotting sheets. The objects should be chosen with regard for their elevation as well as for distinctness of character. For a set of photographs taken on a single flight not along a traverse line, three sets of control distances will usually be employed, one set to join the photographs taken directly ahead and two sets for joining each triad group of photographs together. It is good practice also to range in several objects over as great distances along a flight course as the clarity of the photographs will permit. These ranged-in objects assist materially in maintaining orientation through a set of photographs taken on any course.

256. Comparison with radial-line method.—This method belongs in the limited class of those having to do with individual photographs in so far as plotting of details is concerned. The overlap between photographs serves the purpose of comparing independent plottings from separate photographs. On the other hand, the radial-line method depends on having photographs taken with more than a 50 per cent overlap and making locations by intersection from two or more photographs. The grid method does not provide for making allowances in general for the elevation of objects without considerable additional labor. In the radial-line method of plotting, the effects of relief are almost entirely eliminated. The grid method is undoubtedly cheaper than the radial-line method. It affords a considerably greater area of plotting per unit of photograph surface, and there is little difficulty in obtaining neat and sufficient overlaps over long flight courses. The grid method is very limited in its application, both in types of country and in types of maps. It can be satisfactorily employed only where the terrain has low relief, as in comparatively flat valleys and on plateaus. It is suitable only for preparing small-scale maps, such as those which have the scale 1 inch = 4 miles. The grid method is not accurate enough for use in preparing the standard topographic maps being made in the United States at the scales 1:20,000, 1:24,000 and 1:48,000, but it has value in making exploratory surveys of extensive valleys and plateau lands.

CHAPTER XIII

MOSAIC MAPS AND AERIAL PHOTOGRAPHS AS ILLUSTRATIONS

257. When well made, a *mosaic* (photographic) map of urban areas or of cultivated sections of country serves as a valuable substitute for the regular topographic map. It can be made in a fraction of the time required for, and at a fraction of the cost of, the standard map; and for many purposes it furnishes all needed information. If the terrain is not too hilly and sufficient control is available, it is possible by careful work to make a mosaic that has good scaling quality and regular tone and is free from conspicuous joints. High quality requires expert laboratory work and a full understanding of control and its application to mosaics. It is almost impossible to make completely satisfactory mosaics of rough terrain, although single strips of photographs may be joined together with success in matching details provided the air work has been well done. Flying at a constant height and keeping the camera level at exposure are especially important in aerial photography for mosaics. It is also highly important to secure even tone in printing and to obtain prints that furnish detail in the high lights as well as in the darker areas. There should be sufficient contrast to afford easy reading of details but not enough to sacrifice clarity in some parts. As for other mapping use, the photographs should be taken with 60 per cent end lap, and the side lap between strips should be generous rather than meager. If the terrain is hilly or mountainous, improvement in the mosaic to be made will be obtained by increasing both end lap and side lap. Photographs taken when the sun is high are much to be preferred to photographs taken when the shadows are long. Negatives taken in late spring and summer afford better appearing prints than can be obtained from negatives taken after the leaves have fallen, although more details may appear on the latter. By proper printing, however, satisfactory mosaics may be made from either class of negative. Mosaics are made either by simply bringing together photographs to match as well as possible without employing control or by use of horizontal control to obtain the best possible placement of each photograph with respect to the others, the aim being to secure accuracy rather than perfect match of details but to obtain both as far as practicable. That quality which makes a mosaic suitable to be copied photographically and to be reproduced is also highly desirable.

Proficiency in making controlled mosaics can be attained only by practice in the laboratory, from experience in the use of control, and by acquiring skill in preparing and laying the prints.

258. Treatment of prints.—Prints for mosaics are made on single-weight paper, it being desirable to have them as thin as practicable to avoid the appearance of layers given by thicker paper. Paper with glossy surface is usually employed, for it gives better quality for copying than matte-surface paper. The photographs are laid down as shingles are laid, partly overlapping one another. To prepare them to give a neat appearance with joints as much subdued as possible, they are trimmed by tearing in a manner to feather the edge and to leave the kept part unmarred by the tearing. This requires that the part to be discarded be pulled away from the face of the photograph; *i.e.*, if the photograph is held in one hand face down, the part to be torn off should be pulled with a slight twist upward. This leaves a beveled edge at the back ¼ inch or more in width so that when laid down the photograph will have neither an edge to cast a shadow nor any of the white paper showing. Judgment must be used in selecting the line along which to trim a photograph. It should be selected to obtain as good a match in tone as practicable and so that details will aid in concealment. It gives a better laying edge to trim along a straight or curved line. To get such an edge a razor blade or a thin knife is used to cut through the emulsion without penetrating into the paper, then tearing off the part to be discarded. Tearing without first cutting through the emulsion gives a ragged edge which blends well with the photograph below for concealment but is delicate to handle. Both methods are employed. After the trimming has been done, the photograph is turned face down on a plate of glass or other flat surface and the bevel scraped and smoothed with fine sandpaper. It is good practice to trim and bevel that part of the photograph which is to be covered as well as the part that will be exposed, for this avoids the risk of having objectionable ridges and low places which otherwise are liable to be apparent in the finished mosaic.

259. Preparations for laying a mosaic.—Whether the mosaic is to be laid with control or without control, a careful examination should be made of all the photographs laid out together on a table large enough for the purpose. The table top should be of soft wood to permit the photographs to be pinned down, or else a sheet of moderately soft composition board should be laid on the table and the photographs pinned to it. This examination is to be made to find out how well the photographs match in tone and scale and to discover if any are unsuitable and should be replaced with substitutes. When the overlap is 60 per cent or more, it will not be necessary to use all of them, so there is an opportunity to make substitutions. Where no suitable substitution is available, a badly tilted photograph may have to be corrected by printing in a rectifying projector. This is a time-consuming and objectionable step to have to take, so every effort should be made to avoid it. Often by using lesser parts of all or nearly all the photographs, difficult situations can be surmounted satisfactorily. It will be possible to determine fairly quickly if a satisfactory mosaic can be made from contact prints or if the photographs are not satisfactory for straight use and must be discarded for a retake or corrected by projection printing. It is difficult to match the tone of contact prints by an occasional print made by projection. It is good practice to use all contact prints or all projection prints for a mosaic.

The principles of alignment employed in radial-line plotting (Arts. 233 and 234) should be adhered to as far as practicable in the laying of mosaics; *i.e.*, strips of photographs should be oriented together by center-to-center lines, regardless of the matching of details along the edges, for only by observing this principle is it possible to obtain an assembly that will be reasonably free of warping. The photographs should have the center lines drawn on them with ink that may be readily wiped off. The photographs of each strip should be laid by their center lines; then the strips are brought together for comparison and adjustment. Short strips of Scotch tape or paper clips may be used to hold the photographs together yet permit each strip to be shifted as a unit. Ample study of the temporary assembly should be made to obtain all required information before the photographs are taken apart and trimmed. Temporary removable ink marks may be placed at numerous places to aid in putting them together again for the final assembly. Pin marks also may be used to replace a photograph in the same position if the board used for the temporary layout is used for the final assembly. Reassembling on the same board makes for economy in time and is good practice. Pin marks in the board are easier to find if a sheet of carbon paper is laid against the board before sticking the pin.

260. The final assembly should be laid on a sheet of composition board such as "Masonite Prestwood" or similar board of homogeneous texture, which will permit a pin to be stuck or driven in and will hold it fast. A board that is too dense or too hard to admit a pin is not satisfactory. The board should be between $3/16$ and $5/16$ inch thick, depending on its length and breadth. If a wider board than can be obtained in a single piece is required, two boards may be joined together abutting. Special precautions must be taken in moving a mosaic on joined boards. If the preliminary layout was made on the board to be used for the final assembly and rubber cement is used to fasten the photographs down, pins may be run through the previously made pinholes of the photographs and each brought to its position by placing the pins in the corresponding holes of the board and letting the photograph drop into place. Pinholes made by small pins (size of No. 10 needle) will be closed and obliterated when the photograph is finally laid. Rubber cement or similar substance has no effect on the dimensions of the photographs. These cements are also satisfactory to use because any smears on top of the photographs may be taken off readily by rubbing gently. However, they do not hold the photographs so well as a paste made of gum arabic. Gum arabic paste causes paper to expand, so the pin method of placing the photographs in position cannot be followed when it is used. It then becomes necessary to lay the photographs again by alignment and with the aid of ink lines drawn on the photographs for the purpose. Any such lines drawn on the board, except those for the first photograph, have to be used by offsetting, and it is necessary to determine the amount of enlargement that the solution has caused. Ordinarily it will be about 1 per cent in both dimensions of a photograph. The guide lines on the photographs must be placed where they will not be covered at final laying, and those on the board should extend beyond the area to be covered. Any gum arabic paste squeezed from under a print may be wiped off with damp absorbent cotton. Edges of prints should be pressed down and rubbed carefully until adhesion is complete. After gum arabic cement has set, a mosaic should be thoroughly cleaned with damp cotton.

If it is desired to preserve an original mosaic for use over a long period of time, it is best to lay it with gum arabic. If the mosaic is to be copied within a short time and reproduced, it is advantageous to employ rubber cement, for by its use time can be saved and a more accurate placement of the photographs obtained. If no lettering is to be drawn on the photographs, they may be cemented only at the covered side, the uncovered part being left free for lifting up. A glass plate placed over the mosaic will press it flat for copying. However, this method of tacking down is not satisfactory if lettering or grid lines are to be drawn on the mosaic. If allowed to dry for a few minutes, both surfaces having been smeared, rubber cement holds fast and does not permit adjustment once the two surfaces have made contact. After practice, two persons working, one to hold the photograph in position over the board, the other to adjust the pins, may successfully place photographs in their proper positions, but it is a delicate task. It is safer to place a photograph as soon as the cement has been applied and to weight it down until the cement has dried. Care must be taken to prevent the weights from sticking to the photographs by cement squeezed out, or else the surface may be damaged. The weights should not be heavy enough to squeeze out much of the cement.

261. Controlled mosaics require enough horizontal control points to place each photograph in that position which locates its details as accurately as practicable with the details of the other photographs. As a rule, two or three control points should be available for each photograph. These may be primary points obtained by ground survey or secondary points obtained from a radial-line plot. If contact prints are to be employed, it is necessary to determine the average scale of the photographs. If the photographs have not been well taken, it will be impossible to make a satisfactory layout. In this case either the area should be rephotographed or the prints must be made by projection to bring them to a common scale and to correct the badly tilted ones by rectification.

Contact prints are laid out in strips as explained in Art. 259, and measurements must be made to determine the average scale while then joined together. The average scale of the photographs of each strip is determined from the ratios of distances between control points as measured on a map sheet and on the strip of photographs. For this purpose it is well to employ a distance extending nearly from end to end of the strip and two other distances extending from a mid-point to each end point. The three ratios thus obtained are recorded; and after all the strips have been measured, the several ratios are compared. If they do not vary more than 1 or 2 per cent, it is practicable to make from the contact prints a satisfactory mosaic at the average scale; if the ratios differ more than 2 per cent, it will be necessary to determine scale ratios and reprint by projection, bringing each print to the chosen scale. The ratios of scale between negatives and prints may be determined individually or by strips. In order to obtain satisfactory results, individual determinations require that there be available for each photograph four or more control points, and consideration should be given to their elevations. This is a tedius process and is not usually followed except in special cases of limited and important areas. The usual procedure is to establish a ratio for the photographs of each strip or for several photographs of a strip and print with the projector set for each group. The settings of the projector are calculated as explained in Art. 3.

Excessive tilt requires special treatment, knowledge of the approximate position of the principal line, and a projector equipped for tilting both negative holder and paper holder (easel). Usually there will not be enough information to make an accurately transformed print, so a setting is made to give a print that fits in satisfactorily with those it is to join. Figure 115 shows a projector of this type which is used by the Aero Service Corporation for making bromide prints to be used in mosaics. It is often possible to avoid correcting for tilt by making a few prints from the same negative at different scales and employing small pieces of these instead of a print in one piece, fitting in the parts to fill the spaces as well as possible. In this way an inadmissibly bad joint may be reduced to a few inconspicuous joints. In doing this, care must be taken not to disturb the positions of correctly placed adjacent photographs.

Laying with rubber cement is much to be preferred to laying with gum arabic solution when making controlled mosaics. Control points may be plotted directly on the mounting board at the scale chosen, and the photographs laid by them without having to cope with change in dimensions. If a wet cement is to be used, every precaution should be taken to have the control points plotted at the scale that the photographs will have after application of the cement. Experience with the effect of the cement in changing dimensions is necessary for making an accurate allowance.

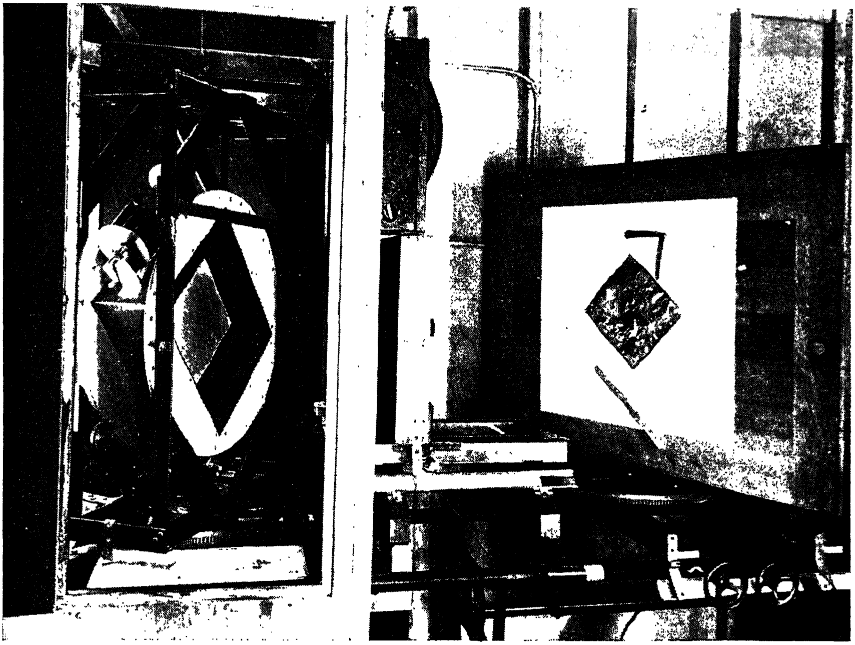

FIG. 115.—Projector used for printing photographs for mosaics. (*Courtesy of Aero Service Corporation.*)

There is no hard-and-fast rule about sequence in laying the photographs. In some cases it may be desirable to lay an entire strip down and build on it; in others it may be better to start at one end and lay photographs abreast of one another across, then build on them. In any case a decision should be made about sequence while the photographs are all laid out together for the preliminary examination and scale determinations so the markings may be placed to avoid covering before the time comes to use them. Before pinpricks are made to be used as guide points for laying, each photograph that the pin goes through should be in that position relative to the others (over or below) which it is to take in the final assembly. Each photograph should be placed and oriented exactly as determined in the preliminary layout. Endeavors to obtain a perfect match of details locally should not be allowed to upset the general adjustment scheme.

262. Finishing a mosaic for copying.—It is better practice to paste a border band over the photographs than to trim the mosaic. This is a simple task if the shape is rectangular. When it is irregular, considerable judgment must be applied to obtain a pleasing and satisfactory effect. Good-quality white paper should be used, and the band should be 2 inches or more wide to give room for marginal notes. Near-white or colored paper will not be satisfactory for copying. Any names, grid lines, or other notation to be reproduced as a feature of a mosaic should be neatly inked on the original in a manner to appear distinctly on all copies of it and so as to obscure no important detail. The only colors available for inking are black and white or colors that register photographically as light or dark. When there is opportunity for choice, white is used over the dark areas and black over areas light in tone. On some, grid lines are drawn with black ink; on others, with white ink. Any mosaic to be copied and used by anyone not already acquainted with the terrain covered should have an appropriate title showing the locality; a graphical scale of miles, feet, or yards as appropriate; date of issue; and source. If the mosaic covers an entire quadrangle, it should have marginal figures showing latitude and longitude or grid numbers depending on the grid drawn. The source, responsibility, and credits should be indicated by appropriate notation. A diagram showing true north, magnetic north, and grid north should appear on all mosaics that are to be used instead of standard maps. When space is available inside border lines, the title and miscellaneous notes may be placed therein as customary for topographic maps. In fact, the finishing of a controlled mosaic should be carried out to conform as closely as practicable to good practice in finishing standard maps. Special care should be given to style of letters and to their appearance. As a rule gothic or block letters will be easier to read than letters in more elaborate style. Civil boundary lines, names, and other data not given by the photographs should be obtained by field work and placed with the same degree of care as for standard maps. Mosaics made under contract should conform precisely to the specifications of the agency that is to pay for them. As a rule, mosaics of lighter than average tone will give better service than those of darker tone. Mosaics and photographs necessarily lose quality by halftone reproduction. Therefore every effort should be made to obtain photographs of highest quality for use in making mosaics.

263. Uses of mosaics.—The largest present use of mosaics in the United States is as substitutes for topographic maps or for use in conjunction with standard maps. For this purpose they are being made at

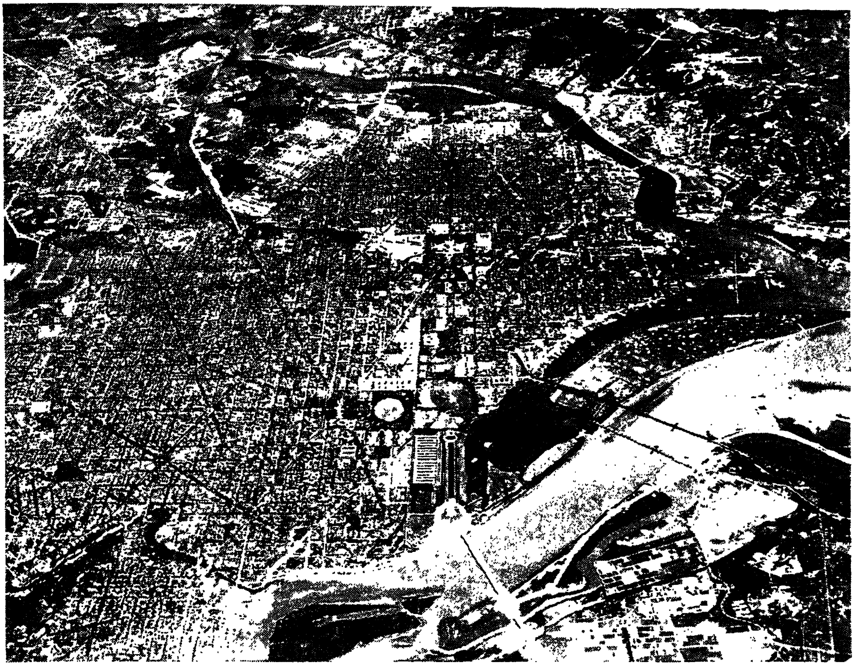

Fig. 116.—Small-scale oblique of Washington, D.C

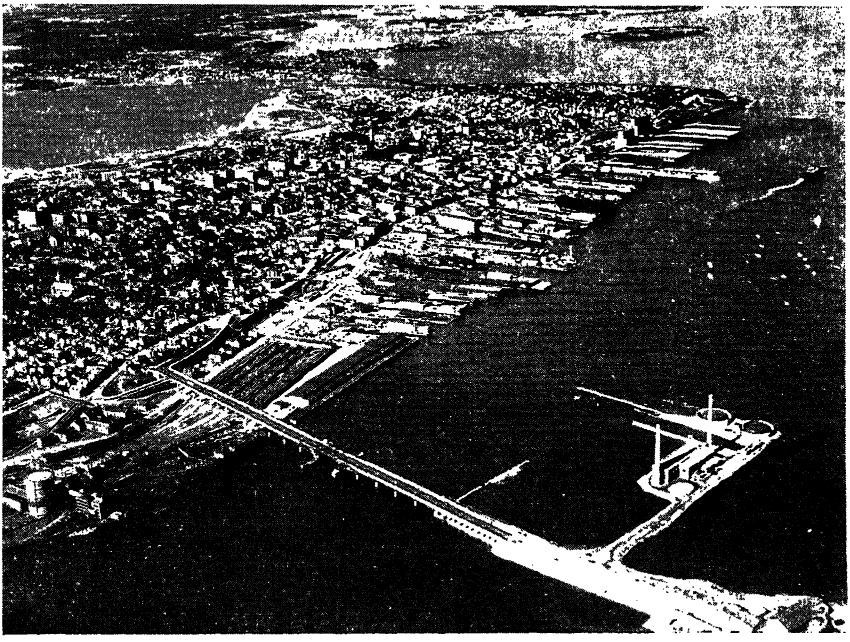

Fig. 117.—Portland, Me., harbor. (*Photograph by Institute of Geographical Exploration.*)

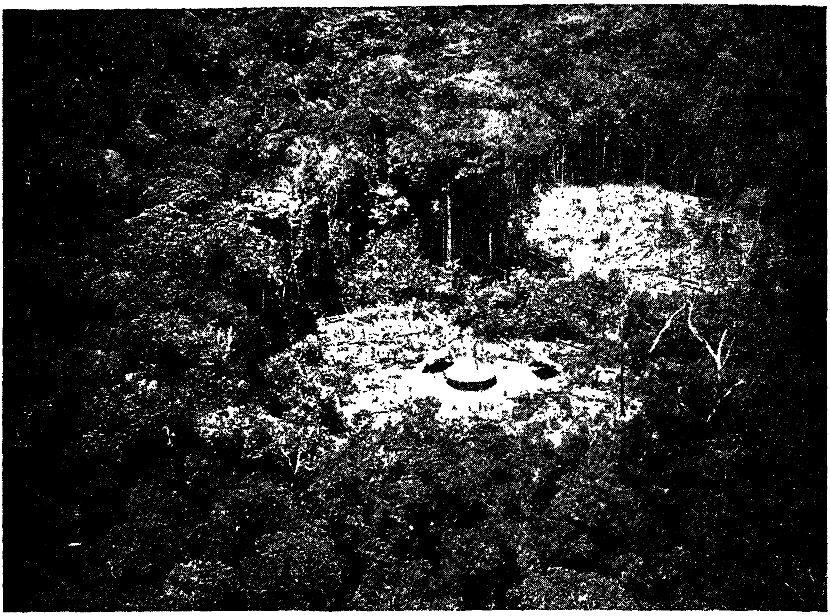

FIG. 118.—Round house of a Maku tribesman at Tokixima-huaite, Rio Parima, State of Amazonas, Brazil; located in latitude 3°45′ N. and longitude 63°45′ W. (*Photograph by Hamilton Rice Expedition,* 1924–1925.)

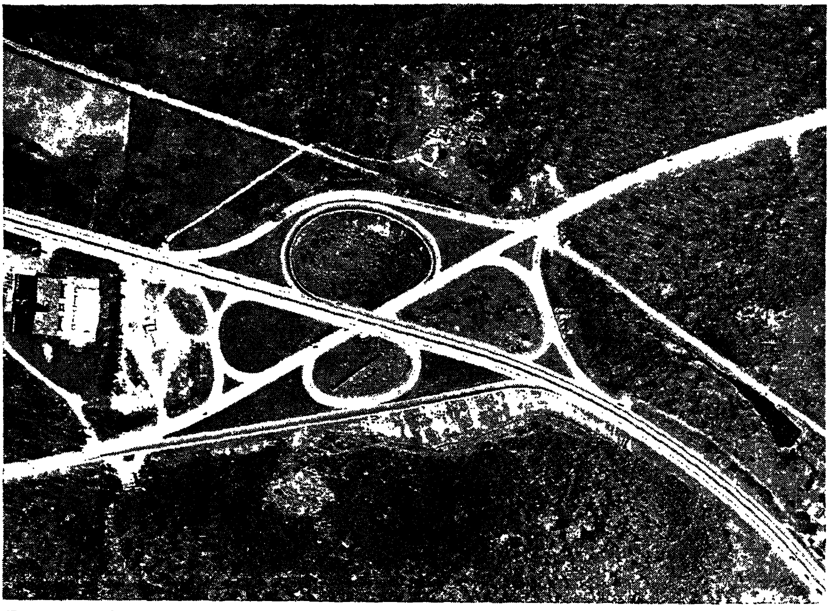

FIG. 119.—Junction of two important highways near Worcester, Mass. (*Photograph by Institute of Geographical Exploration.*)

standard map scales—1:20,000, 1:31,680, and in some instances 1:10,000—and they conform to 7½-minute quadrangles or embrace areas 10,000 by 15,000 yards or have other appropriate limits. These are being made in large part for the Army under contract and are being reproduced at the Engineer Reproduction Plant and by private lithographic printing concerns. Mosaics are also being made and used by other public agencies and by private concerns engaged in field investigations and land-utilization surveys. For many years industrial organizations have employed mosaics and aerial photographs individually for surveys and studies pertaining to extension of their activities. Notable uses are in road, railroad, pipe-line, transmission-line, and telephone-line surveys. They have a valuable use in connection with water-power development; timber-land surveys; and hydrographic, harbor-line, and harbor-improvement surveys and for studies of proper utilization of port, terminal, waterway, and other transportation facilities. An example of effective use of mosaics and oblique aerial photographs as illustrations is found in the Port Series of Reports issued jointly by the Corps of Engineers of the United States Army and the U.S. Maritime Commission.

264. Aerial photographs as illustrations.—The interest in aerial photographs as illustrations has spread to all phases of human life. The daily newspapers, national weekly magazines, monthly periodicals, scientific bulletins, and special reports all carry reproductions of aerial photographs as a staple product of illustration. The oblique is more commonly used than the vertical, but the latter, too, is becoming familiar to school children all over the country. There are included here a high oblique of Washington, D.C. (Fig. 116), and an oblique of Portland, Me., harbor (Fig. 117), both of which take in extensive adjacent areas. In contrast to these is the Amazon tribesman house in the great tropical jungle of Brazil (Fig. 118) and a close-up vertical of a clover-leaf junction of two important highways near Worcester, Mass. (Fig. 119). These are examples of the value of aerial photographs in a popular sense as well as in engineering and explorations and for purposes of study. The value of aerial photographs, particularly obliques of regions of great mountains, in making physiographic maps, such as the splendid ones prepared by Dr. Raisz[1] at the Institute of Geographical Exploration, should not be overlooked.

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[1] Raisz, Erwin, "General Cartography," McGraw-Hill Book Company, Inc., New York, 1938.

CHAPTER XIV

METHODS OF OBTAINING HORIZONTAL AND VERTICAL CONTROL BY GROUND SURVEYS

265. Control established by ground surveying is essential to aerial mapping. It is therefore necessary to understand how it may be obtained and the degree of its reliability as obtained by various methods. Success in making use of aerial photographs in mapping depends on having available a sufficient number of stations of proper quality in the area concerned and on their distribution. In order to appraise quality of control it is necessary to become acquainted with the instruments used in ground surveying and to understand how they may be employed to best advantage. Although it is not necessary for the individual engaging in aerial mapping to become proficient in ground surveying, he must know where to look for existing data, how surveys to obtain satisfactory control are conducted, and how to make effective use of it when obtained.

266. Standards of accuracy for control surveys.—A few years ago the Federal Board of Surveys and Maps established the following standards of accuracy for horizontal and vertical control surveys which have been generally adopted in the United States:

Class	Average triangle closure in triangulation	Allowable error of closure, feet	
	Horizontal	Horizontal	Vertical (leveling)
First order	1 sec.	1 in 25,000	0.017 multiplied by the square root of the length of the line in miles
Second order	3 sec.	1 in 10,000	0.035 multiplied by the square root of the length of the line in miles
Third order	5 sec.	1 in 5,000	0.05 multiplied by the square root of the length of the line in miles

The accuracy of third-order horizontal survey is about 1 foot per mile. For an area 20 miles long or for stations 20 miles apart, the allowable corresponding error is 20 feet. This means that the cumulative error in traverses may not be greater than 20 feet for a line 20 miles long. If the traverses are joined to stations of first- or second-order accuracy, the error in any station of a line after adjustment will probably be less than 1 part in 5,000.

267. Accuracy required for aerial photographs.—Third-order accuracy is adequate for mapping with aerial photographs at all scales from 1:10,000 downward, because the error at any station cannot be detected even when the test measurements are made with the greatest possible refinement. It is possible to plot distances that span $\frac{1}{200}$ inch, on a sheet of map paper, and discrepancies of that amount are practically at the limit of plotting with pencil and straightedge. At the scale 1:10,000, 0.005 inch corresponds to a distance of about 4 feet. Vertical photographs are usually at a scale not less than the scale of the map being made with them. For making a map at the scale 1:10,000 a single-lens vertical would seldom span more than 3 miles; a multilens vertical, not more than 6 miles. For the single-lens vertical no two stations of third-order accuracy would be displaced with respect to each other on the map more than 3 feet; for the multilens vertical no two would be in error more than 6 feet. Third order of accuracy, therefore, stands at just about the allowable limit for mapping at the 1:10,000 scale.

For mapping at the scale 1:20,000, smaller scale photographs are ordinarily employed. They cover much greater areas than do those required for 1:10,000 scale plotting. As a consequence, although errors as great as 8 feet can scarcely be detected, the greater distances over which plotting is done from an individual photograph are sufficient to counterbalance the lack of refinement; hence third-order accuracy is necessary in this case also.

For mapping at all scales larger than 1:10,000, horizontal control should be of higher accuracy than third order. If a few second- or first-order stations are available in an area to be so mapped, transit traverses run with great care between them will produce results of sufficient accuracy after adjustment. It is difficult to run transit traverses of higher than third-order accuracy, but a line run with a highly trained party along a favorable route, started on a first- or a second-order station and closed on a second station of the same quality, will produce stations accurate enough for even the most exacting requirements likely to be encountered.

For mapping at scales such as 1:40,000, 1:48,000, and 1 inch to the mile, third-order accuracy is necessary only to assure that all localities will be correctly shown in position to the limit of scaling and practical usage. In mapping at these smaller scales, it may not be practicable always to obtain third-order accuracy in control stations. Terrain difficulties, lack of suitable routes, or lack of funds or time may make it necessary to accept a lower grade. In these cases every reasonable endeavor should be made to attain the accepted standard and the best possible results under the conditions obtained. Local accuracy of 1 part in 2,000 should be the lowest acceptable for mapping at these scales.

For general or reconnaissance mapping of large areas at quite small scales where the cost must be kept low, horizontal and vertical control cannot be provided in quantity and quality as for standard mapping. In regions suitable for triangulation and trigonometric leveling, adequate control may be provided by establishing one or more astronomic stations (latitude and longitude), laying out a base and expanding from it with a transit. Once a distance of several miles between two stations has been well determined, further expansion may be accomplished with the plane table and the telescopic alidade. The most favorable layout to provide control for the aerial photographs should be carefully considered. This should include both distribution and means of identification of stations or location of objects near stations that can be identified in the photographs. In timbered valleys without roads, control is exceedingly difficult to establish. Usually the best that can be done in such regions is to establish control on each side of the valley and extend secondary control across it by careful radial-line plotting with the aerial photographs. Every possible use of the photographs themselves should be made to span uncontrolled areas, keeping the errors within plotting limits.

In reconnaissance mapping, careful consideration should be given to the method to be employed to obtain control and a plan made to establish the control and the taking of the aerial photographs to produce the best results at the least cost in the time available. Traverses should be supplemented by or based on triangulation stations wherever practicable. The ideal horizontal control is obtained from transit traverses tied into a sufficient number of triangulation stations distributed about 10 to 12 miles apart.

In some types of country where roads are scarce or lacking altogether, but bare summits are plentiful, only triangulation is practicable. In flat, wooded country well provided with roads, control must be established either by traverses or by triangulation with high towers, and the method by traverses not only is less expensive but ordinarily affords a more effective density and distribution of stations.

The elevations of all permanent bench marks established in an area should be of third-order accuracy or higher whenever practicable. Elevations of points used locally for control should as a rule be within 1 foot of their true elevations. This is not always practicable, so under certain conditions tolerances must be widened. In general, accuracy in points used locally should be suitable for the method to be employed to obtain contours. To have the elevations of such local control points true within one-tenth of the contour interval will provide sufficient accuracy for all except the most exacting purposes.

268. Triangulation under favorable conditions gives more reliable results than traverses, but it is usually much more expensive per station, and stations usually require special targets for identification with certainty on aerial photographs. The cost of triangulation per station varies greatly and depends on the character of the terrain, density of roads, and number of stations to be occupied on a single project or in one season. Instruments of good quality are required to measure the angles. Except for extensive work over large areas or for special mapping of exceptional refinement, third-order surveys are as high in class as needed, provided stations of higher order are near enough to the area to start from and tie to. Less time and effort are required to train a party for triangulation than for traverses. Control of accuracy is concentrated in one man—the observer—and checks are obtained on closure of each triangle.

The smallest party for efficient operation consists of the chief, who makes the observations; a recorder; a signalman for each station to be observed when signal lamps or heliotropes are employed; a truck driver; and enough laborers to clear obstructions and erect towers. Following is a list of instruments and other articles required for measurements at stations:

One micrometer theodolite or transit theodolite which can be read to 1 second
One case for instrument with shoulder straps for transporting to stations.
Two pairs of field glasses
One protractor
One rule graduated to inches and tenths
One 50-foot steel tape
Two electric hand lamps
One 6-foot steel tape
Three to six signal lamps depending on the number of stations to be sighted from any station
One plumb bob and long line or vertical collimator
One sun umbrella
One wind screen of canvas
Notebooks for recording angles and station data
Forms for computing geodetic distances
Notebook with blank leaves for making field computations and plotting figures of stations
"American Ephemeris and Nautical Almanac" or "General Land Office Ephemeris of the Sun and Polaris" for the year
Seven-place logarithmetic tables
A good watch or chronometer
Climbing irons, if country to be worked over is wooded

Miscellaneous tools and equipment required for auxiliary work and repairs are: station markers (bronze disks or plugs), cement in cans, axes, hatchets, saw, nails, tacks, signal cloth, wire, stone drills, drill hammer, posthole digger, wire cutter, brace and bits; for minor repairs of instruments, small pliers, screw drivers, small files (flat and round), spool of soft copper wire, bottle of instrument oil, bottle of liquid shellac, cocoon of spider web, vial of plaster of Paris, and a bit of beeswax.

For expeditions going beyond the ready reach of articles required for a survey, thorough preparations should be made before departure and full attention given to details to see that no essential article is omitted. The equipment should include everything needed to carry on in spite of wear and tear and breakages often unavoidable in long journeys and in the execution of surveys in frontier country.

269. General procedure in a triangulation survey.—In every case the best practicable layout of figures should be adopted. Field conditions will usually prevent obtaining ideal arrangement of stations, but full study of possible locations will result in the best under the circumstances. As a rule it is more expedient to select stations at moderate distances apart rather than at great or short distances. In regions of good visibility, stations may be placed 20 to 30 miles apart from one another without entailing excessive handicaps to observations and working of signals. In regions of low visibility, greater speed in the work will result if stations are kept within 5 to 12 miles of one another. As far as practicable, stations should be carefully selected by reconnaissances, and necessary targets and towers placed before starting observations. Quadrilaterals in which both diagonals are observed or central-point figures with from four to seven sides are most desirable. Avoid angles less than 30° or greater than 120°. Numerous objects presenting good targets for observation which can be readily identified should be sighted as secondary stations from primary stations, being observed from three or more occupied stations.

Best results are obtained with signal lamps at night; but in remote regions signal lamps are not so practical to maintain with the necessary batteries as in more accessible regions, and they are expensive to operate. The heliotrope or common targets of cloth or painted rods will often have to be employed exclusively. For daytime work, late afternoon or early morning often give the best atmospheric conditions. It is necessary, however, to avoid observations in the sectors toward the sun unless heliotropes are used as signals.

A small-scale plot should be kept by the observer as the work progresses. This enables him to keep complete track of stations to be observed and the approximate angle to turn off to each. It also serves as a record of angles read to check closures. Frequent azimuth determinations should be made on Polaris at or near elongation. There should certainly be not less than one determination for each 200 to 300 square miles of area to be mapped.

Particular attention should be given to the keeping of the record in the notebooks. Entries should be made in ink or with a pencil of medium hardness. Erasures should never be made; but if corrections are necessary, the first entry should have a line drawn through it and the second (correct) entry made alongside with explanation if appropriate. Eccentric stations should be avoided as far as practicable; but when necessary, reductions to center should be made immediately after completion of observations so that closure errors may be determined. Field computations of distances should be made and of coordinates also if required in carrying out the mapping work.

FORM OF TRIANGULATION RECORD—DIRECTION METHOD

Station occupied, Rex
Date: May 16, 1937 (Left page) (Right page)

Stations sighted	Micrometer *A*	Micrometer *B*	Mean	Angle	Summaries of angles (both direct and reversed)	Remarks
Telescope direct	° ′ div.	° ′ div.	° ′ div.	° ′ ″	Dome-pinnacle ° ′ ″	Time, 5 P.M. Eccentric point occupied, distant from station mark, 5.61 ft.
Dome.........	61 42 07	241 42 9	61 42 08		41 45 13	
Pinnacle.......	103 27 18	283 27 24	103 27 21	41 45 13	18	
Table..........	154 33 12	334 33 18	154 33 15	51 5 54	15	
Station mark...	236 56 16	56 56 20	236 56 18	82 23 03	11	
Dome.........	61 42 06	241 42 12	61 42 09	267 8 54	10	
Telescope reversed					Pinnacle-table 51 5 54	
					56	
					52	
					50	
					53	
					Table-dome 267 8 54	
					58	
					52	
					56	
					50	

NOTE: Record is continued in this manner through five readings round the horizon with instrument direct and five readings with instrument reversed.

The repeating method of measuring each angle separately may be employed if preferred.

Eccentric positions should be occupied only when station itself cannot be occupied. Reduction should be computed immediately after completion of reading of angles to afford data on triangle closures.

For third-order work with an instrument of proper quality and in thorough adjustment, five sets of readings direct and five reversed, each set made on a part of the horizontal circle not used for the other sets, will ordinarily be enough to gain the desired accuracy in angle readings.

There should be enough, however, to assure closures within the allowable limit stated in Art. 266. It is important that the telescope wire be brought on to the target by movement in one direction always—the direction against the slow-motion spring.

A sample triangulation record is shown in the table on page 267.

270. Transit traverses.[1]—A great percentage of the horizontal control in the United States has been established by running transit traverses. It is the most economical and most satisfactory method to obtain horizontal control when roads and railroads are available for use as routes. The party must be well trained, and each member must exercise caution at all times to avoid accidental errors. Important advantages in favor of transit traverses compared with triangulation are as follows: (*a*) Stations can be placed where they may be readily reached for later use; (*b*) the stations may be used as bench marks in lines of levels; (*c*) it is practicable to locate many road crossings and other points that appear distinctly in aerial photographs; (*d*) stations may be distributed to good advantage.

The party should consist of an instrumentman (chief of party), a recorder, two tape men, two rodmen, and a truck driver. A truck or a station wagon large enough to transport the party, instruments, and materials is required. In sparsely settled country special equipment for camping is required, and usually a cook must be added to the party. The instruments required are

Transit graduated to 30 seconds (preferably to 10 or 20 seconds) having stadia wires
300-foot steel tape graduated to feet through its entire length
100-foot steel tape
Two red and white transit rods (range poles)
One stadia rod
Stakes
Two plumb bobs
Eleven tally pins
Two tape holders
One spring balance
One thermometer
Two electric hand lamps
Polaris and sun table
Carpenter's level or small spirit level

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[1] The articles on transit traverses herein are based on the practice of the U.S. Geological Survey. To those responsible for carrying out surveys by transit traverses, *Bulletin* 788-C, "Transit Traverse," issued by the U.S. Geological Survey, is recommended.

Hatchet
Eight-penny nails
Tacks
Notebooks
Keel (or crayon) red and blue

Additional equipment useful in the work is listed below:

Two or three hand recorders
One tape-repair outfit
Tape clips for making temporary repairs
Book bag
Canteens

271. Permanent stations.—Every permanent station should be set in advance of the actual survey, and all of them should be occupied as regular stations of the traverse line. The usual form of permanent marker is a bronze disk, with twisted stem, bearing the name of the survey, a centering point, and space for the number of the station. A disk may be set in place with cement in a concrete post or in rock by drilling a hole large enough to receive the stem. Copper, brass, or monel-metal plugs may be used; or a drill hole may be left as the mark, if disks are not available. Permanent stations should be placed at intervals of 3 to 4 miles along the line. For setting permanent marks at stations, the following articles are required:

Standard disks (or metal plugs)
Rock drills, hatchet, hammer, and posthole digger
Cement in cans or bags, as appropriate
Set of steel dies, numerals 0 to 9
Set of steel dies, capital letters

272. Procedure for survey.—Wherever practicable a previously established station should be occupied to begin the traverse and another one should be occupied to close the line. In those instances where it is not practicable to close a line on a second previously established station, a circuit should be run round the area to be mapped, and the traverse closed on the starting station.

Approximately in the order of use the following important points should be carefully followed:

(1) See that all parts of the transit are functioning properly and attend to correction of any looseness or derangements requiring repairs before starting the survey.

(2) Check and make necessary adjustments of the transit each morning before beginning work on the traverse. Use the instrument maker's handbook of instructions. The several adjustments to be made are:

plate levels, collimation, standards, object glass slide if provided for, eyepiece tube, telescope level, vertical circle.

(3) At the initial station the start will be made by setting the *A* vernier to read the correct azimuth of the object sighted plus 180° for turning the angle to the front rod. This object will be one whose azimuth with respect to the initial station is known. In the course of running the traverse, the reading of the *A* vernier on sighting the rear rod at occupation of any new station should be the same as its final reading on the front rod from the next previous station.

(4) Having taken the initial sight (on the object whose azimuth is known or on the rear rod), transit the telescope, turn off the angle to the front rod at the next following station, and read both verniers. With the two plates clamped together turn the instrument about the vertical axis, and sight the rear rod again; retransit the telescope, and sight the front rod a second time. Compare the two readings of the deflection angle. If they do not agree within 1 minute (30 seconds is a preferable criterion), repeat the double measurement.

(5) Whenever practicable, measure distances along uniform slopes, reading the slope angle with the transit. Differences in elevation may in some cases be easier to obtain than the direct slope angle. (Read on stadia or level rod.)

(6) Always stretch the tape for full-length measurements with a 20-pound tension on the spring balance. Short distances on sloping or rough ground where "breaking" the tape is required should be measured to even feet, the tape being held horizontal and stretched at an estimated pull of 20 pounds and the plumb point obtained with the plumb bob and line. The tape should be drawn out full length, and each part of the course measured with an adjacent part of the tape, so that when the end of the tape is reached, the sum of the short distances will equal the entire length of the tape. A check on the sum of the short distances is thus obtained.

(7) Select stations on firm ground elevated enough to provide for sighting the pole, held at each adjacent station, near its bottom. This is important to avoid errors in deflection angles from failure to plumb the pole accurately. Sites for permanent stations should be out of the way of probable future widening or repair of road or railroad.

(8) Record the temperature every hour. A temperature correction of taped distances can then be made before computations are started. The average correction to apply is 0.02 foot for a 300-foot length for each 10° change in temperature. The tape should be standardized at about 70°F., and the correction applied to that length.

(9) One of the duties of the front tape man is to select the sites to be used as transit stations. He must use proper judgment to assure good views ahead and back to the previous station. He is required to keep a

record of distances measured between stations and of the number of each station and to check distances with the rear tape man. Both tape men count the tape lengths and additional feet of each distance between stations and report failures to check to the party chief who then has the distance remeasured.

(10) Every precaution should be taken to avoid making either small or gross errors. The party chief and all concerned with measuring the line must be alert at all times to keep the work up to the standard required.

(11) In rural sections all road crossings, junctions, and other surface features readily recognizable on aerial photographs should be located along the line. In towns and suburbs judgment must be used to locate a sufficient number of street intersections without overcrowding where they will not be mistaken by those who are to use the aerial photographs for plotting the map. The center of the intersection, whether of two roads, two streets, or a road crossing a railroad, is the most satisfactory point for controlling aerial photographs. Crossings of political boundary lines or boundary monuments and important land monuments should likewise be checked and duly recorded.

(12) The instrumentman should measure each distance between stations by stadia readings to provide checks on the taped distances. Comparison between the stadia distance and the taped distance should be made at the completion of each measurement, and discrepancies reconciled before going forward.

(13) Distances between stations should never exceed the distance that can be measured by stadia. This will ordinarily be between 2,000 and 2,500 feet.

(14) Whenever long straight stretches occur in the line, the instrumentman should sight an object ahead and record its reading, making a note of it and its approximate distance from the station. This is particularly applicable to a long tangent of a railroad, and in this case a rail at the end of the tangent should be sighted.

(15) Wherever practicable, sights should be taken to prominent objects along the line to be located by intersection. Water tanks, large chimneys, cupolas, church spires, and other objects that may be readily identified on the aerial photographs should be chosen for this purpose.

(16) In general, choice of the route should be given to the following: first choice, railroads, because measurements may be made along rails with higher accuracy than along roads; second choice, broad highways with room at the side for the party to operate without interference from traffic; third choice, secondary roads with little traffic.

(17) An azimuth observation on Polaris or the sun should be made daily along the line or at distances apart not greater than 10 miles where long sights are obtainable or about every 100 stations where sights

between stations are short and at all abrupt changes in direction of the line, as at corners of the area. Observation on Polaris at elongation is best, for correct time is then not required and a period of about 20 minutes is available for the observation. By practice and careful preparation Polaris can be observed late in the afternoon before sundown. Considerable practice is required to become proficient in making daylight observations. It is necessary to know in advance the altitude of the star within about 10 minutes of arc and the azimuth bearing within 1°. The "American Ephemeris" affords the required information for making preparations and computations.

(18) When it is not practicable to observe Polaris for azimuth, observations may be made on the sun, using a piece of white paper held a few inches from the eyepiece to catch the image. Observations should not be made on the sun within 2 hours of noon or when the sun is lower than 15° above the horizon. Several repetitions (five or more) should be made of observations at each azimuth station, and all should agree within 1 minute of arc or be rejected. The "American Ephemeris" should be available to furnish required data.

(19) At all azimuth stations of the traverse the mark should be not less than 500 feet or more than about ¼ mile from the station. At each permanent station and each azimuth station several distant objects such as church spires, cupolas, or chimneys should be observed, and their azimuths from the station computed.

(20) By following the procedure indicated in (3) and (4) above, the transit man can readily compute azimuths brought forward with the line and at any time check against an azimuth determination on Polaris or the sun. To do this it is only necessary for him to proceed as follows: Take the difference between the second reading of the *A* vernier for any preceding foresight of known azimuth and the corresponding second reading of the *A* vernier for any sight whose azimuth is wanted, and divide it by 2. Add this quotient to the known azimuth if the reading *a* of the sight of unknown azimuth is greater than the reading *b* of the sight of known azimuth; subtract it if reading *a* is less than reading *b*. Convergence of the meridians must, of course, be taken into account. At latitude 30° this amounts to ½ minute for each mile east and west. For latitude 49° the amount per east-west mile is 1 minute. The formula is as follows: Convergence in minutes at any latitude is the difference in minutes of longitude from end to end of the line, multiplied by the sine of the medial parallel of latitude. Proper regard must be given to plus and minus signs.

(21) At least three reference (witness) marks should be placed about each permanent traverse station. The bearing and distance to each must be carefully measured and recorded. Bronze disks similar to those

marking the stations themselves are the best markers to use, but any substantial form of mark may be employed.

(22) Concrete posts used as permanent stations should be at least 24 inches deep in mild climates and not less than 36 inches deep in regions corresponding in climate to that of the northern part of the United States. A depth of 48 inches is recommended for New England and the northernmost states. Stove pipe 8 or 9 inches in diameter and 1½ or 2 feet long is convenient to shape the upper part of a concrete post, the lower part of the hole being first filled with concrete and the pipe set on it to come flush with the ground. It is well to end the post at the surface of the ground or let it extend an inch or two above.

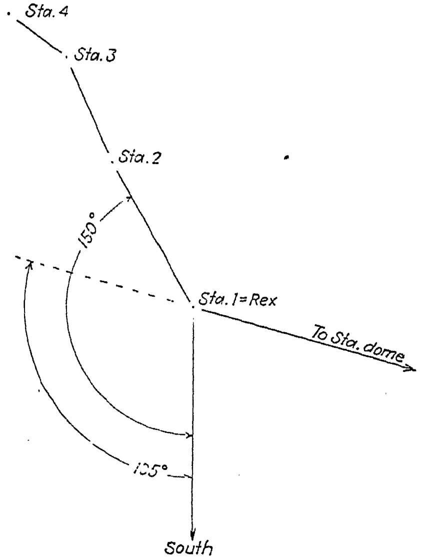

Fig. 120.—Plot of a segment of a transit traverse.

(23) As far as practicable, temporary transit stations between the permanent stations should be placed at even foot marks of the tape, thereby avoiding fractions of a foot in the measured distances. It is the duty of the front tape man to attend to this point. By this procedure the work of computing the coordinates of stations is simplified and the ultimate cost of the work reduced.

(24) Special care should be given to descriptions of permanent stations, not only to provide for recovery of marks at the site but also to be adequate for approach without delay by anyone in the future. Distance and general direction from the nearest town, village, or center; distance from the nearest road, road crossing, or intersection; and easiest route of approach for those stations off highways will ordinarily be adequate for general directions. Locally the distances given to reference objects

should be taped, and the bearings should be those read with the transit from true, or magnetic, north.

(25) It is improper to record a permanent station as located unless it is actually a station of the traverse line. In those cases where a mark is so placed that it cannot be satisfactorily used as one of the main-line stations, a spur line closed back on the main line should be run and the station occupied on this spur line, even though it involves sighting and measuring a single distance, in which case the distance should be measured in both directions and the azimuth of the sight obtained from both stations concerned.

273. Record of traverse notes.—The survey notes should be kept in a systematic form. Below is given a sample page which will serve as a guide.

The recorder must check the two deflection angles before the instrument is moved; if a discrepancy appears, the angle is reread starting with the vernier set as at beginning the first reading. He also sets down the

TRANSIT TRAVERSE RECORD FORM

Locality:
Date: Line from Station Rex to Station Skeet

Station number	Distance, feet	Vernier *A*	Vernier *B*	Mean	Deflection angle	Azimuth	Stadia distance, feet	Vertical angle
		° ′ ″	° ′ ″	° ′ ″	° ′ ″	*a*° ′ ″		
1		105 00 00	285 00 30	105 00 15	....	105 00 15		
= Rex		150 00 00	330 00 20	150 00 10	44 59 55	*b*		
		195 00 00	15 00 30	195 00 15	45 00 05	150 00 15		
	2,200			+	45 00 00	*a*(adj.)	2,210	
		c						
		195 00 00	15 00 30	195 00 15				
2		200 40 00	20 40 30	200 40 15	5 40 00	*b*		
		206 20 00	26 20 30	206 20 15	5 40 00	155 40 15		
	1,500			+	5 40 00	*a*(adj.)	1,505	
		c						
		206 20 00	26 20 30	206 20 15				
3		176 30 00	356 30 30	176 30 15	29 50 00	*b*		
		146 40 00	326 40 30	146 40 15	29 50 00	125 50 15		
4	910			−	29 50 00	*a*(adj.)	910	

a Written in red ink.
b Written with black pencil.
c From actual readings; not copied from preceding record.
(adj.) Final adjusted azimuth.

Azimuth check on course Station 3 to Station 4

146–40–00	105 00 00	
−105–00–00	+ 20 50 00	2/41–40–00
41–40–00	125 50 00	20–50–00

mean deflection angle and gives it the proper sign. Figure 120 is a plot of the segment of the line recorded in this article, in which south is the zero line for azimuths. It assists in explaining how the azimuth of any sight may be obtained by the process stated.

274. Field computations.—In order to assure maintenance of the required degree of accuracy it is necessary to make field computations of azimuth, latitudes, and departures and, if geographic positions of stations are to be obtained, of latitude and longitude. Azimuth is carried from station to station by computing deflection angles and adding them algebraically to an initial azimuth of a line, azimuth angles being reckoned from due south as zero. Allowance must be made for convergence of meridians in order to obtain a proper check between an azimuth brought forward by computation and one obtained by a new observation. Convergence of meridians amounts to about 30 seconds per mile of running east or west at latitude 30° and to about 1 minute at latitude 49°. The formula to apply to determine convergence is as follows: Multiply the total number of minutes of longitude, spanned by the line, by the sine of the mid-latitude. Going eastward an azimuth carried forward by computation should be less than one obtained by observation; going westward, greater. If the closure error is not too large to be acceptable (does not exceed a few minutes), the difference between the computed and observed azimuth at the closing station is divided by the number of stations of the line, and the error distributed proportionately to the recorded azimuths, the final accepted values being written in red ink.

275. Computation of latitudes and departures.—In this sense a latitude is the north-south component of a traverse distance; a departure, east-west component. The length of a latitude is therefore the distance multiplied by the cosine of the azimuth of the sight; the length of a departure, the distance multiplied by the sine of the azimuth. It is customary to consider latitudes going northward as minus, going southward as plus; departures going westward as plus, going eastward as minus. Computations are made daily in the field on a form like that shown below, which conforms to the record of Art. 273.

Form for Computing Latitudes and Departures
Line from Station 1 to Station 4

Station	Azimuth	Distance, feet	Sine	Cosine	N	S	E	W
	° ′ ″							
1 to 2	150 0 15	2,200	+	−				
2 to 3	155 40 15	1,500	+	−				
3 to 4	125 50 15	910	+	−				

In this instance the direction in which the traverse progressed is northwestward; so all the sine products are positive, and all the cosine products minus, and the products will be entered in the north and west columns. Five-place logarithmic tables are suitable for making the computations. If Crelle's[1] tables are available, the computations can be made more expeditiously, using natural sines and cosines.

For mapping small areas this set of computations would be sufficient to determine the error of closure, make an adjustment, and obtain the coordinates with respect to rectilinear axes. In this case it would be well to employ the central meridian as the axis of departures. The survey of extensive areas requires that the horizontal datum be on one of the accepted projections which will give a compensation for the sphericity of the earth and permit the establishment of positions by latitude and longitude or with reference to rectilinear axes whose origin may be placed entirely outside the area concerned (Art. 291).

276. Computation of latitude and longitude.—In systematic surveys by mapping organizations it is desirable for several reasons that computations of latitude and longitude be done in the field by the survey party: (*a*) It gives a conclusive check on the accuracy of results being obtained; (*b*) it avoids misunderstanding in the selection of those points which are to be fixed in latitude and longitude and listed for control purposes; and (*c*) it relieves the office computers of a considerable burden. The points to be chosen will depend on the availability of features that are satisfactory without marking for control use with aerial photographs, the number and distribution of points needed, and the number of permanent stations established along the line. In urban and suburban areas having many streets and roads, a point every half mile or mile along the route may be taken. In rural sections having few roads there may be a scarcity of suitable points, so all that have been located along the line are used. Sums of latitudes and departures are taken from the computations of latitudes and departures and entered in a form provided for the computations to be made. Special tables are available for shortening the work involved, and an established procedure lessens the danger of error. Checks are applied and closures must be obtained within the limit of allowable error. A plot is made for each group of traverse notes, and an entry made of the length of the circuit and the closing error.

277. Selection of routes.—For mapping a rectangular area containing between 200 and 300 square miles, it is generally necessary to have control stations distributed around its borders and others along a line through its middle. Routes must therefore be selected to provide stations so distributed. It is inadvisable to have the traverse line extend beyond the reach of the aerial photographs at any part of its course, but in some areas where roads and railroads are not plentiful it becomes necessary to deviate from the area. Wherever this is the case, the line should be taken back to the area over the nearest practicable way. By extending the photographs short distances beyond the border of the area it often becomes possible to connect them with routes where control stations may be most favorably established, thereby effecting a saving in cost of the survey. Those planning the mapping work should take into account the most feasible routes for control and have the photographs taken to cover them or in cases where control stations are already available to have ample extensions of photography made to embrace the existing stations.

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[1] Crelle, A. L., "Tables de calcul," U.S. Department of the Interior, U.S. Geological Survey, Washington, D.C., 1918.

278. Azimuth determinations.—During the period of time extending from 20 minutes before elongation to 20 minutes after elongation at latitude 42° north, the change in azimuth of Polaris amounts to only about 12 seconds; *i.e.*, at both 20 minutes before elongation and 20 minutes after elongation the angle between Polaris and the north pole is about 12 seconds less than it is exactly at elongation. In latitudes north of 42° the period of time in which the discrepancy is not greater than 12 seconds is shorter; in latitudes south of 42° the corresponding period of time is longer.

In making azimuth determinations on traverse lines with a transit graduated to 20 or 30 seconds, an observation made within 15 or 20 minutes of the exact time of elongation is of sufficient refinement for all practical use that can be made of it. With the instrument set in position well ahead of time and the signal ready at the reference mark, it is possible to obtain a required set of readings with the instrument, both direct and reversed, within the period of time during which there will be no perceptible change in the direction to Polaris. The great advantage in getting observations at elongation comes from the comparative simplicity of computations required to determine the azimuth of Polaris and freedom from having to secure exact time. It is necessary only to know by ordinary watch the time of elongation and the difference between the local time and sun time.

It is safe practice for a beginner to observe Polaris for azimuth at elongation. After experience has been gained and assurance is established, observations may be extended to other times or to the sun when it becomes difficult to sight Polaris.

It is often inconvenient to be at the station at the time of elongation; and if the instrument permits the making of daylight observations, it may be more suitable to provide correct time and go through with a more elaborate computation in order to avoid a delay or a return to the station. Therefore, it is highly desirable to be prepared to make both daylight and night observations and the required computation in either case. Whenever choice of time for an observation may be made without too much inconvenience, it is best to select the time near elongation rather than near culmination, for an error in time has less effect then than near culmination.

The equation for use to compute the azimuth of Polaris at elongation is

$$\sin A = \frac{\cos \delta}{\cos \alpha} \tag{33}$$

in which A is the angle between Polaris and the north pole, δ is the declination of Polaris, and α is the latitude of the station. The latitude of the station need be known only approximately. An error of 1 minute in latitude at or near 42° north will cause an error in the determination of the azimuth of slightly less than 1 second. The "American Ephemeris and Nautical Almanac" includes a table (Table V) that gives the azimuth of Polaris at elongation during the entire year of issue for the zone of latitude stretching from 10 to 70° north at various declination angles. It also furnishes the right ascension and declination of 35 circumpolar stars (including Polaris) under the heading "Apparent Places of Stars (year of issue)" for each day in the year.

The data given above are adequate with the tables mentioned to obtain satisfactory azimuth determinations at elongation, either eastern or western. In determining the azimuth of the mark on the ground observed from a station, it is, of course, necessary to observe the sign plus or minus applicable to the situation. (For example, if the mark is to the east of Polaris and observation is on Polaris at eastern elongation, the azimuth of Polaris should be added to the angle between Polaris and mark to give the azimuth of the mark with respect to north. Add 180° to this result to obtain the azimuth of the mark with respect to south. It is customary to use the true south as the zero line for geodetic azimuths.)

An excellent form of signal for use at night is a small box, with a slit about ¼ inch wide and a few inches long, having a lighted candle or lantern within. A set of observations should consist of at least five readings on star and mark with instrument direct and five with instrument reversed. Special care must be given to adjustment of the horizontal axis of the telescope and to its leveling. The striding level should be read at each pointing on the star and recorded.

Concerning Polaris, it is convenient to remember the following: Each year upper culmination occurs at noon near the middle of April and thereafter about 4 minutes earlier each day; its declination is now increasing at the rate of about 18 seconds a year; the annual range of variation of its declination is about 32 seconds; and its azimuth at elongation in middle latitudes (circa 40°) is gradually decreasing at the rate of about 24 seconds a year. Table VI of the "American Ephemeris and Nautical

Almanac" gives useful information for finding, by observation relative to Mizar and δ Cassiopeia, when Polaris passes the meridian.

279. Azimuth observations from triangulation stations.—Azimuth observations from triangulation stations must be made with more refinement and care than is necessary on traverses. The reason for this is that greater distances between stations are involved. For third-order accuracy the probable error should not be greater than 5 seconds. Time must be known accurately at each pointing on the star. Observations should be made near elongation whenever practicable, starting before elongation and continuing afterward. Either a signal at another triangulation station or one set at a mark at least ½ mile away should be used.

It is also necessary to determine the angle between the mark (if other than one at a station) and one of the triangulation stations with the same refinement used in measuring the angles between stations.

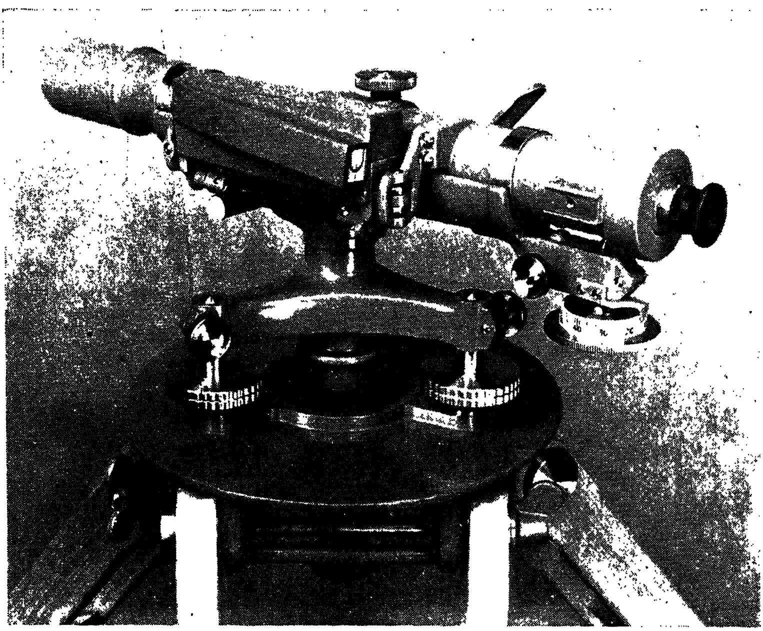
Satisfactory procedure in carrying out azimuth observations of various refinements is given in the reference publications listed in Art. 287, which should be studied by those undertaking surveys involving transit-traverse and triangulation.

280. Spirit leveling.—In general, lines of levels should follow transit-traverse lines, and the permanent transit stations should be used as the permanent bench marks. For a 15-minute quadrangle or equivalent area it is ordinarily sufficient to have a circuit line around the area as close as practicable to the borders (as for the traverse) and a line through the middle of the area. If additional leveling is required, it may be run between stations of the first-mentioned lines. If the area is highly developed and densely inhabited, conditions may require that such auxiliary lines be of the same order of accuracy as the main lines, in which frequent permanent bench marks should be established. Otherwise it will be adequate for mapping to set only temporary marks and execute the work with closure error no greater than 1 foot over a distance of a few miles.

Each city, town, township, or other important district should have at least two permanent bench marks. In rural sections bench marks should be set so that the average distance between them is not greater than 3 miles. The locations should be chosen to assure a long undisturbed life and be where the mark can be readily reached for use. It is necessary to set concrete posts to extend below the frost line in firm ground. Ledge rock or large boulders may also be used.

As far as practicable, permanent marks should be set before the level line is run, and the marks used as turning points in the line. Wherever it is not practicable to get the permanent marks set ahead of the instrumental work, a temporary bench mark should be left near by on each side of the site for the permanent mark, and the elevation of the permanent mark obtained by starting from one of the temporary marks and closing on the other one.

Temporary or supplementary bench marks should be established at intervals of ½ to 1 mile along the line. They should in every case be turning points of the line. Chisel marks in concrete, ledge rock, or boulders and copper nails in roots of trees are excellent forms of marks.



A B

FIG. 121.—Berger prism tilting dumpy level. Diagram *A* shows the bubble images as they appear when the instrument is nearly level, *B* when level. (*Courtesy of C. L. Berger and Sons.*)

Every bench mark, whether temporary or supplementary, should be adequately described as to location and form so that ready recovery will be possible. The description should include the locality with reference to highways, railroads, or other well-known permanent features.

Miscellaneous elevations of summits, lowest parts of terrain, road intersections, railroad crossings, and bridges over streams should be obtained all along level lines for the purpose of aiding to the greatest possible extent in completing the topographic map.

A level instrument of high quality should be used. The 20-inch wye level or a prism tilting level like the Berger instrument shown in Fig. 121 is satisfactory. The New York type of level rod is commonly used as well as the yard rod. The invar rod is the most reliable. Maximum care should be taken to keep the level in first-class adjustment.

The allowable closure errors in lines of the various orders are given in Art. 266. In each case the length in miles is the distance between bench marks of the same or higher order of accuracy or, in the case of third-order leveling, the distance round a circuit.

Balancing of sight lengths should receive full consideration. Special precautions must be taken in crossing streams or other obstacles requiring sights longer than 300 feet. Sights longer than about 300 feet should not be taken except where necessary to overcome obstacles.

The third-order level party should consist of a levelman, a rodman, and a chauffeur, who may serve as extra rodman or to set bench marks in advance of the instrumental work. In some cases of extreme heat an umbrella man is required to shade the level.

A list of equipment follows:

Wye level (or prism level)
Two New York rods (yard rods if prism level is used)
Two plumbing levels
Two steel turning pins
25-foot steel tape
Invar testing tape
Thermometer
Bench-mark tablets or metal bolts
Copper nails
Cement
Paint in can and brush
Notebooks
Cold chisel
Stone drills
Hatchet
Drill hammer
Posthole digger (if concrete posts are to be set)

281. Trigonometric leveling.—In mountainous or open hill terrain, leveling may be impracticable to obtain the vertical control for mapping. Trigonometric leveling or the determination of elevations of stations by reading vertical angles will often serve as a substitute for, or to supplement, spirit leveling. Where employed to carry levels from station to station, the vertical angles should be read forward and backward between stations. Where several triangulation stations of a closed figure are occupied, vertical angles should be read from each station to the others. Supplementary elevations of objects not occupied as stations should be based on at least three determinations from three separate stations.

For third-order work, use should be made of an instrument (theodolite, transit, or telescopic alidade) having a vertical circle permitting readings to 10 seconds. For control of a particular area a vertical circle reading to 20 seconds will give acceptable results. Figure 122 shows the standard U.S. Geological Survey telescopic alidade with micrometer eyepiece. This instrument has a vertical circle that may be read to 1 minute and estimated to 30 seconds. The micrometer eyepiece has a special value for determining distances in open country. It is equipped both with stadia wires and a movable wire that may be used on a base of fixed length in lieu of the stadia wires.

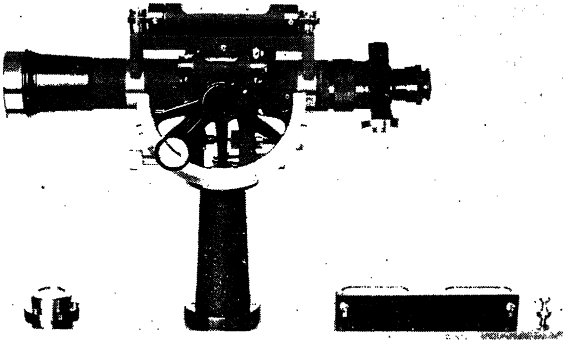


FIG. 122.—Standard U.S. Geological Survey telescopic alidade with micrometer eyepiece. (*Courtesy of U.S. Geological Survey.*)

The telescopic alidade is best employed on a plane table with a light board 18 by 24 inches in size, the tripod having a Johnson head such as illustrated in Fig. 123. The head provides for leveling the board and for turning it in azimuth. It gives a light but rigid tripod which is well braced against torque by long bolts in the head. A telescopic alidade and a plane table such as here shown are the most satisfactory instruments for obtaining vertical angles, because all adjustments required to assure accuracy may be quickly made at any time during the work. It is important that the telescope be rigged for rotation about its own axis, or else a quick check of the line of collimation and the level bubble index cannot be made while at stations. Dependable work cannot be assured without this facility.

Tables are available for making rapid computations of elevations by vertical angles. A table with distances in miles and correction for curvature and refraction to accord may be used or a table with distances in feet. (Both these tables can be obtained in convenient form from the U.S. Geological Survey, Washington, D.C.) Trigonometric leveling is applicable to traverses, triangulation, or plane-table surveys wherever horizontal distances are available. The correction for curvature of the earth and refraction of light should be applied in every case where sight distances cannot be balanced in traverses and always from triangulation stations. The correction can be calculated if tables are not available, using the formula: Correction in feet equals 0.574 distance-in-miles squared.

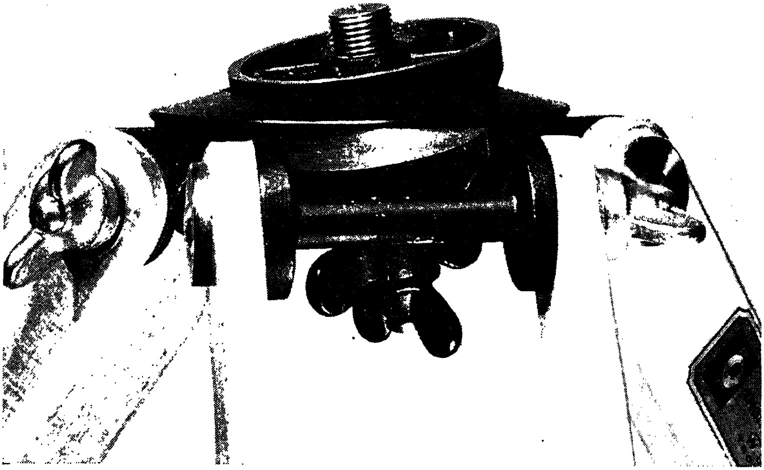


FIG. 123.—Head of Johnson-type tripod.

VERTICAL-ANGLE RECORD

Date Station occupied Elevation

Object observed	Point reading	Level reading	Vertical angle	Distance, miles	Elevation difference, feet	Elevation of object	Remarks

The record form given on page 283 is appropriate for a notebook to be used to record vertical angles and to enter the results of computations of elevation differences.

If the telescopic alidade is equipped with an auxiliary level bubble, it will be necessary only to set the telescope at level position at the start of observations at a station, record the vernier reading, then check it at completion of observations. This saves considerable time over making observations without an auxiliary bubble, for then the level reading has to be entered for each sight taken. The instrument of Fig. 122 is equipped with an auxiliary level bubble.

282. Magnetic declination.—Unless the results of previous magnetic surveys are available, it is necessary to make numerous determinations of the magnetic declination over the areas being mapped. The mean declination of the year of the survey should appear on the map as finally drafted. If the area is large, and if there is considerable variation, the declination should be shown for several places distributed widely over its extent. If available, the annual change in declination should also be shown. Every plane-table sheet of the survey should carry a magnetic north line.

Accurate magnetic surveys are made with the magnetometer at azimuth stations or, if dip is not required, with a compass declinator. However, the instruments will not ordinarily be available in carrying out topographic surveys. Observations should be made at traverse or triangulation stations with a properly pivoted needle several inches long, such as those with which transits and theodolites of good quality are equipped. The observations should be recorded along with traverse or triangulation notes.

283. Positions of latitude and longitude determined astronomically.—When a survey is to be made in a remote region where no existing control station is available, it is highly desirable that at least one station within the area be fixed in latitude and longitude. Some surveys have to be carried out under conditions which make it impracticable to take along the necessary extra equipment; but if feasible to transport the instruments, and time permits, a prismatic astrolabe, or theodolite with prism attachment, together with a radio receiving set to catch time signals, the required star data, and necessary accessories, should be taken and the required observations made. The Institute of Geographical Exploration has prepared and published a complete 60° star list for position fixing by the equal altitude method, embracing the zone from 60° south to 60° north latitude. This volume also includes notes for the adjustment and use of the 60° prism attachment for theodolites.

It is not intended to give instructions for the use of the prismatic astrolabe but merely to explain the purpose of this instrument and to give some of its characteristics. Figure 124 shows one of the instruments owned by the Institute of Geographical Exploration, as designed by Claude and Driencourt and made by the Société d'Optique et de Mécanique de Haute Précision of Paris. It is the geodetic model, having a 60° prism placed in front of the objective; a bracket to support a basin of mercury which furnishes the artificial horizon; and two eyepieces, one of which magnifies 80 times, the other 30 times. The objective has a diameter of 2.1 inches and a focal length of about 15 inches, giving a 36-minute field for the higher magnification, which is used for actual observation, and about 1½° for the lower magnification, which is used as a star finder.

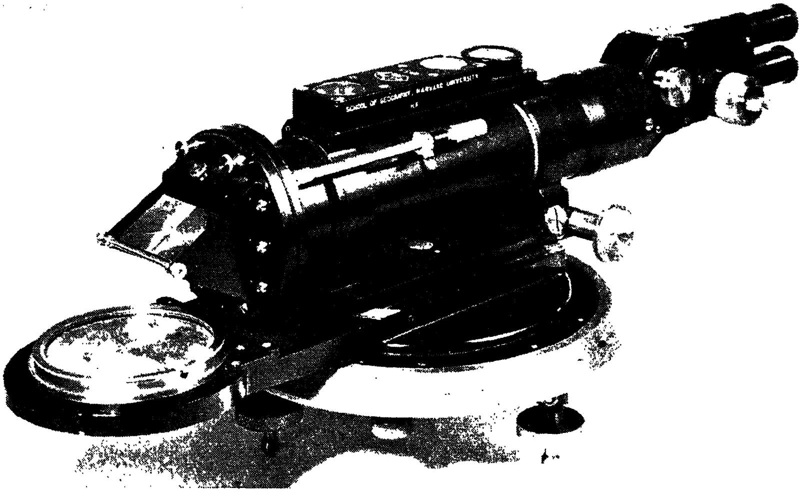


FIG. 124.—Sixty-degree prismatic astrolabe—geodetic model. (*Photograph by Institute of Geographical Exploration, Harvard University.*)

The faces of the prism are 2.3 by 2.3 inches. A horizon circle which may be turned independently of the telescope, a vertical axis for the telescope, and a slow-motion screw provide for pointing the telescope in any azimuth. The instrument proper weighs about 30 pounds and its over-all length is 25½ inches. In its case with accessories the weight is 41 pounds. The tripod weighs 10½ pounds.

The purpose of the instrument is to observe the exact instant that a star crosses a given almucantar (altitude or zenith distance circle). Use of a prism of high quality keeps this altitude invariable. By observation of a number of stars, differing in azimuth, at a station and taking the time of passage of each star, the results are applied to the method of equal altitudes to locate the station. The reduction process ordinarily used is a combination of computations and a graphical plotting.

Possible accuracy of the prismatic astrolabe (geodetic model) is rated in latitude to 0.2 second of arc, in time to 0.04 second in latitudes between 30 and 45°, which corresponds to about 0.6 second of arc of longitude. To attain such precision in results it is necessary to observe expertly and to utilize a relatively large number of stars—12 to 30. The personal equation in catching the time varies considerably with individuals, so that the degree of precision indicated above is not obtained as a rule. It may be said that under favorable conditions and by proper care position determinations are reliable to within about 1 second of arc. It should be understood, however, that this does not mean that a location will be obtained with that degree of precision. Deflection of the plumb line, caused by masses of the earth's crust in the vicinity having unequal densities, will derange the observations according to the amount of such deflection. It is known that over the earth's surface deflection of the plumb line varies from zero to many seconds of arc.

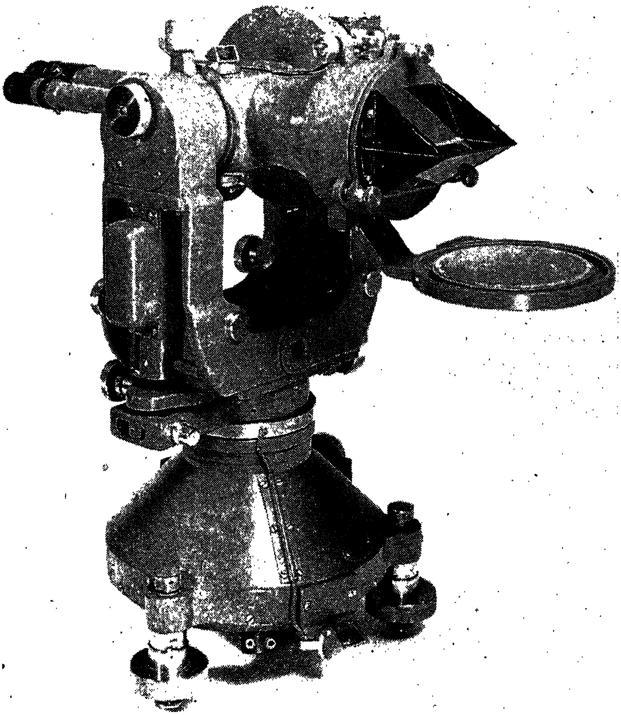


FIG. 125.—Theodolite with 60° prism attachment. (*Photograph by Institute of Geographical Exploration, Harvard University.*)

In making an observation two images of a star are seen, one appearing to travel downward, the other upward. Time is taken at the instant that the two images appear to pass each other. Actually the images travel obliquely across the field, some with a more pronounced slant than others. It is usually necessary to adjust the telescope in azimuth after a star has been picked up, so that the passage of the images will take place near the center of its field. One of the images is transmitted through the upper surface of the prism; the other, through the lower face after reflection from the artificial horizon of mercury.

Figure 125 is a photograph of a theodolite with a 60° prism attached. So rigged, the theodolite may be used as a prismatic astrolabe. Since it has somewhat less magnification in the telescope than is given by the instrument of Fig. 124, results obtained with such a theodolite are not quite so accurate as those obtained with the astrolabe proper. However, the weight is greatly reduced, and two instruments in one are obtained with but a few small accessories.

284. Laying out and measuring a triangulation base.—For establishing a number of triangulation stations in a region where no previous survey has been made, it is necessary to lay out and measure a suitable base from which the net of triangulation stations may be established. Usually a reconnaissance to find suitable terrain not only will prove valuable in disclosing a satisfactory site but may save considerable labor and time in establishing the base and more than repay its cost by increasing the accuracy of results. Terrain should be sought where the base ends will be favorably situated with respect to adjacent sites for stations so that the expansion figures will not have angles too small, and the differences in elevation of stations will not require elevating or depressing the telescope through large vertical angles.

It is essential that the tape used be standardized (an invar tape is best to use) and that it be uniformly supported and pulled at the proper tension when measurements are being made. Supporting and marking stakes should be set accurately along the line at measured distances, and levels run to make the tops accord with gradients along the terrain. Stakes falling at the ends of tape lengths should be flat on top for accurate marking of distances. The line must be well enough cleared to free the tape entirely from obstruction and permit the final measurements to be made with all possible accuracy. Measurements being made along slopes and with tape suspended between supports make it necessary to correct for slope and sag. The base should be carefully measured twice; and if results in stretches do not agree within 1 part in 100,000, a third measurement of the stretches concerned should be made.

285. Amount and distribution of control required for mapping.—The required number and distribution of control stations (both horizontal and vertical) will depend on the type of aerial photographs employed, the scale of the map, and the method of using the photographs. In general, it may be stated that for radial-line work with single-lens photographs there should be at least the equivalent of a traverse around the borders and a line through the middle of an area equivalent to the 15-minute quadrangle. For radial-line work with five-lens photographs fewer actual stations will be needed than for single-lens photographs, but the distribution required is about the same as for single-lens photographs, so the number of miles of traverse will be the same. If the control is to be established by triangulation, the number of stations required for five-lens photographs will be about half the number required for single-lens photographs. In addition to bench marks and spot elevations along traverse lines there should be also a number of points known in elevation distributed through the area.

The foregoing statement is applicable to the method of contouring by ground survey or to contouring from five-lens photographs with a simple stereoscope, elevations of numerous objects being obtained from the photographs by ground survey.

For contouring with the multiplex or any of the stereo-plotting machines, the amount of control required will probably be greater than that indicated above. It is in general desirable to have three or four points known in elevation in the zone of overlap of each pair of photographs. However, by systematic procedures it is possible to bridge considerable distances between control stations with some of the more elaborate instruments. Secondary vertical control is probably more readily obtained from multilens photographs employing the upright-planes method (Art. 219).

286. Sources of existing control data.—Before beginning a survey, inquiries should be made to ascertain whether existing control data are available of the area to be mapped and, if so, to obtain all that can be used. The chief sources of control data in the United States are:

The U.S. Coast and Geodetic Survey, Washington, D.C.
The U.S. Geological Survey, Washington, D.C.
The Chief of Engineers, U.S. Army, Washington, D.C.
The various state, county, and city survey organizations
Private organizations engaged in making surveys, and large landholders who have had surveys made of their properties

Many of the states cooperate with the Federal bureaus, and some have their own mapping organizations. The Massachusetts Geodetic Survey has a vast amount of control data relating to the state and adjacent areas, which are obtainable on request.

Similar organizations in foreign countries have control data which in many instances can be obtained for worthy explorations or surveys.

287. Reference publications.—Before undertaking productive field work, the individual should study the practice of organizations and others extensively engaged in such operations and follow their practice to the extent applicable to the task at hand. The following publications are recommended for study and consultation. Those issued by government bureaus may be obtained from the Superintendent of Documents, Govern- t, Printing Office, Washington.

and Hosmer, "Higher Surveying," 5th ed., Vol. II, John Wiley & Sons, Inc., w York, 1938.

neral Land Office: "Ephemeris of the Sun and Polaris," issued annually.

ual of Reconnaissance for Triangulation," U.S. Coast and Geodetic Survey, *Special Publication* 225.

"Manual of Second and Third Order Triangulation and Traverse," U.S. Coast and Geodetic Survey, *Special Publication* 145.

J. J. Nassau, "Textbook of Practical Astronomy," McGraw-Hill Book Company, Inc., New York, 1932.

"Topographic Instructions," U.S. Geological Survey, *Bulletin* 788, in several parts.

U.S. Naval Observatory, "The American Ephemeris and Nautical Almanac" for the current year; issued annually.

CHAPTER XV

MAP PROJECTIONS, GRIDS, AND FORMS

FINISHING A MAP FOR REPRODUCTION

288. Standard maps are drawn on certain projections that have been adopted because of their suitability for a series of maps made in units, each of which covers a small part of a large region. That projection which is most satisfactory for this purpose may not be the most suitable one for an atlas map of a continent. Only a few of the scores of possible projections[1] of the spheroidal surface of the earth are used for standard maps and charts made in the United States.

A small area may be mapped on an orthographic projection, *i.e.*, on a plane tangent to the earth's surface near the middle of the area, without having any appreciable error from flattening the curved surface, but for extensive areas this simple projection gives large displacements of features in the outer parts of the map. In order to keep errors from this cause from rendering a map unsatisfactory for scaling, projection to the surface of a right cone tangent to or intersecting the earth's surface is usually employed. The cone is then imagined as being split along an element and rolled out into a plane. In some instances the projection is based on the surface of a cylinder modified to give improvements, this surface also being rolled out into a plane. For special reasons in navigation the Mercator projection is employed for charts of oceans and coastal areas.

The projections of chief interest in standard mapping in the United States are the polyconic, the Lambert conformal conic, and the transverse Mercator. Practically all unit quadrangular maps made by government organizations are drawn from the polyconic tables,[2] which are available for the entire surface of the earth. Limited areas of a few hundred square miles drawn on any of these three projections are practically the same in so far as one could judge by measuring with means ordinarily available. For this reason the polyconic tables serve for all. Only when a comparatively large area, such as a state, is to be drawn on a single sheet does it becomes necessary to employ the table for the projection used. Although all of these are based on conic or cylindrical surfaces, none is a simple projection to the surface chosen. Owing to the spheroidal shape of the earth, all involve complicated formulas so that for accuracy tables prepared by competent mathematicians and computers are employed. The polyconic tables relate to a different tangent cone for each minute of latitude and have been prepared by a great volume of work. The Lambert conformal conic projection has been adopted for some states whose length stretches from east to west; a transverse Mercator projection, for others whose length runs from north to south. As these projections give a high degree of accuracy for limited areas, they are particularly valuable for referencing horizontal control data and make it feasible to extend lines of control from existing stations locally by plane coordinates. The Commonwealth of Massachusetts has officially adopted a Lambert projection for the geodetic surveys of the state and has published the positions of a large number of control stations by rectangular coordinates based on that projection.

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[1] The U.S. Department of Commerce, *Special Publication* 68, entitled "Elements of Map Projection," is recommended to those who wish to study the general subject. *Bulletin* 809, issued by the U.S. Department of the Interior, entitled "Formulas and Tables for the Construction of Polyconic Projections" and compiled by Col. C. H. Birdseye, gives a mathematical analysis of the polyconic projection.

[2] "Tables for a Polyconic Projection of Maps," U.S. Department of Commerce, *Special Publication* 5, or U.S. Department of Interior, *Bulletin* 809, mentioned in footnote 1.

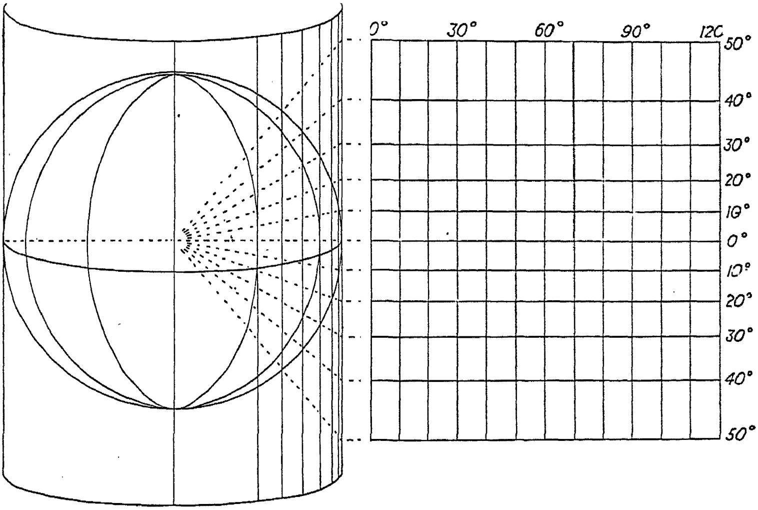

Fig. 126.—Projection to a tangent cylinder, the basis of the Mercator projection.

289. The Mercator projection is based on a cylinder tangent to the earth at the equator, the axis of the cylinder being coincidental with the axis of the earth. Without modification such a projection gives great distortion in high latitudes. In order to reduce distortion the projection is modified by equalizing the stretching in latitude and longitude; hence the common Mercator projection cannot be constructed graphically as shown in Fig. 126 but must be drawn from a table. Even so constructed the projection is not a satisfactory one to represent polar regions. On it both meridians and parallels of latitude are represented by straight lines. For charts this projection is favored above all others by mariners, but it is a poor projection to represent land features except for small areas or for aerial navigation. The transverse Mercator projection is based on a cylinder that is tangent to the earth along a meridian, so its axis is turned at 90° to the earth's axis. With the cylinder in this position a projection of great accuracy for a narrow area in the east-west direction is obtained. The area may have any length from pole to pole. States such as Vermont, New Hampshire, New Jersey, Delaware, Mississippi, Indiana, and Illinois can be well represented on the transverse Mercator projection.

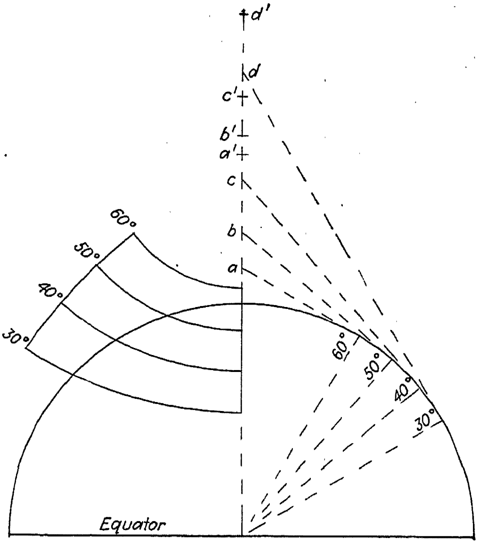

FIG. 127.—Polyconic projection of a 30° zone of a globe.

290. The polyconic projection as used in the United States is based on a series of right cones whose bases are tangent to the spheroid along parallels of latitude starting at the equator and extending at intervals of 1 minute of latitude to the pole. The north half of the earth being assumed similar to the south half, a table extending from the equator to a pole is sufficient for the entire globe. This is an excellent projection to represent on one sheet an area of great length which is narrow in the east-west direction, but it is a poor one to represent an area that extends a great distance east and west. The reasons why this is necessarily the case may be understood from an examination of Fig. 127. It should be kept in mind that when comparatively small areas are drawn separately as units of a great region, or indeed of the entire globe, the stretching along the outer meridians is not incorporated in the sheets but left between them. Owing to the lack of true sphericity of the earth, this projection cannot be accurately constructed graphically from a drawing like Fig. 127 but must be drawn from tables.

Although simple to construct, a polyconic projection must be drawn with great care in a certain way, or else it is liable to be unsatisfactory. To illustrate the construction it will be assumed that an area embracing 30 minutes of latitude and 30 minutes of longitude (a 30-minute quadrangle) is to be mapped and a sheet with 15-minute latitude and longitude lines is to be prepared, the latitude of the southern parallel being 35° north and the scale to be 1:90,000. The longitude is immaterial, for the projection can be used anywhere in the latitude zone 30°00′ to 30°15′ north. A data sheet is first made from the tables referred to in Art. 288. It should be made in the form of a miniature sheet like the one to be constructed as shown in Fig. 128. The projection will be constructed from a central meridian as the initial line, which will be drawn entirely across the middle of the map sheet in the north-south direction, usually the long dimension. Using a beam compass, a perpendicular is carefully erected at its mid-point and another at the point where the southernmost parallel crosses it and a third one where the northernmost parallel crosses it. These are to serve as the reference lines for obtaining the other required intersections of meridians and parallels. The trueness of the perpendiculars can be tested by measuring the distances between them at their extremities. The meridional distances between parallels should be laid off with the beam compass as taken from a standard metal scale. As

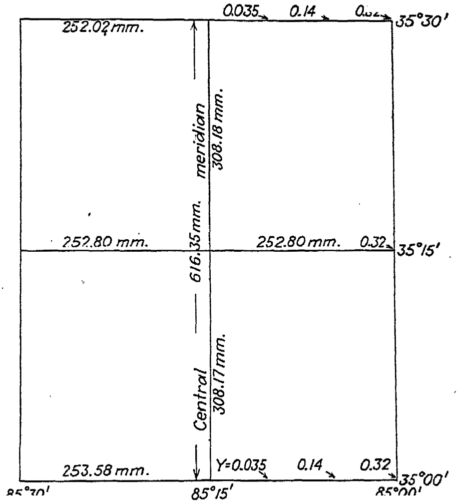

FIG. 128.—Data sheet for constructing a polyconic projection of a 30-minute quadrangle map sheet at the scale 1: 90,000 for the latitude zone 35°00′ to 35°30′.

a rule an over-all distance should be measured off, then divided into parts as required. It is not good practice to lay off a long distance by steps.

On page 81 of the tables in the Department of Commerce *Special Publication* 5 (Art. 288), seventh column, which is headed "Continuous sums of minutes from latitude 35°00′," opposite latitude 35°30′ is found the figure 55,471.1, which is the actual length in meters of the middle meridian to be drawn on the map, *i.e.*, it is the distance from the 35° parallel to the 35°30′ parallel. This is divided by 90,000 to obtain 616.35 millimeters, the length of this meridian as it should be drawn. Similarly opposite 35°15′ the figure 27,735 meters is found, and this divided by 90,000 gives 308.17 as the meridional distance from latitude 35°00 to 35°15′, leaving 308.18 as the meridional distance from latitude 35°15′ to latitude 35°30′. There is a slight difference between these two meridional distances, which is as it should be, for on this projection the meridional length of a unit of latitude gradually increases from the equator to the pole. In the next to last column of the same page, in the line opposite 15′ will be found the figure 22,822.5 meters as the X quantity for 15 minutes of longitude; this divided by 90,000 gives 253.58 millimeters. In the last column opposite 15′ the figure 28.6 meters is found, which divided by 90,000 gives the Y quantity, to be laid off to obtain the intersection of the 35° parallel and the meridian 15 minutes east or west of the central meridian. That is to say, the X quantity is to be laid off on the line that is perpendicular to the central meridian and the Y quantity parallel to the central meridian to obtain the desired intersection. The same X and Y quantities are laid off on each side of the central meridian to give the southeast and southwest corners of the quadrangle. It is necessary also to obtain the X and Y quantities for the northeast and northwest corners of the quadrangle. Turning to page 83 of the same tables, the X quantity at latitude 36° is found to be 22,541.4 meters for 15 minutes of longitude. A mean between 22,541.4 and 22,822.5 (22,681.95 meters) is the X quantity for latitude 35°30′, and a mean between 28.6 and 28.9 (28.75) is the corresponding Y quantity. These are both divided by 90,000 to give 252.02 and 0.32 millimeters, respectively. All the Y quantities for a given meridian on this sheet are so nearly the same that they may be taken as equal.

The X and Y values found are now marked on the data sheet where they belong as shown in Fig. 128. The X and Y values for the 35°15′ parallel may be interpolated and also marked on the data sheet. In order to draw the parallels it is also necessary to obtain additional Y values. It will be sufficient to get them for 5- and 10-minute longitudinal distances. For 5 minutes, the last column of page 81 gives 3.2 meters; for 10 minutes, 12.7 meters. Divided by 90,000, these give 0.035 and 0.14 millimeter, respectively. The first is too small to be measured; the second, only about the width of a fine line. All the necessary intersections having been located, the accuracy of the construction is tested by measuring the diagonals, which in any latitude zone should be equal. The beam compass is used for this purpose. The plotting is not satisfactory if any discrepancy can be detected. Another check is obtained by laying a straightedge along the points marking the outer meridians. All points of a meridian should be on a straight line. If satisfactory, the projection is completed by drawing the meridians straight and the parallels with a broken line which should pass through all plotted intersections. For a limited area such as that here considered there is so slight a curvature of meridians that they cannot be drawn except as straight lines, whereas the curvature of parallels is well enough indicated for practical purposes by drawing them as broken lines, with straight segments between located intersections. The distances between the central and outer meridians may be subdivided proportionately to obtain the positions at which the intermediate Y quantities may be laid off. If it is desired to subdivide the quadrangle into smaller sections, the parallels should be first subdivided, the additional meridians drawn, then the meridians subdivided, and the required parallels drawn. In constructing a projection, the standard to obtain should be such that no error can be detected by a rigid test with the beam compass.

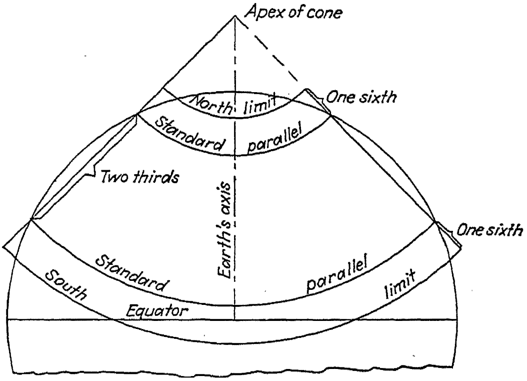

FIG. 129.—Lambert conformal conic projection.

291. The Lambert conformal conic projection is illustrated by Fig. 129. A single cone is employed, so not more than one zone of latitude can be accommodated by a particular cone. That cone is selected which gives the most satisfactory representation, and this has been found to be the one whose axis coincides with the axis of the earth and whose surface intersects the spheroid so as to have about two-thirds of the area inside the spheroid and one-sixth outside at the northern and southern sides as the figure shows. The lines of intersection are denoted standard parallels. It is evident, then, that, if points on the earth's surface are projected along lines radiating from its center, they will be drawn closer together between the standard parallels and stretched apart outside, and all points along the standard parallels will be in their true positions. The magnitude of the contraction or stretching depends on the width of the zone to be taken in. The projection for Massachusetts affords an example. On it the contraction of points at the middle of the zone shortens their distances from the standard parallels about 1 in 28,000, and the stretching at the outer limits of the zone causes displacements from the standard parallels of about 1 in 26,000. The zone provided for by the projection is about 100 miles wide. Therefore the largest displacement amounts to less than 7 feet, or at the rate of about 2½ inches per mile.

The origin for the zone is taken to be on the 41° parallel and far enough to the west to give positive values to the coordinates of all points lying within the state. X values increase going eastward, and Y values increase going northward. The coordinate grid system of the projection is oriented with the meridian at longitude 71°30′ west of Greenwich. The X value of points on that meridian is taken to be 600,000, so the origin is 600,000 feet west of that line, which is termed the *prime meridian.* Comprehensive tables in feet published by the Massachusetts Geodetic Survey are available for plotting the intersections of parallels and meridians on a grid at any scale, so latitude and longitude lines also may be shown in their true positions. A formula for converting geographic coordinates into the grid coordinates accompanies the tables.

This projection covers the mainland region of the state. Martha's Vineyard, Nantucket, and adjacent islands are provided for by another Lambert projection whose zone is only about 20 miles wide. In it the maximum displacement of points amounts to only about 6 inches.

On the Lambert conformal conic projection, meridians are straight lines (elements of the cone), and parallels of latitude are concentric circular arcs. The projection is therefore very simple to construct at a small scale for a large area. Chord distances along the standard parallels and meridians may be computed, and the intermediate lines interpolated. Tables for the projection are given in U.S. Coast and Geodetic Survey, *Special Publication* 52. However, for projections at the scales ordinarily employed in topographic mapping, the polyconic tables are used, because they provide for the entire globe, and there is no appreciable difference between the polyconic and Lambert projections at such scales for limited areas.

The Lambert conformal conic projection is not so flexible as the polyconic projection. In order to make it apply all over the earth it would be necessary to divide the hemispheres into numerous zones of latitude and prepare a table for each zone. This would necessarily involve areas of overlap and require the use of equations to interrelate the zones.

292. Grid system for military maps.—The United States Army has adopted a rectangular grid for military maps. It is placed on maps as a square grillage of yards, the squares representing 1,000 or 5,000 yards, depending on the scale of the map. In order to avoid excessive divergence of the grid lines from meridians and parallels the United States is divided into seven grid zones, each of which covers 9° of longitude and extends across the country from south to north. The zones are chosen to overlap one another 1° of longitude in order to avoid having to employ more than one grid on any border sheet. The zones are designated by letters of the alphabet starting with *A* for the northeastern states and continuing through *G*, which embraces the Pacific seaboard. Basic data of the zones are given in Table 11.

TABLE 11.—ZONES OF MILITARY-GRID SYSTEM

Zone	Central meridian, degrees	Limiting meridians
		° ′ ° ′
A	73	68 30 to 77 30
B	81	76 30 to 85 30
C	89	84 30 to 93 30
D	97	92 30 to 101 30
E	105	100 30 to 109 30
F	113	108 30 to 117 30
G	121	116 30 to 125 30

An origin that lies south of the southernmost limit of the country and west of a zone has been chosen so that all coordinates have positive values, the *X* quantities increasing going eastward, the *Y* quantities increasing going northward. The origin for each zone is fixed in the same relative position with respect to the central meridian, and its intersection with parallels of latitude, so a table for one zone is sufficient for all, the same coordinate numbers being repeated from zone to zone. A table prepared by the U.S. Coast and Geodetic Survey[1] gives the intersections at 5-minute intervals of the meridians and parallels within the borders of the United States.

Included in this publication is a full explanation of the military-grid system, instructions for placing the grid on a map, and formulas for converting geographic coordinates to grid coordinates.

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[1] "Grid System for Progressive Maps in the United States," U.S. Coast and Geodetic Survey, *Special Publication* 59, 227 pp.

The military grid is being indicated in the margins of maps made and printed by the U.S. Geological Survey, the projection being drawn with latitude and longitude lines. These marks consist of short lines and the proper grid numbers so that, if desired, the user may complete the grid by joining the marks with straight lines. On the 1:31,680 scale topographic maps now being made of quadrangles in Massachusetts, both the 1,000-yard military grid and the state grid in feet are indicated.

Recently there have been prepared by L. G. Simmons, geodetic engineer of the U.S. Coast and Geodetic Survey, special tables for conversion of geographic coordinates to military grid coordinates, comprising but four pages and, to accompany them, sheets of instructions for using the tables and a sample computation. Use of a calculating machine accelerates the work of making computations. Results are accurate to 1/10 or 2/10 yard.

293. Map forms.—To prepare a map in good form requires careful consideration and expert drafting. In large mapping organizations many experts have a hand in producing each map, and it is by this method that uniformity is obtained. The beginner cannot do better than to select a representative map turned out by the U.S. Geological Survey and follow its pattern as far as practicable. The example chosen should be one approximately at the scale of the map to be made, and the character of its topography should match that of the area concerned. It is customary to draft maps at a scale a little larger than the reproduction, or printing, scale. For example, maps to be printed at the scale 1:31,680 are drawn at the 1:24,000 scale; those to be printed at 1:62,500, at 1:48,000; those to be printed at 1:125,000, at 1:90,000 or 1:96,000; those to be printed at 1:250,000, at 1:180,000. Larger scale maps such as those to be printed at 1:20,000 or 1:10,000 are usually drawn at the reproduction scale. A field scale smaller than 180,000 is seldom used in surveys. Maps smaller in printing scale than 1:250,000 are generally compiled from existing larger scale maps. The scales mentioned here are for standard maps as made in the United States. Scales commonly used for navigation charts along coasts are 1:5,000, 1:10,000, 1:20,000, 1:40,000, 1:80,000; for charts of the Great Lakes, 1:5,000, 1:10,000, 1:15,000, 1:30,000, 1:40,000; 1:80,000, 1:400,000 and 1:500,000.

Although the scale for drafting a particular map may have been decided before the field survey was begun, it is well to reconsider the question of scale before actually beginning to prepare the map. The scale of the aerial photographs, their types, and the method of employing them will count. If more than one sheet is required, the most satisfactory layout should be obtained. Ample space must be provided for control points that lie outside the borders but are to be employed. The ratio between draft scale and printing scale should be about 3:2 or 4:3. If a ratio as large as 2:1 must be taken, special attention must be given to the size of letters and figures so that they will be legible and have appropriate dimensions on the printed map.

294. Inking and finishing a map.—Standard symbols should be employed throughout. A sheet of symbols may be obtained from the U.S. Geological Survey, Washington, D.C. A War Department training regulation on the subject of conventional signs may be procured from the Superintendent of Documents, Washington, D.C. One or the other of these should be followed as the guide. Weights of lines should be appropriate to the scale of the map and such that distinctness and clarity may be obtained without excessive spreading. It is better practice to employ letters of simple style than ornate letters. The standard aimed at should be ease of reading rather than embellishment. As a rule, Gothic or block letters are easier to read than Roman letters, and they may be neatly made by practice.

Reproduction of maps in colors requires that a separate printing plate be made for each color. Two methods are employed for doing this: (*a*) Each color is drawn on a separate sheet; (*b*) the original map is drawn in colors on one sheet, and separation is effected on photographic negatives (glass plates). There are advantages in both methods, but the first does not necessarily provide for obtaining a good photographic copy or lithographic prints in one color in a short time. It also requires that "blue-line" prints be made on as many sheets as the number of colors, a process involving special laboratory facilities and technical skill. It is therefore best to prepare a complete map with all colors inked together and leave to the printer the job of separating colors. An important point is to have the finished map suitable for copying photographically. To this end great care should be taken in the selection of the inks. India ink is the best black; Prussian blue is satisfactory but is improved by adding a dash of black in the mixture; and burnt sienna is the standard brown. Reds require particular attention. Weber's waterproof carmine-pigment drawing ink is satisfactory. Transparent inks must be avoided. There must be enough pigment to give a solidly opaque line, and all lines should be drawn with sufficient body to leave no thin parts.

Maps of quadrangles are usually filled out to the borders with topographic data. In order to maintain complete coverage of the area, the title and other essential notes are placed in the margins outside the borders. These notes should be carefully compiled and arranged. The list of them will ordinarily include the title, the scale of the map (graphical and representative fraction), the contour interval, the magnetic declination with diagram showing true north (and grid north if map is gridded), the projection, the vertical datum, the agency or persons responsible for the survey and issuance of the map, sources of information with credits for contributions, the method and the year of the survey, the date of issuance, the latitude and longitude figures, grid marks and numbers, and any required special legend.

Exploratory surveys seldom end with a regular-shaped area mapped, so in many cases a blank space is available inside the borders where the title and nearly all other essential data may be brought together in attractive arrangement. It is more satisfactory to the user of the map to find the notes together than to have to search round the area to find any particular item.

CHAPTER XVI

ERRORS AND MEASURES FOR COUNTERACTION; CALIBRATION OF CAMERAS

295. Theoretically it is not possible to obtain absolute accuracy from parallax measurements made on a pair of photographs unless the stereo base is parallel to the plane that includes the photographs and the photographs have the same focal length. Perspective defects are here left out of consideration. It follows, then, that for perfect results a camera of fixed focus should be set up at the ends of a base, leveled, and pointed in the same direction at right angles to the base each time to make the exposures. For the purpose of obtaining contours the base should be horizontal. It is difficult to impose such strict conditions for photography on the ground. In aerial photography it is impossible with the instruments now available. Therefore measures must be employed to counteract the effects of departures from the theoretical setup. Errors are also brought into the results obtained by other methods. Since no method is strictly accurate and no instrument is perfect, it is necessary to obtain as much information as practicable about the magnitude of errors and the effectiveness of the measures used for overcoming them. Some of these will be examined.

296. Photographs taken obliquely to a stereo base.—This condition is common in aerial photography and is imposed in photographs taken on the ground in order that as much terrain as practicable may be surveyed from a single base. Obliquity to the base has the effect of disturbing the scale equality of the photographs. It will be assumed that the planes of the photographs are parallel. A pair of verticals taken at different heights above a datum plane will be used for the examination. Compensation is accomplished by enlarging the photograph taken at the higher level or by reducing that taken at the lower level. The ratio of enlargement or reduction is computed with reference to a particular datum plane, and compensation will be complete for measurements made of objects lying in that plane. There will be an error in measurements made of image points of objects not in that plane, and the farther an object is from that plane the greater will be the error. In Fig. 130 let S_1 and S_2 be the air stations of two photographs forming a stereo pair, b the effective length of the stereo base, D the height of the left air station above the datum plane, f_1 the focal length of each original photograph, f_2 the focal length of the right photograph after enlargement, r the height difference between S_1 and S_2, h the height of an object above the datum plane, and p_1 and p_2 parallax distances as indicated. S_2' is the position that the right air station would have, had it been at the elevation of S_1. The distance p_1 would be the parallax reading of the point B had the right photograph been taken from S_2'. Then to obtain compensation in the datum plane the photograph taken from S_2 must be enlarged in the ratio $(D + r)/D$, or the left photograph reduced in the ratio $D/(D + r)$. In the figure it is assumed that the right photograph has been enlarged. The parallax reading of the image of a point A in the datum plane will be correct, but of a point B at the height h above A the parallax reading will be too small by an amount equal to $p_1 - p_2$. The value of p_1 may be obtained from the equation $p_1 = f_1b/(D - h)$; and p_2 from the equation $p_2 = f_2b/(D + r - h)$ [Equation (20) Art. 200].

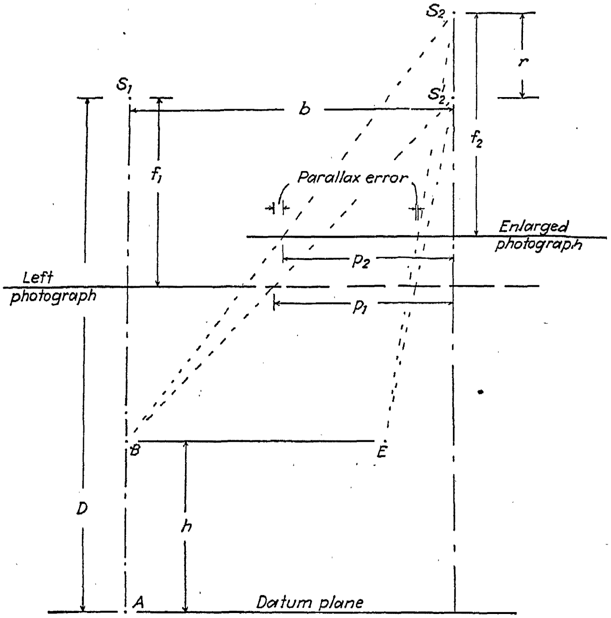

FIG. 130.—Error in parallax readings when one photograph of a stereo pair is enlarged.

For example, let

f_1 = 200 millimeters
D = 4,000 meters
r = 100 meters
h = 1,000 meters
b = 1,000 meters

Then

$$f_2 = f_1 \frac{(D + r)}{D} = \frac{200 \times 4{,}100}{4{,}000} = 205 \text{ millimeters}$$

$$p_1 = \frac{200 \times 1{,}000}{4{,}000 - 1{,}000} = 66.67 \text{ millimeters}$$

$$p_2 = \frac{205 \times 1{,}000}{4{,}000 + 100 - 1{,}000} = 66.13 \text{ millimeters}$$

The parallax error therefore is 0.54 millimeter, which would give an error in the determination of the height of point B of about 140 feet. The error in the determination of the point E, at the same level as B, would be much less.

The example illustrates an extreme case in which the air-station heights differ much more than is common, and the relief is exceptional. However, a satisfactory estimate of the error in any particular case may be obtained by direct proportion from the result given. For instance, if the station-height difference is 10 meters, the error would be but about 5 feet for $h = 1{,}000$ meters and about 0.9 feet for $h = 100$ meters.

It will be noted that this is approximately the maximum error that could occur. In other parts of the overlap the error would be less, and for objects directly under S_2 there will be no error from this cause. For moderate relief and small differences of air-station heights the error is negligible provided the photograph has been properly enlarged or reduced as required. Satisfactory further compensation can be obtained by applying corrections according to the relative positions of image points on the photograph affected. Along any line at right angles to the stereo base the error is the same for all points. Therefore a practical method of making corrections is to select bands across the area of overlap and apply the correction for each band. If a photograph has been enlarged to obtain equality in scale, the correction should be added; if a reduction has been made, the correction should be subtracted.

297. Residual tilt in a stereo pair of photographs.—Even under the most favorable conditions, residual tilt will remain in aerial photographs after the steps of rectification have been completed. The effect of tilt is to deform the stero model obtained. A plane may appear to be a curved surface, simple or complex, or it may seem to have been changed from a horizontal to a slanting position, depending on the relative orientation of the principal lines of the photographs concerned. These forms are too varied to present in full, but two cases will be considered. Figure 131 illustrates the effect when the principal planes of the photographs are included in a common plane that also includes the perspective centers S_1 and S_2 and the photographs are tilted equally upward and toward each other. The locus of elevations of points of the datum plane as

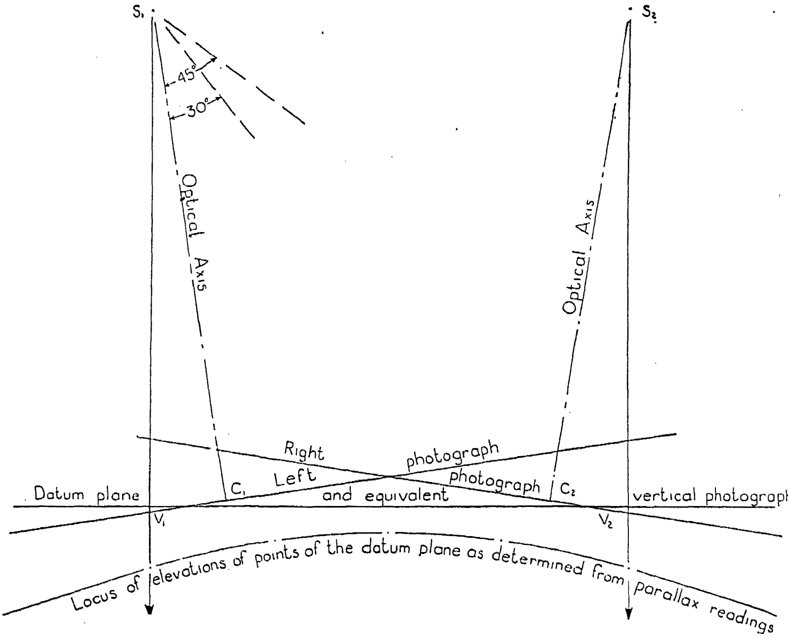
S_1
S_2
45°
30°
Optical Axis
Optical Axis
Right photograph
Left photograph
C_1
C_2
Datum plane and equivalent vertical photograph
V_1
V_2
Locus of elevations of points of the datum plane as determined from parallax readings

FIG. 131.—Diagram showing the effect of tilt on elevations determined by parallax from aerial photographs when the photographs are tilted upward toward each other.

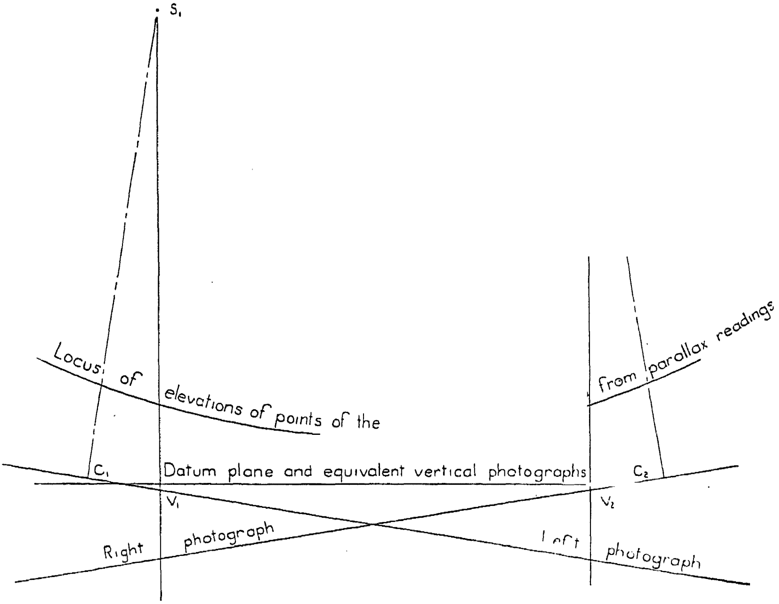
S_1
Locus of elevations of points of the
from parallax readings
C_1
Datum plane and equivalent vertical photographs
C_2
V_1
V_2
Right photograph
Left photograph

FIG. 132.—Diagram showing the effect of tilt on elevations determined by parallax from aerial photographs when the photographs are tilted downward toward each other.

determined from parallax readings, or the apparent position of that plane in the stereo model, will be a curved surface like the ceiling of an arched vault, and it will appear to be lower than it actually is. Other horizontal planes will be similarly distorted.

In Fig. 132 the principal planes are also included in a common plane that includes the perspective centers S_1 and S_2, but the photographs are tilted equally downward and toward each other. In this case the datum plane will appear to be higher than it actually is and curved upward from the middle. Other horizontal planes will appear as similarly curved surfaces.

The extent of deformation will depend on the amount of residual tilt. It is difficult to determine the degree of tilt of a photograph even from an image of the horizon closer than a few minutes of arc. Suppose the focal length of the photograph to be 8 inches; and neglecting the effects of error in the height of the air station or the height of land giving the horizon image, it is unlikely that measurements from the center of the photograph to the horizon image can be made closer than about 1/100 inch because lack of definition always makes the position of the image a little uncertain. The probable error is the angle whose tangent is 1/800, or about 4 minutes. In determining from accurate control stations the tilt of photographs taken with the five-lens camera, the probable error is not quite so low, whereas for single-lens photographs tilts similarly determined may be in error as much as 10 or 12 minutes unless exceptionally large-scale photographs are employed and special instruments are available for making the required measurements.

Errors from residual tilt in determining elevations or from erroneously determined tilt may vary from zero in one part of the overlap (model) to a maximum in other parts. Figures 131 and 132 are drawn fairly carefully to scale; and as the error for small tilt is practically proportional to the tilt, they may be measured to obtain an idea of the magnitude of the errors in height. As drawn, the figures show a tilt of about 9°, or 540 minutes. From center to center of the photographs the errors range from about 5 to 15 per cent of the height of the air stations above the datum plane. If that height is taken at 10,000 feet and errors are computed proportionately to tilt, it is found that for tilts of 10 minutes in each photograph they would under similar orientation of the photographs range from nearly 10 to nearly 30 feet, whereas for tilts of 5 minutes the errors would be about half as large. A more accurate result would be obtained by computation or by drawing figures with less tilt (3 or 4°) and prorating from measurements, but the results here given are true within a few per cent.

There is no way to counteract errors from this cause except to make adjustments on control points whose elevations are accurately known.

Fortunately this can be and is done successfully. Nevertheless it means that such control points must be provided and the expense incurred in order to obtain accuracy. The importance of having accurate means of determining tilt suggests the value of a horizon camera of ample focal length.

298. Error caused by deviation of upright planes from the vertical in the displacement method of finding the elevations of objects pictured in vertical aerial photographs.—Figure 133 shows a tilted photograph and a horizontal plane intersecting the photograph in a line that passes through the principal point *C*. This line of intersection necessarily is parallel to the axis of tilt. Let *S* be the air station and *V* the vertical point of the photograph. Then *W* is the point where the plumb line pierces the horizontal plane. *SCG* is any upright (slightly slanting to the vertical) plane that contains the optical axis *SC* of the photograph. Any object in space, as *H* at a height *RH* above the horizontal plane, will appear in the photograph at *G*. With respect to the vertical point *V* of the photograph, the image of *H* at *G* is displaced an amount equal to *QG* whose magnitude depends on the height *RH*, the angle α, and the distortion caused by the tilt component along the line *VG*. If, therefore, the vertical point were known and it should be used instead of the principal point *C*, the process of finding elevations by the displacement method would give directly the value of the true height of any object dealt with, as *RH* for the point *H*. However, because the position of the vertical point is not known to begin with, and because of the complications that would arise from the unknown angle between *SV* and *GV*, it is necessary and desirable to employ the principal point *C* where all lines on the photograph are perpendicular to the line *SC*. Although *C* is the most favorable point to use, it does involve a height error because heights must be measured in slanting planes such as *SCG*. It is evident that the slant height *TH* is obtained, not the vertical height *RH*.

The greatest slant always occurs in that upright plane which intersects the photograph in a line (*CF* in Fig. 133) parallel to the axis of tilt. When the upright plane coincides with the principal plane (*SVC*), which is vertical, no error occurs. Relationships to derive formulas for use in determining the difference between the true height *RH* and the slant height *TH* of an object in any given slanting plane, such as *SCG*, can be established as follows:

Complete the constructions shown in Fig. 133, and apply the characters as indicated, i representing the angle of tilt, k the angle between the photograph and the horizontal plane cut by the slanting plane *SCG*, and φ the angle *VCG* in the plane of the photograph. Let α represent the angle *VSG* and β the angle *CSG*. *TH* is drawn perpendicular to the line *CJ*, and *HL* perpendicular to *CG*. Hence plane *HRT* is vertical, and

HL is parallel to *SC*. Therefore, angle *LHG* equals β and angle *THL* equals k, so angle *THG* equals $\beta - k$. This relationship is more readily seen in Fig. 134, which represents the slanting plane *SCG* and shows the angles involved laid out in true proportions. As this figure is drawn, the line *CG* is below the horizontal plane. On the opposite side of *C*, *CG* extended is above the horizontal plane, and there the angle corresponding to *THG* (or *THJ*) equals $\beta + k$ (β_1 and β_2 in Fig. 134 correspond to β in Fig. 133). It is necessary to keep track of the position of *G* and apply the proper sign to k. In general, therefore, the angle *THG* equals $\beta \pm k$. The significant relationships are

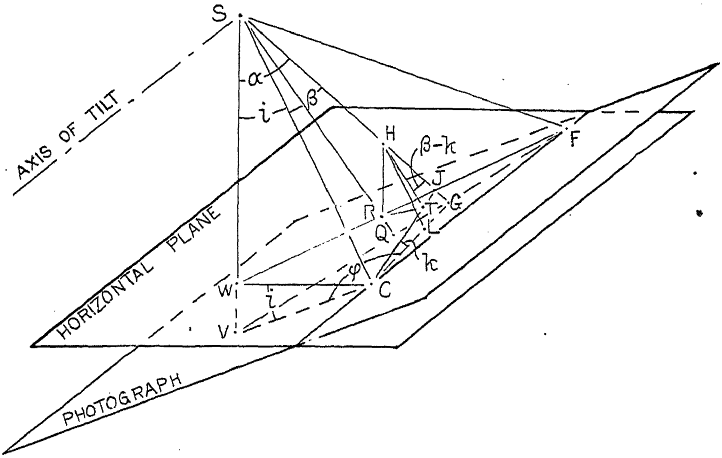

FIG. 133.—Diagram showing an upright plane by which the elevation of an object is determined.

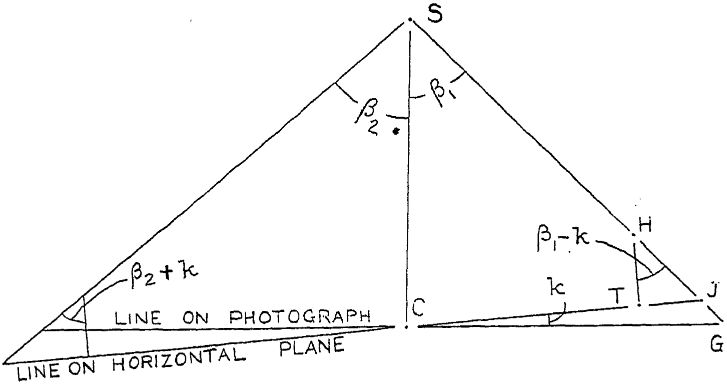

FIG. 134.—Upright plane laid out.

$$\cos \alpha = \frac{RH}{} \qquad \cos \quad \pm k) = \frac{TH}{}$$

whence

$$TH = HJ \cos(\beta \pm k) = \frac{RH \cos(\beta \pm k)}{\cos} \tag{34}$$

It has been shown (Art. 170) that, for any given value of i, k depends on the cosine of the angle φ. It is evident that for any line from C, making the angle φ with VC, β depends on the focal length SC and the distance of the image point from C, whereas α depends on the value of β and other relating factors, so a solution of equation (34) is not possible until the value of α has been determined for any given value of β. To find α it is necessary to consider the triangles CVG and SVG as well as the relationships that bind them together. In the triangle CVG the angle at C equals φ, $CV = f \tan i$, and CG may have any value within the scope of the photograph. Knowing, then, the two sides and included angle, the triangle is solved to obtain the length of the third side VG. In the triangle SVG, $SV = f/\cos i$, and $SC = f/\cos \beta$, so having the three sides α can be determined by computation. The computations are tedious.

Table 12 shows that between the slant height and the vertical height for any given angle of tilt, the difference varies with angle φ and that the maximum difference occurs when $\varphi = 90°$. The difference in any given slanting plane remains constant. This means that the error is independent of the value of β. Maximum errors for various angles of tilt can be determined readily from equation (34) by substituting zero for cos $(\beta \pm k)$ and i for α (H of Fig. 134 will now lie in the line SC). Table 13 shows the maximum height errors (differences between slant height and vertical height h for plane SCF) for differing angles of tilt. It is evident

TABLE 12.—INFLUENCE OF φ

φ, degrees	i, degrees	β, degrees	h, feet	Height difference, feet
0	2	0	1,000	0
30	2	60	1,000	0.24
45	2	60	1,000	0.34
90	2	60	1,000	0.60
45	2	45	1,000	0.34

TABLE 13.—MAXIMUM HEIGHT ERRORS FOR $h = 1{,}000$ Feet ($\varphi = 90°$)

Angle of Tilt, Degrees	Error, Feet
1	0.2–
2	0.6
3	1.4
4	2.44
5	3.83

that the error is so slight that it may be neglected for all heights not exceeding a few thousand feet when the tilt does not exceed 3° or for quite large angles of tilt if the heights are not great.

299. Glass plates in cameras.—A glass plate in the focal plane of a camera distorts the image by displacing points toward the center. The effect is nil at the optical axis and increases toward the borders. If i is the angle of incidence of a ray, t the thickness of the glass plate, μ its index of refraction, and r the angle after refraction, the displacement of any image point may be expressed by

$$d = t(\tan i - \tan r) \qquad (35)$$

The relationship between i, μ, and r is

$$\sin r = \sin i \qquad (36)$$

Linear displacements may be computed from these two equations; but if the focal length f of the camera is known, equation (35) may be stated in the following form:

$$d = f \tan i - (f - t) \tan i - t \tan r \qquad (37)$$

which is somewhat more direct in application.

Table 14 gives the linear distortions caused by a glass plate ⅛ inch thick in a camera having a focal length of 210.9 millimeters, the index of refraction of the glass being 1.528,[1] at 5° intervals through a field of 45°.

TABLE 14.—NEGATIVE REFRACTION DISPLACEMENTS CAUSED BY A GLASS PLATE ⅛ INCH THICK IN A CAMERA WHOSE FOCAL LENGTH IS 210.9 MILLIMETERS

$\mu = 1.528$

Angular Distance from Center, Degrees	Displacement, Millimeters
5	0.096
10	0.195
15	0.305
20	0.426
25	0.567
30	0.734
35	0.948
40	1.195
45	1.517

A plate ⅛ inch thick is about as thin as can be employed in a camera, which has a 7- by 9- or 9- by 9-inch field, without running considerable risk of breakage, and to assure flatness of field. The distortion is far too great to neglect in photogrammetry. Practically perfect correction can be obtained if prints are made by projection with a lens having the same characteristics as the lens in the aerial camera and a glass plate is used to support the negatives. To secure complete correction in 1-to-1 printing it is necessary to employ a lens whose focal length is half that of the aerial camera and to use a glass plate of the same thickness and same index of refraction as of that used in the aerial camera. It is possible to impose such conditions, but it is probable that the lens would have to be made on order and would be expensive. Something less than the ideal arrangement is usually employed. This consists of using a custom-made projector and selecting a plate of glass that gives a satisfactory compensation.

---

[1] Tests made of several plates of glass by Prof. E. S. Larsen of Harvard University, employing the immersion method, gave a mean index of refraction of 1.528, the range being from 1.525 to 1.530.

In order to determine the quality of compensation obtainable by this method it is necessary to make a series of computations such as required to obtain Table 14, trying several thicknesses of glass so that a selection may be made of that one which gives a correction within the allowable error. Table 15 is the result of computations of a plate of glass that for 1-to-1 printing gives a fair compensation of the refraction displacements listed in Table 14.

TABLE 15.—POSITIVE REFRACTION DISPLACEMENTS CAUSED BY A GLASS PLATE 6.03 MILLIMETER THICK

Plate used to support the negative in a printing camera whose lens has a focal length of 183.28 millimeters and is set to make 1 to 1 prints, $\mu = 1.528$

Angular Distance from Center, Degrees	Displacements, Millimeters
5	0.182
10	0.375
15	0.580
20	0.820
25	1.080
30	1.385

To see whether or not the compensation will be satisfactory, the negative distortion curve and positive distortion curve should be plotted together to make a graph. For this purpose linear distances from the center of the field (principal point), corresponding to the angular distances of Tables 14 and 15, as obtained from the computations, are employed. Figure 135 is a graph that shows the curves drawn from plottings of the values given in Tables 14 and 15; the negative distortion curves and angular distances pertaining to it, in dash lines; the positive distortion curve and its data, in solid lines. This graph shows that excellent compensation will be obtained through about 35° of the field of the negatives (the residual displacement being less than 0.03 millimeter) but that beyond that point the two curves separate rather widely and compensation is not satisfactory.

It is possible that better results might be obtained by use of a different combination of focal length, thickness, and index of refraction.

Perhaps a lens of shorter focal length could be found to give a satisfactory compensation as far as 45° from the center of the field of the negative.

Glass plates may be employed in this manner to compensate for negative distortion in lenses if that distortion follows the same pattern as given by the glass plate. It is possible also to obviate the necessity of vacuum in the paper holder of a projection printer by balancing glass plates, one placed in the negative holder, the other in the paper holder.

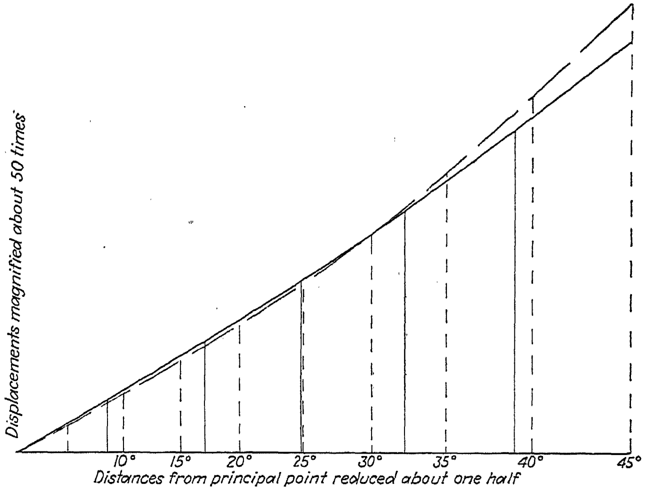

FIG. 135.—Graph of negative and positive refraction displacements from glass plates in the focal planes of cameras.

300. Error in determining the height of an air station.—By taking temperature readings and making computations for temperature corrections it is possible to obtain heights of air stations above a datum plane from altimeter readings with surprisingly close approximations when the weather is stable (Art. 54). Heights so obtained, however, are somewhat uncertain and under usual circumstances can be used only for low-order mapping. After elevations of a few air stations of a photographic run have been determined by a more exact method, the altimeter or statoscope record may be used with considerable satisfaction to determine the elevations of intermediate stations. Using a sensitive altimeter or statoscope, which indicates changes in elevation of 5 to 10 feet, the accuracy obtainable for intermediate stations is well within the requirements for standard mapping.

Air-station heights determined from control points on the ground are the most reliable with the means now available. Accuracy of results will depend on the accuracy of the control and the refinement with which measurements are made. No consideration will be given in this connection to camera errors or to errors in the photograph because of dimensional changes; these items are dealt with in Arts. 6 and 36. It has been shown in Art. 267 that third-order control is accurate enough for standard mapping at moderate scales. Therefore if third-order control is available, no consideration need be given to that item, provided all the stations have been exactly spotted on the photograph. Control points obtained from the photographs themselves are liable to be in appreciable error; but if locations are made with care by the radial-line method based on sufficient third-order stations, positions of points can be kept within plotting accuracy, *i.e.*, within a few feet of their true positions. If displacement is as great as 10 feet, it may be assumed that the error in determining the height of an air station will be about the same amount. Errors in the elevations of control points will also be carried at unchanged magnitude into determinations of the heights of air stations. Elevations of ground points will usually be known accurately enough to permit this item to be neglected.

The probable magnitude of the error in finding the height of an air station by computation from measurements made on a photograph that includes several control stations is indicated in Art. 219. The result of 20 feet as there stated was obtained on the assumption of having photographs ranging in scale from 1:15,000 to 1:20,000. Use of larger scale photographs printed on glass plates from glass negatives will materially lessen that figure, although the accuracy is dependent to a considerable extent on the distribution of the control points with respect to the principal line of a photograph. If their distribution is favorable, the effect on the result will be small; if unfavorable, it may vitiate the advantage from glass plates and larger scale. As a rule, under the practical limitations on scale and under the most advantageous conditions with an accurate and well-calibrated camera and by making refined measurements, air-station heights may be determinable with error ranging from 0 to 5 or 10 feet.

301. Miscellaneous errors.[1]—There is no practicable method of determining the accuracy of the mounting of a lens except with proper laboratory equipment and by a competent physicist. Obliquity of the optical axis in mounting a lens gives errors similar to those caused by tilt. Lens distortions are best eliminated through projection with a lens that gives distortion opposite to that given by the lens in the aerial camera. Glass plates are also effectively used for this purpose. In the aerocartograph and similar instruments the correction for distortion is highly successful, because projection is accomplished with the transparent plate set at its focal distance (for infinity) from the lens which is either the same one used in the aerial camera or one of the same type having the same or similar distortion characteristics.

---

[1] An excellent article entitled "Source and Correction of Errors Affecting Multiplex Mapping," by Russell K. Bean, was printed in *Photogrammetric Engineering*, April-May-June, 1940.

An abstract of the *Journal of Research* of the National Bureau of Standards, February, 1939, has been printed separately as *Research Paper* 1177 and is entitled "Relation of Camera Error to Photogrammetric Mapping" by Dr. Irvine C. Gardner.

302. Calibration of cameras.—It is highly important to have an accurate determination of the focal length of an aerial camera, because many uses of the photographs require its value to be known. If the focal length is not given by the manufacturer, it is necessary to obtain it by a proper method. If the focal length is given by the manufacturer, it is well to make an independent check anyhow. Calibration consists in obtaining the focal length, determining the position where the perpendicular dropped from the center of the lens strikes the focal plane, having the optical axis coincide with that perpendicular, and placing the reference marks in the frame of the focal plane (or on the glass plate of the focal plane if there is one) where they will accurately indicate the principal point of the negative. In cameras whose photographs are to be used in special printers or in special plotting instruments it is also desirable to have the focal-plane reference marks placed to give two lines at right angles to each other and to have those lines oriented precisely with respect to basic reference lines or planes. In multiple-chambered cameras construction and calibration should be such as will place the optical axes of the lenses by groups in a common plane and a line in each focal plane in the same common plane. The angles between the optical axes must also be determined. These should be equal to the dihedral angles between the corresponding focal planes.

303. The position of the foot of the perpendicular dropped from the center of the lens to the focal plane of a camera may be found either by a laboratory method or in the field. The laboratory method is easier to carry out and is usually employed. It requires a stand, a simple special leveling table which has a hole near its center, a glass plate a little smaller than the focal-plane space, a pan of mercury, a small spirit level, and a light. Figure 136 shows a setup. The camera is placed on the leveling table with the lens immediately above the hole, the glass plate on the focal plane, and the pan of mercury directly below the lens. The glass plate should have a small cross drawn or etched near its center. This cross should be the intersection of two lines stretching across the plate at right angles to each other. It is necessary only to have the ends of these lines drawn on the glass plate. They serve to indicate the positions for the marginal marks which later are to be made in the focal-plane frame and will be registered in every negative obtained with the camera. The face of the glass plate on which the cross and other lines are drawn is turned down against the focal plane. The glass plate should be selected for flatness and uniform thickness and cut to the proper dimensions to allow for slight shifting in the focal plane. The plate in place, the camera is leveled to bring the plate into a horizontal position. The cross is illuminated properly, the light being held a little to one side and shifted until the image reflected from the mercury surface can be seen on the glass surface. The plate is shifted until the image coincides with the cross. The positions of lines at the focal-plane frame are now marked on the frame or noted with reference to temporary marks or scales placed on the frame for the purpose. On removal of the glass plate the marks may be scribed on the frame with a sharp needle drawn along a straightedge. If registration marks have already been placed in the frame, the test will show whether or not they are correctly placed.

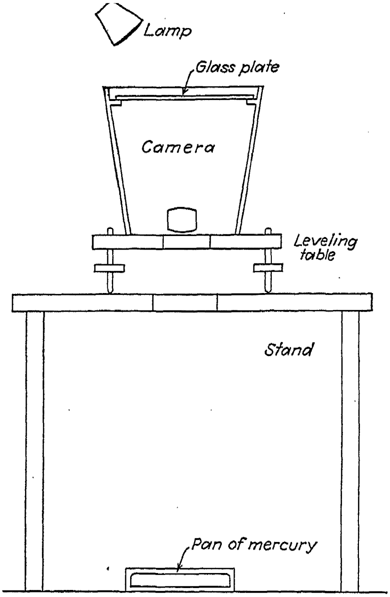

FIG. 136.—Setup for determining the foot of the perpendicular dropped from the center of the lens to the focal plane of a camera.

A few trials of the light in different positions may be necessary before the observer is able to see the image of the cross. The closer the mercury is to the lens the brighter the image will be, but after a little practice it should be possible to catch an image with the mercury pan placed 3 or 4 feet below the lens. By careful work the position of the principal point should be in error by not more than $\frac{1}{10}$ millimeter.

The *field* method requires that the camera be placed with the focal plane in a vertical position and held rigidly in this position while an exposure is made; in fact, facilities must be available for placing the camera first on one face, then on an adjacent face, *i.e.*, to revolve the camera 90° about the optical axis for the second exposure. Targets that will give a well-defined image are placed across the field several hundred yards (far enough to represent distant objects) from the camera and at the same level as the camera. There should be at least three targets, one near the middle and one near each outer limit of the field of the camera. An exposure is made for each position of the camera, the center target being used for aiming, the negatives developed, and the horizon line drawn on each. By superposing one negative over the other, making them register by the border lines, the two horizon lines will intersect at the principal point. Measurements can then be made to place marginal marks in the camera; or if already in the camera, a check is obtained by comparing the position of the principal point thus found with that given by the marks of the camera.

If a suitable pond is available and the camera can be placed near the level of its surface, a shore line may be used in place of targets. This method is tedious, and the steps are not so easily kept under certain control as in the laboratory method. Consequently it is seldom used.

304. Determining the focal length of a camera.—The field method given by Gen. E. Deville[1] is still used and is highly satisfactory. It requires that the camera be set up with its optical axis horizontal (plate vertical) and that a horizontal angle be measured with a transit or a theodolite from the position occupied by the front nodal point of the lens. If the targets are a mile or so away, the error brought in by a difference of 1 or 2 inches between the positions occupied by the lens and the vertical axis of the transit will be inappreciable. The top of a building that is higher than those near by makes an excellent place for the test. Two distant objects, such as spires, chimneys, or vertical walls of buildings that are suitably distant, are selected for the targets. The horizontal angle between them, as measured at the station, should be a few degrees less than the angular field of the camera. It is measured with the transit and recorded, a mean of several readings being taken as the angle. The camera is then set up at the station and leveled, and an exposure made. It is necessary to aim the camera so that both targets will be included in the field. When leveled, the photographic plate should be vertical, and marks in the frame should give the principal line in the negative. After development, the distance of each target image from the principal line is measured. Let these be a and b, and let A be the horizontal angle between the two targets as measured with the transit and f the focal length of the negative. Then

$$f = \frac{a+b}{2\tan A} + \sqrt{\frac{(a+b)^2}{4\tan A} \qquad ab} \tag{38}$$

---

[1] DEVILLE, E., "Photographic Surveying," Ottawa, Can.

If there has been shrinkage in the negative, its percentage must be applied to give the focal length of the camera. The accuracy of results will depend on the refinement of measurements. If the angle A is measured with error no greater than 30 seconds and a and b are measured within $\frac{1}{10}$ millimeter, the focal length will be true within $\frac{1}{10}$ or $\frac{2}{10}$ millimeter. If a comparator giving readings to $\frac{1}{100}$ millimeter is available to measure a and b, and commensurate accuracy in determining f is desired, angle A should be measured with a probable error not greater than 10 seconds. Under these conditions the focal length can be determined with a probable error of not more than $\frac{1}{100}$ or $\frac{2}{100}$ millimeter. If the images of the targets are not sharply defined, additional error will be involved.

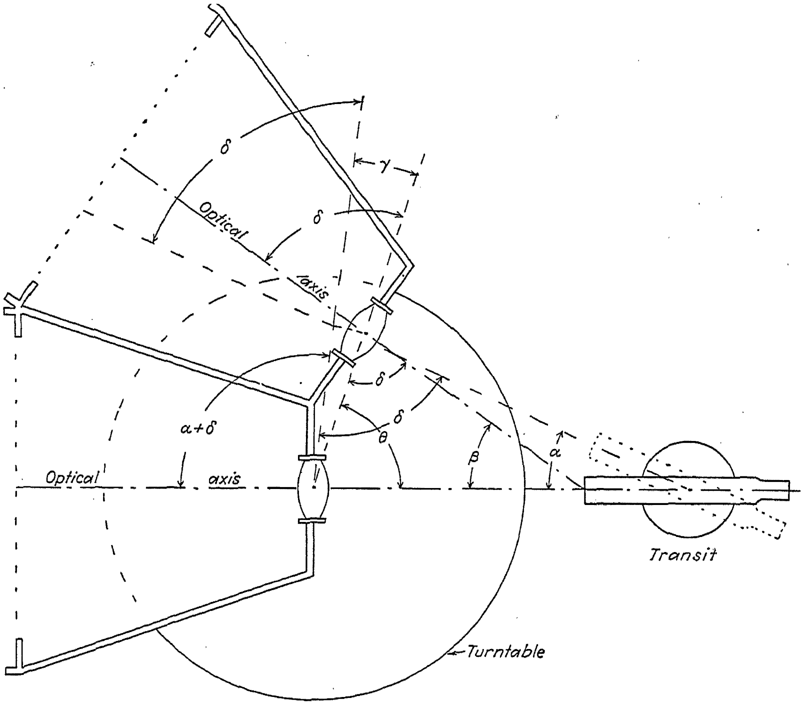

FIG. 137.—Determining the dihedral angle between the focal planes of multiple-chambered cameras.

305. The dihedral angle between the focal planes of two chambers of a multiple-chambered camera may be determined if a suitable turntable and transit are available. The turntable must have a graduated horizontal circle so that the amount that it is turned may be measured. It is assumed that the plane containing the optical axes is perpendicular to both focal planes. Figure 137 shows a setup. The camera is mounted on the turntable with the front nodal point of one lens in the vertical axis of rotation and the focal planes vertical. The transit is set up near by and leveled. The turntable is moved up or down to bring the centers of the lenses to the height of the line of collimation of the transit, then rotated until the principal point of the focal plane of the first chamber is on the line of sight of the transit. A reading is made on the horizontal circle of the turntable. Both table and transit are rotated horizontally in opposite directions until the principal point of the second chamber comes onto the line of sight of the transit. The sum of the two horizontal angles turned off is the dihedral angle sought, because in Fig. 137

$$\theta + \gamma + \alpha + \delta = \theta + \delta + \beta = 180°$$

and δ is a fixed angle of the camera, whence

$$\gamma + \alpha = \beta$$

It is assumed that the camera has been accurately constructed and that each optical axis is perpendicular to its focal plane. If the camera should be set up on the turntable with the intersection of its optical axes on the vertical axis of rotation, the transit telescope would not have to be revolved; in that case the angle sought would be the horizontal angle through which the turntable was revolved to change the line of sight from one principal point to the other. However, the position of the nodal point of the lens is usually known in advance, but the intersection of the optical axes (in space) would have to be determined.

The angular field of a camera or of any chamber may be determined in the same manner.

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